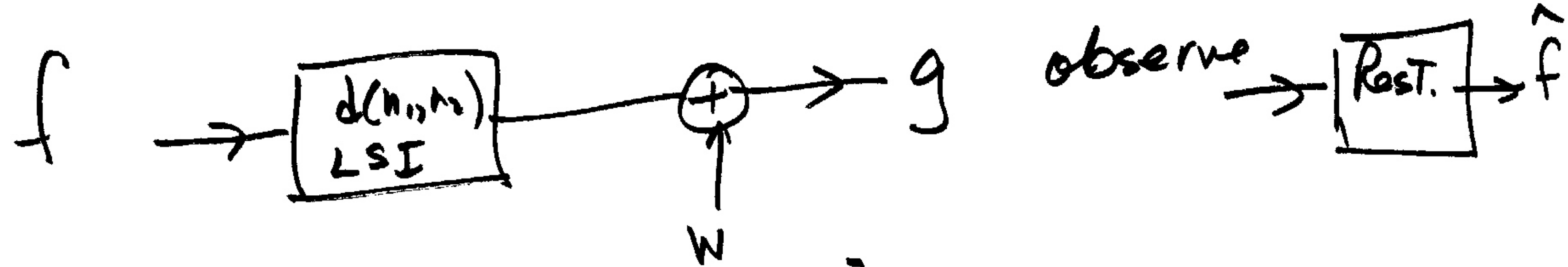
Image Restoration

Blind Deconvolution

Homomorphic filtering.



Minimize $E[(f-\hat{f})^2]$.

3 methods:

1. Inverse filters.
2. Least Squares filters → Weiner / Constraint least square.
3. Iterative Technique.

Key Assumption : We know d.

blurring fn.

How about if we don't know d.

$\Longrightarrow$ Blind Deconvolution.

2 Step process : 1 Estimate d

2 use "Classical" Restoration.

How To do this?

1 Estimate parameters of The blur function

Knowing what kind of Blur

→ Atmospheric

→ out of focus

→ motion.

Approach: Look at F.T of g.

Zeros of $G(\omega_1, \omega_2)$ $\longrightarrow$ Deduce the parameters of blur fn.

$\rightarrow R$ in out of focus

$\rightarrow L, \phi \rightarrow$ motion blur

② Maximum Likelihood Blur Estimation.

ML $\longrightarrow$ used for parameter estimation.

Given observation, find parameters of the model for that observation that maximize the likelihood.

~~Blind Deconvolution~~

Parameters in our case: $\Theta = \{ \sigma_w^2, d(n_1, n_2), \sigma_v^2, a_{ij} \}$

a_{ij}, σ_v^2 relate to the autoregressive model for f:

$$f(n_1,n_2) = \underline{a_{01}}\, f(n_1, n_2-1) + \underline{a_{11}}\, f(n_1-1, n_2-1) + \underline{a_{10}}\, f(n_1-1, n_2) + v(n_1,n_2)$$

$v(n_1,n_2)$ → Gauss. σ_v^2

σ_w^2 = variance of the added noise w → Gauss.

Maximize log likelihood function:

$$(*)\quad L(\theta) = -\sum_{\omega_1}\sum_{\omega_2}\left(\log P(\omega_1,\omega_2) + \frac{|G(\omega_1,\omega_2)|^2}{P(\omega_1,\omega_2)}\right)$$

where $P(\omega_1,\omega_2) = \sigma_v^2 \dfrac{|D(\omega_1,\omega_2)|^2}{|1 - A(\omega_1,\omega_2)|^2} + \sigma_w^2$

$A(\omega_1,\omega_2) \longrightarrow$ 2D DTFT of a_{ij}

Issues associated with solving (c)

(1) Must ~~[illegible]~~ apply regularization Techniques to make it "well conditioned".

Add constraints.

(a) Energy conservation

$$\sum\sum d(n_1, n_2) = 1$$

i.e D.C value $D(\omega_1, \omega_2)$ is 1

$$\left[D(\omega_1, \omega_2)\right]_{(\omega_1, \omega_2)=(0,0)} = 1$$

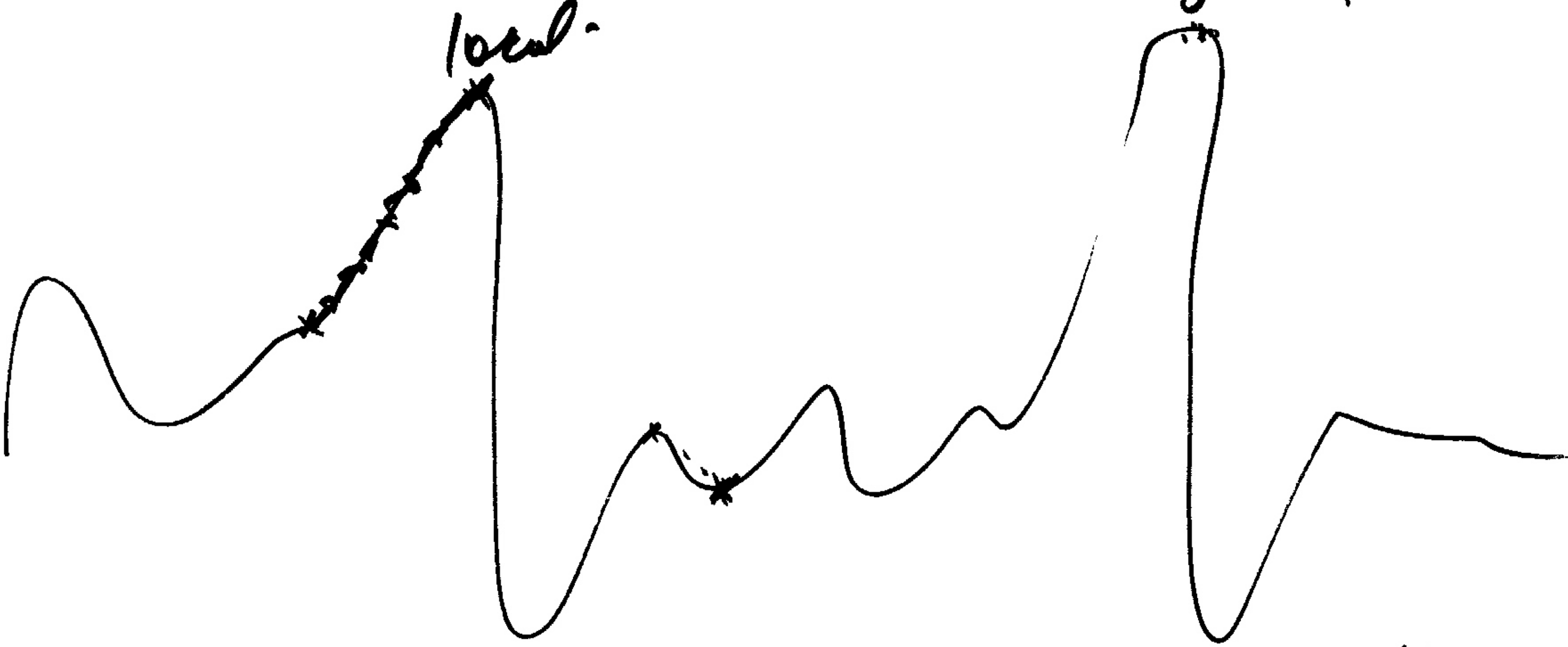
$\Rightarrow$ ~~[illegible]~~ Blurring was passive process no energy was either generated nor absorbed.

(b) $d(n_1, n_2) = d(-n_1, -n_2)$, i.e. blur p.s.f is symmetric.

② optimization of a nonlinear fn.

⟶ steepest descent ⟹ local minima.

⟹ initial condition global



One. gradient descent technique ⟶ EM

EM = Expectation Maximization.

G given $\theta, \hat{f}$

Assume $\triangleright$ know θ $\rightarrow$ using weiner filtering

estimate $\hat{f}$

estimate a_{ij}, σ_v^2

$g(n_1, n_2) \simeq d * \hat{f}$

estimate $d(n_1, n_2)$

Blind deconvolution

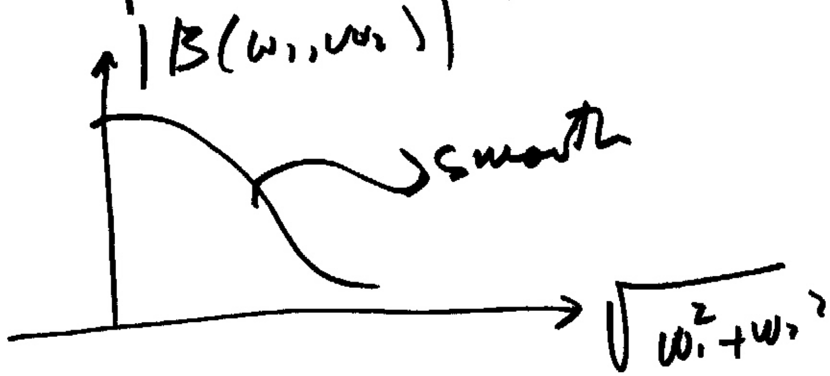
$$F(\omega_1,\omega_2) \longrightarrow \boxed{B(\omega_1,\omega_2)} \longrightarrow G(\omega_1,\omega_2)$$

Blur fn.

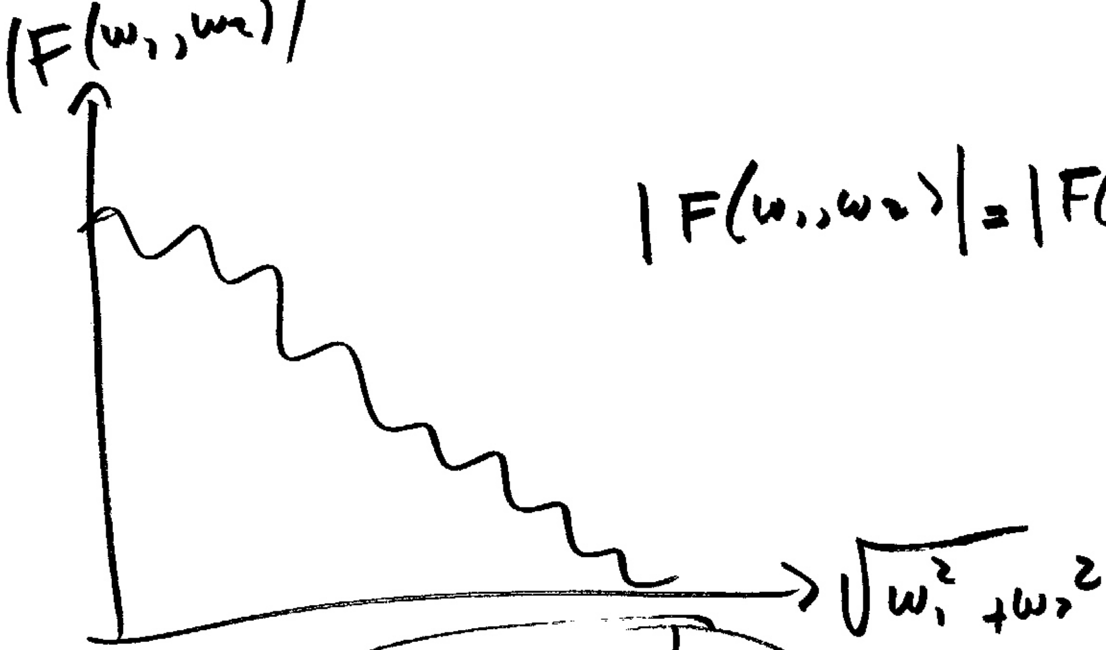
Assumption $|B(\omega_1,\omega_2)|$ is smooth.

$$G(\omega_1,\omega_2) = F(\omega_1,\omega_2)\, B(\omega_1,\omega_2)$$

$$|G(\quad)| = |F(\quad)|\,|B(\quad)|$$

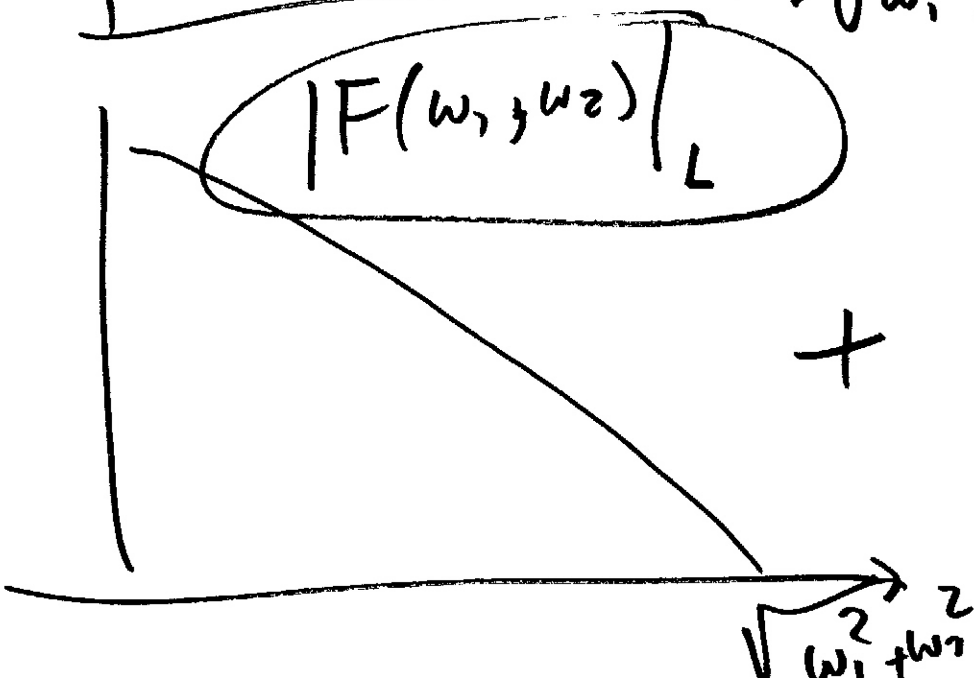


Decompose. $|F(\omega_1,\omega_2)|$ into $\begin{cases} \to \text{slowly varying.} \\ \to \text{fast varying component.} \end{cases}$



$$|F(\omega_1,\omega_2)| = |F(\omega_1,\omega_2)|_L + |F(\omega_1,\omega_2)|_H$$

$$=$$



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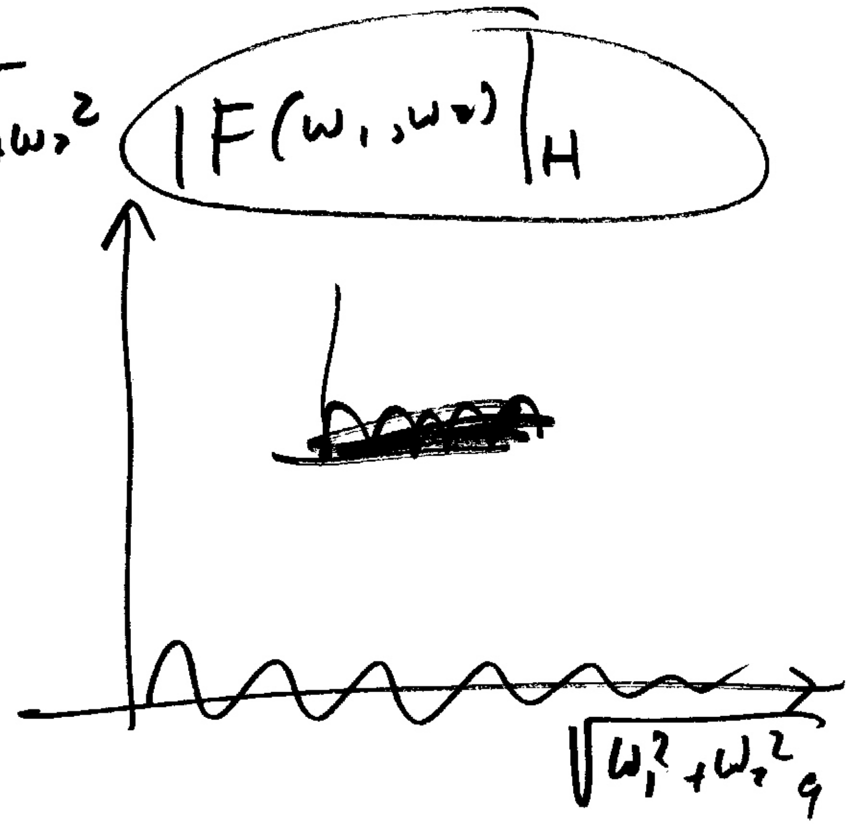
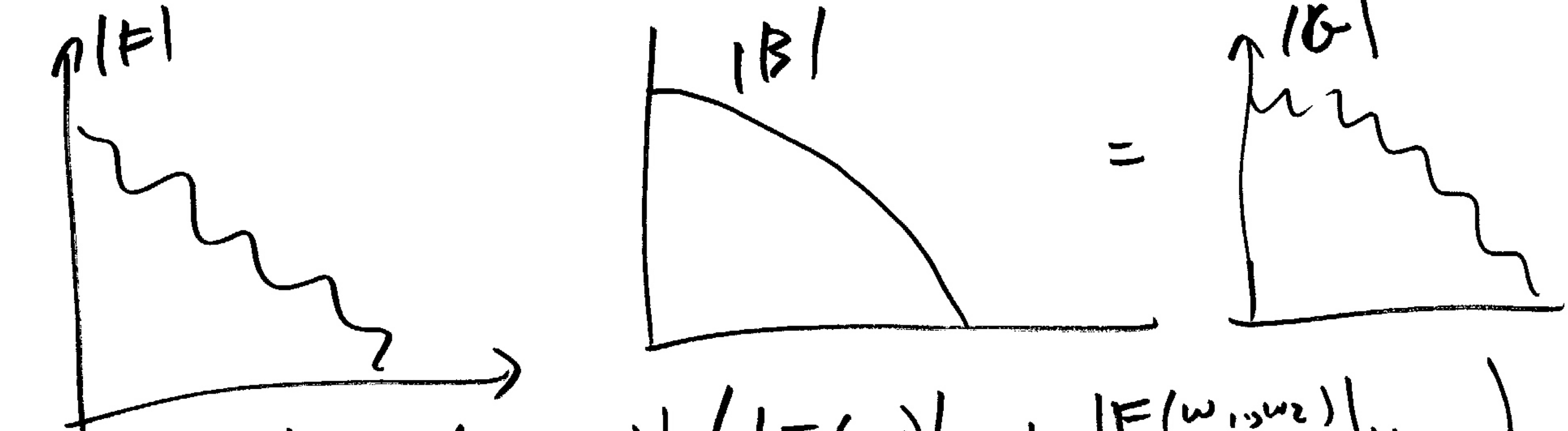


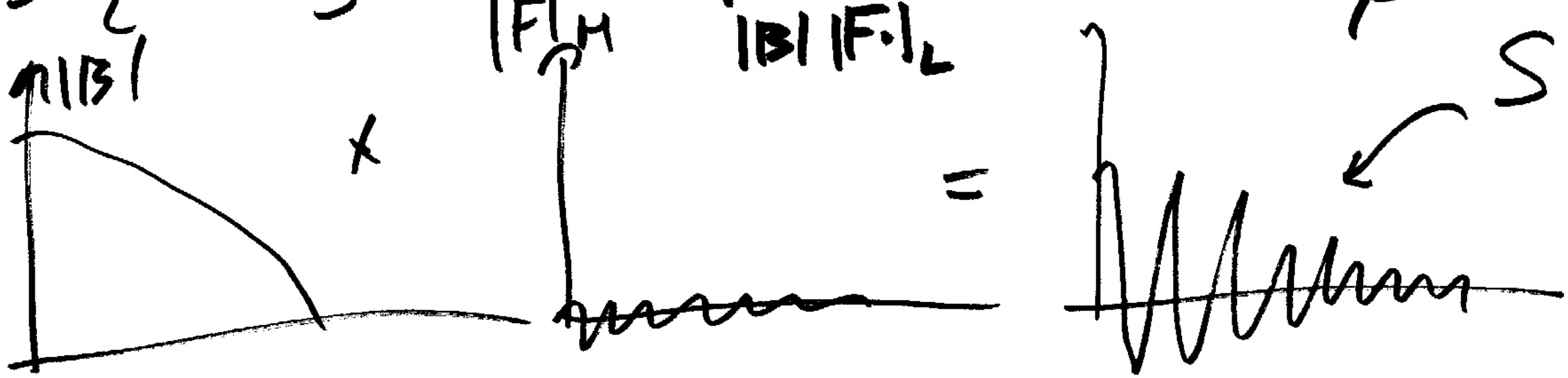


$$|G(\omega_1,\omega_2)| = |B(\omega_1,\omega_2)| \left(|F(\omega_1,\omega_2)|_L + |F(\omega_1,\omega_2)|_H \right)$$

$$|G| = |B|\,|F|_L + |B|\,|F|_H$$

Apply "Smoothing Operator" to Both sides $\longrightarrow S$

$$S\{|G|\} = \underbrace{S\{|B|\,|F|_L\}}_{|B|\,|F|_L} + \underbrace{S\{|B|\,|F|_H\}}_{\emptyset}$$



$$S\{|G|\} \simeq |B|\ |F|_L$$

$$\Longrightarrow |B| \simeq \frac{S\{|G|\}}{|F|_L}$$

Estimate $|F|_L$ by using an ensemble of images $|F'|_L$

$$|B(\omega_1, \omega_2)| \simeq \frac{S\{|G|\}}{|F'|_L}$$

How about Phase.

$\angle B(\omega_1, \omega_2) \simeq 0 \longrightarrow$ zero phase filter.

Fig 9.21 $\longrightarrow$ shows Blind deconvolution to estimate B
+ Iterative.)

$$\hat{f}_{K+1}(n_1, n_2) = \hat{f}_K(n_1, n_2) + \lambda\left(g(n_1, n_2) - \hat{f}_K(n_1, n_2) * b(n_1, n_2)\right)$$

Homomorphic Processing

$$f \longrightarrow \oplus \longrightarrow g \rightarrow \boxed{\text{Rest}} \rightarrow \hat{f}$$
$$\uparrow w$$

Film grain noise is multiplicat.

$$f \longrightarrow \otimes \longrightarrow g \rightarrow \boxed{} \rightarrow \hat{g}$$
$$\uparrow w$$

$$g \simeq f \cdot w.$$

$$\log(g) \simeq \log(f) + \log(w)$$

$$g' \simeq f' + w'$$

Apply any additive noise Restoration
Alg to restore $\hat{f}$

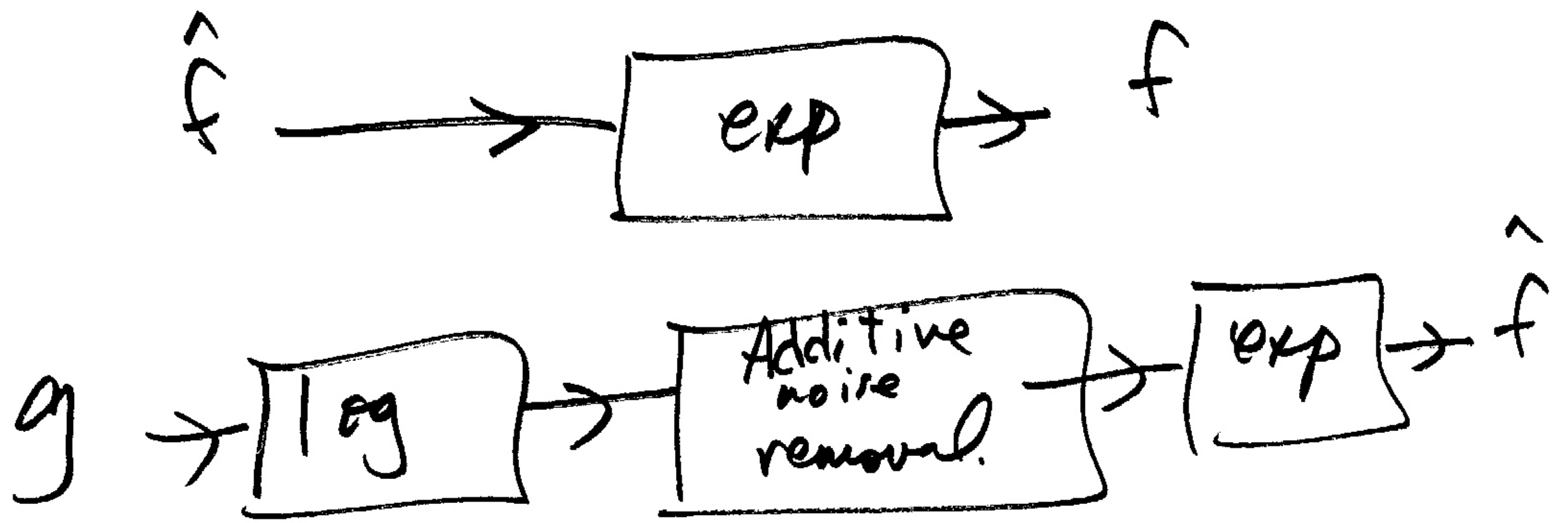
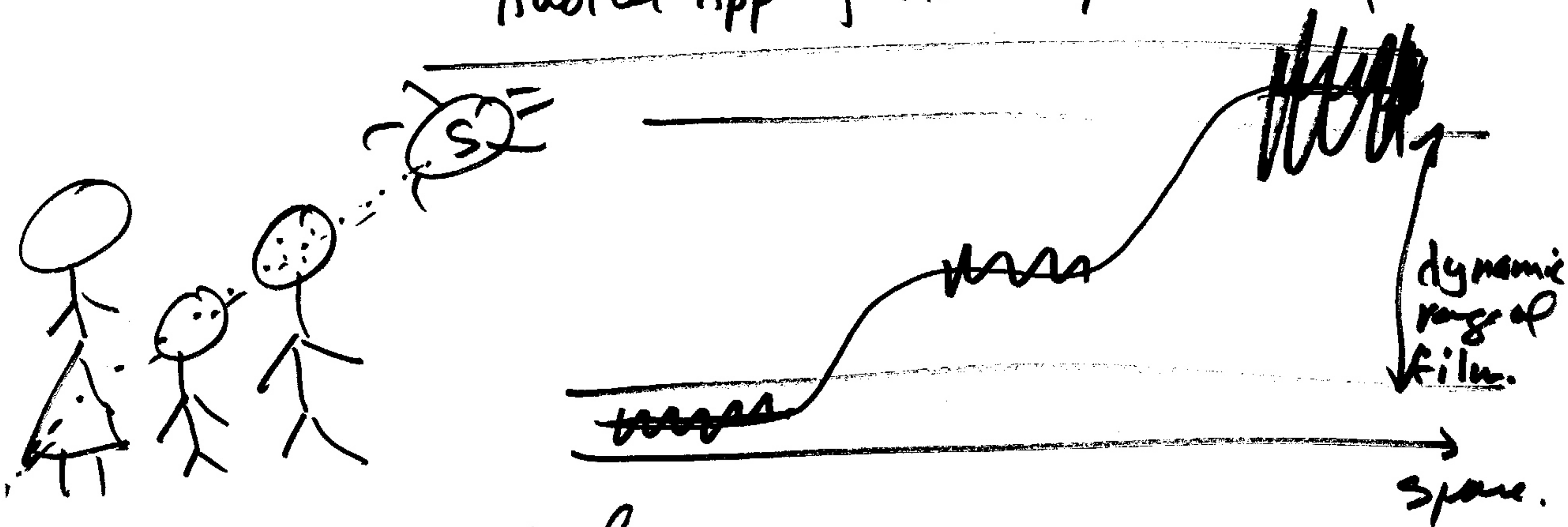


Fig 9.27 S. Lim

Another App of Homomorphic filtering



recorded signal

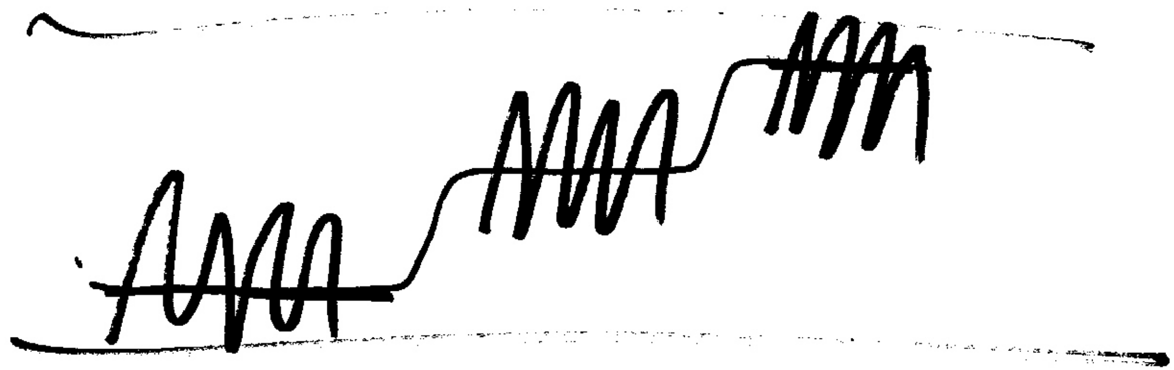
$$f(n_1,n_2) = r(n_1,n_2) \cdot i(n_1,n_2)$$

$r(n_1,n_2)$: reflectance due to objects → varies fast

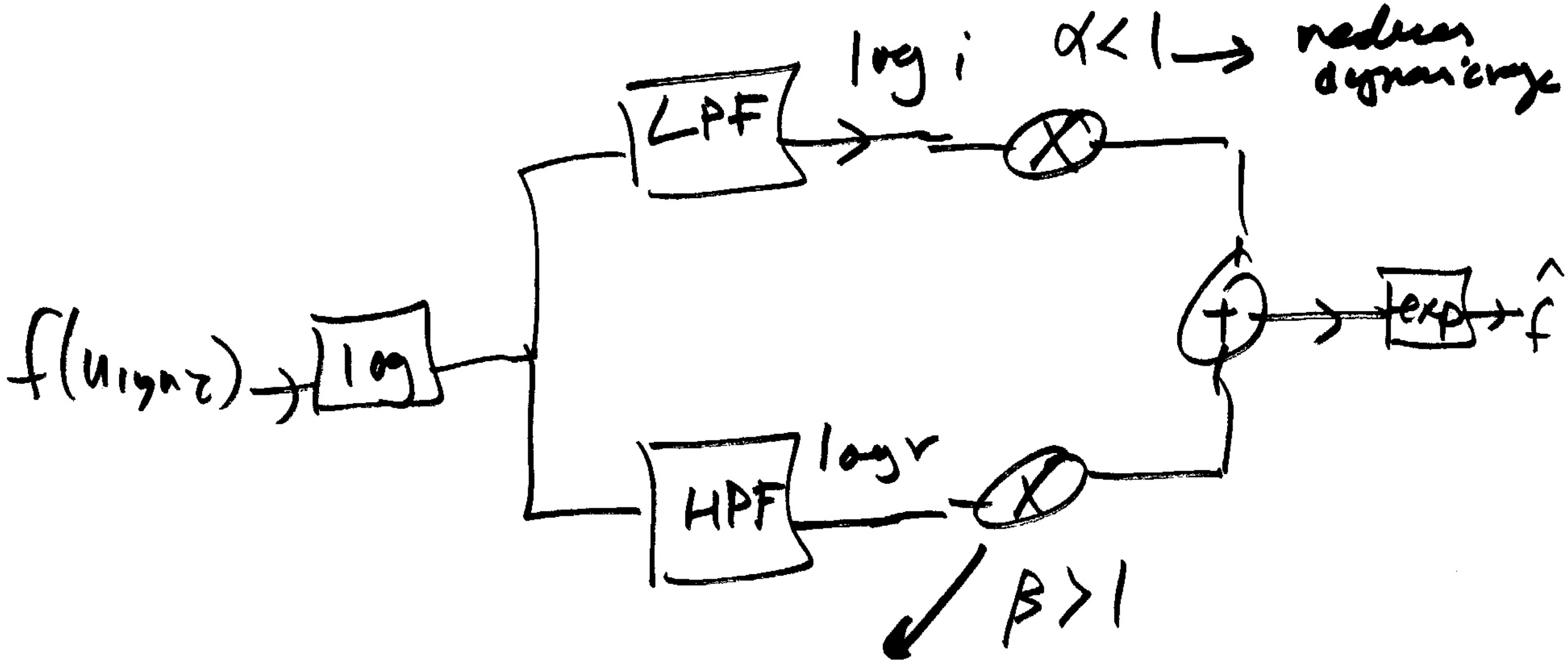
$i(n_1,n_2)$: illumination due to light source. slowly varying as a fn of space.

⟹ dynamic range

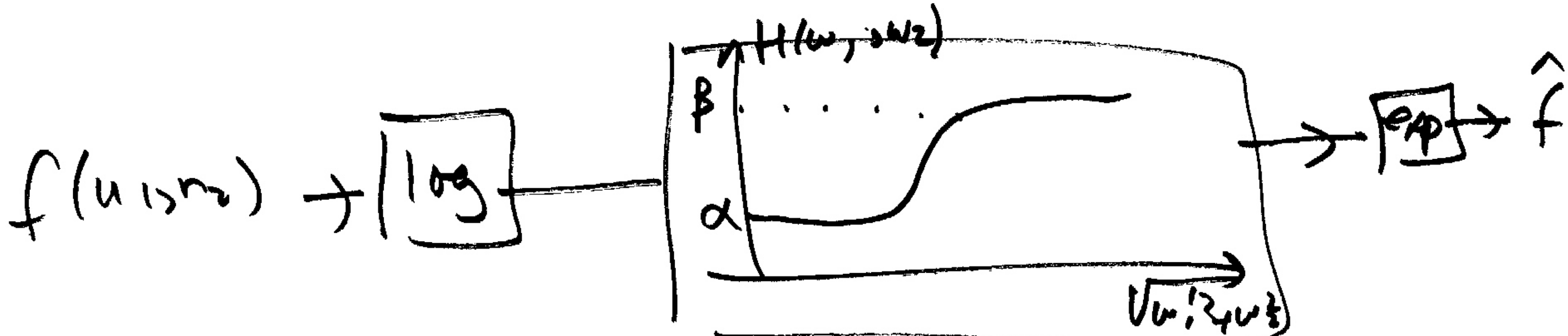
local Contrast enhancement + dynamic range reduction.



$$\log f(n_1,n_2) = \underbrace{\log r(n_1,n_2)}_{\text{rapidly varying}} + \underbrace{\log i(n_1,n_2)}_{\text{slowly varying}}$$



f(n1,n2)
log
LPF
HPF
log i
log r
α<1 → reduce dynamic range
β>1
increase local contrast
exp
f̂



f(n1,n2)
log
H(ω1,ω2)
β
α
exp
f̂

Fig 8.11 J. Lim