04/14/06

Image Coding

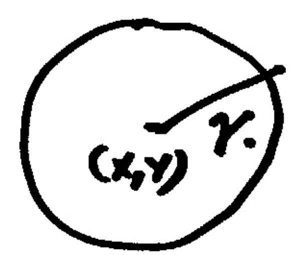
1. Objective: compress

2. What to code:

   a). Image intensity:

   b). Transform coefficients:

   c). Model parameter.



2. Quantization

a). Uniform spacing,

b). non-uniform. spacing

3. Bit assignment.

a). equal - length

b). unequal - length.

Uniform Quantization

Scalar Case:

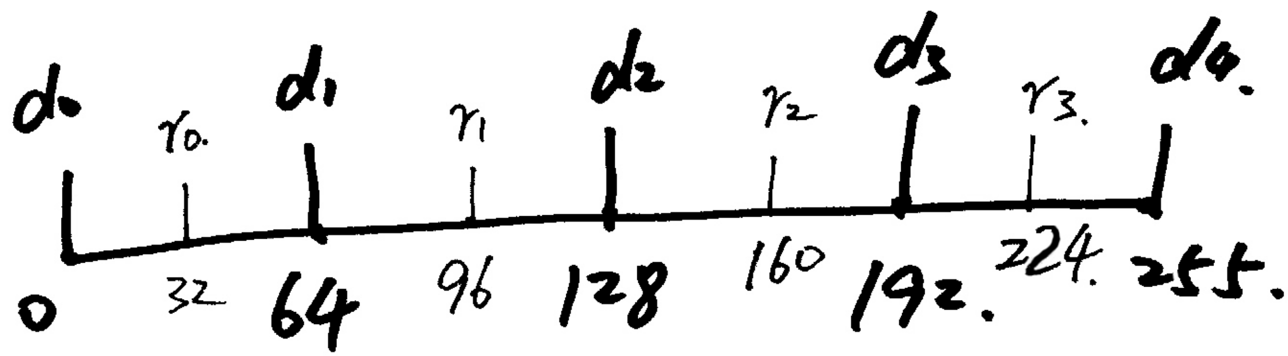
1. uniform.

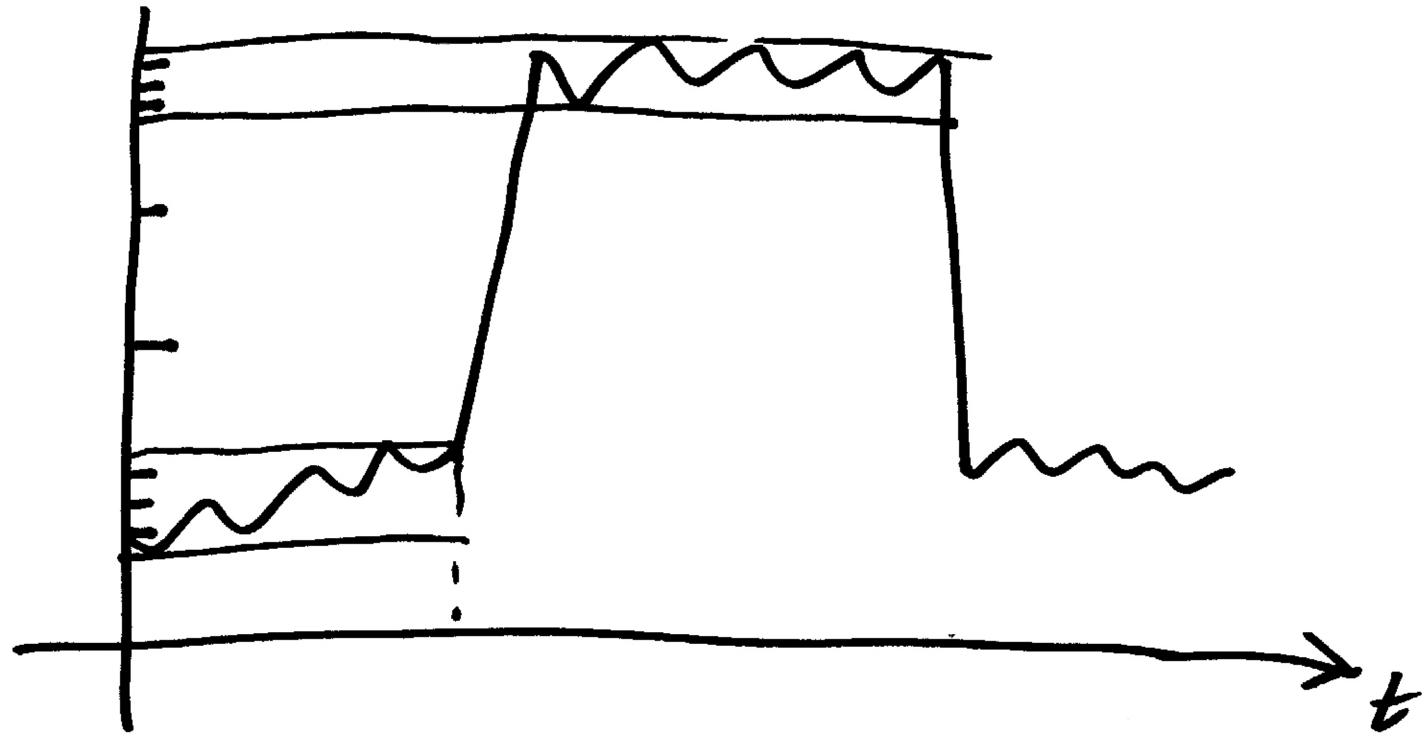
$f: 0 \sim 255$

L: num. of reconstruction levels.

$$\Delta = \frac{f_{max} - f_{min}}{L}.$$

$$\begin{cases} d_i - d_{i-1} = \Delta \\ r_i = \dfrac{d_i + d_{i-1}}{2} \end{cases}$$

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2. non-uniform : error criterion.

MSE : Mean square Error.

1). f : random variable

$P(f)$: prob. density func.

2). r_i : reconstruction level

d_i: Decision boundary.

3) J: # of recon. levels

$$\varepsilon = E[(f-\hat{f})^2]$$

$$= \int_{f_{min}}^{f_{max}} (f-\hat{f})^2 \cdot P(f)\,df. \qquad (1).$$

Min. ε, by choosing r_i, d_i:

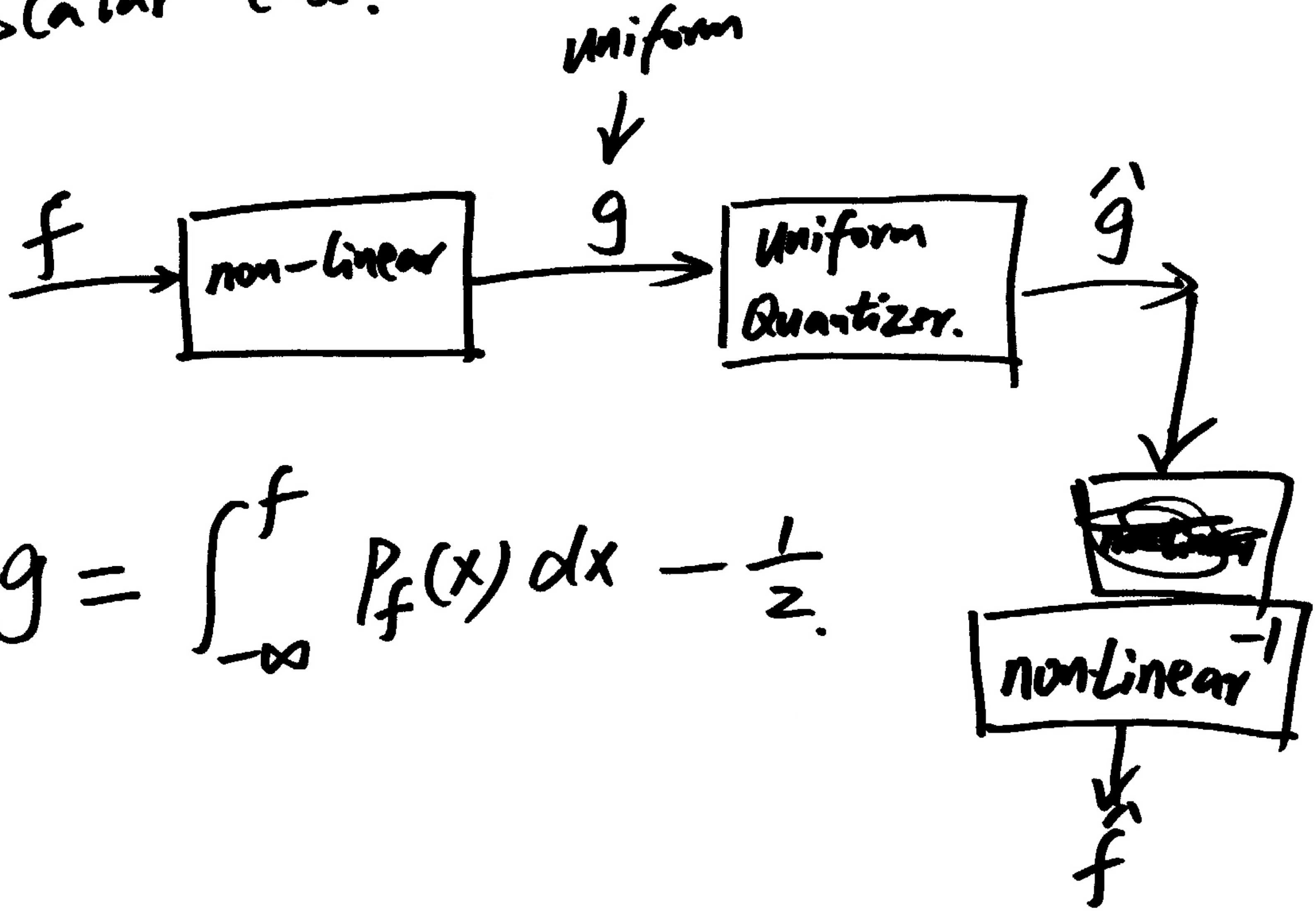
$$= \sum_{j=0}^{J-1} \int_{d_j}^{d_{j+1}} (f - r_j)^2 \cdot P(f)\,df. \qquad (2).$$

$$\frac{\partial \varepsilon}{\partial d_j} = 0 \implies d_j = \frac{r_j + r_{j-1}}{2}.$$

$$\frac{\partial \varepsilon}{\partial r_j} = 0 \implies \underset{\substack{\uparrow \\ \text{centroid.}}}{r_j} = \frac{\int_{d_j}^{d_{j+1}} f \cdot p(f)\,df}{\int_{d_j}^{d_{j+1}} p(f)\,df}.$$

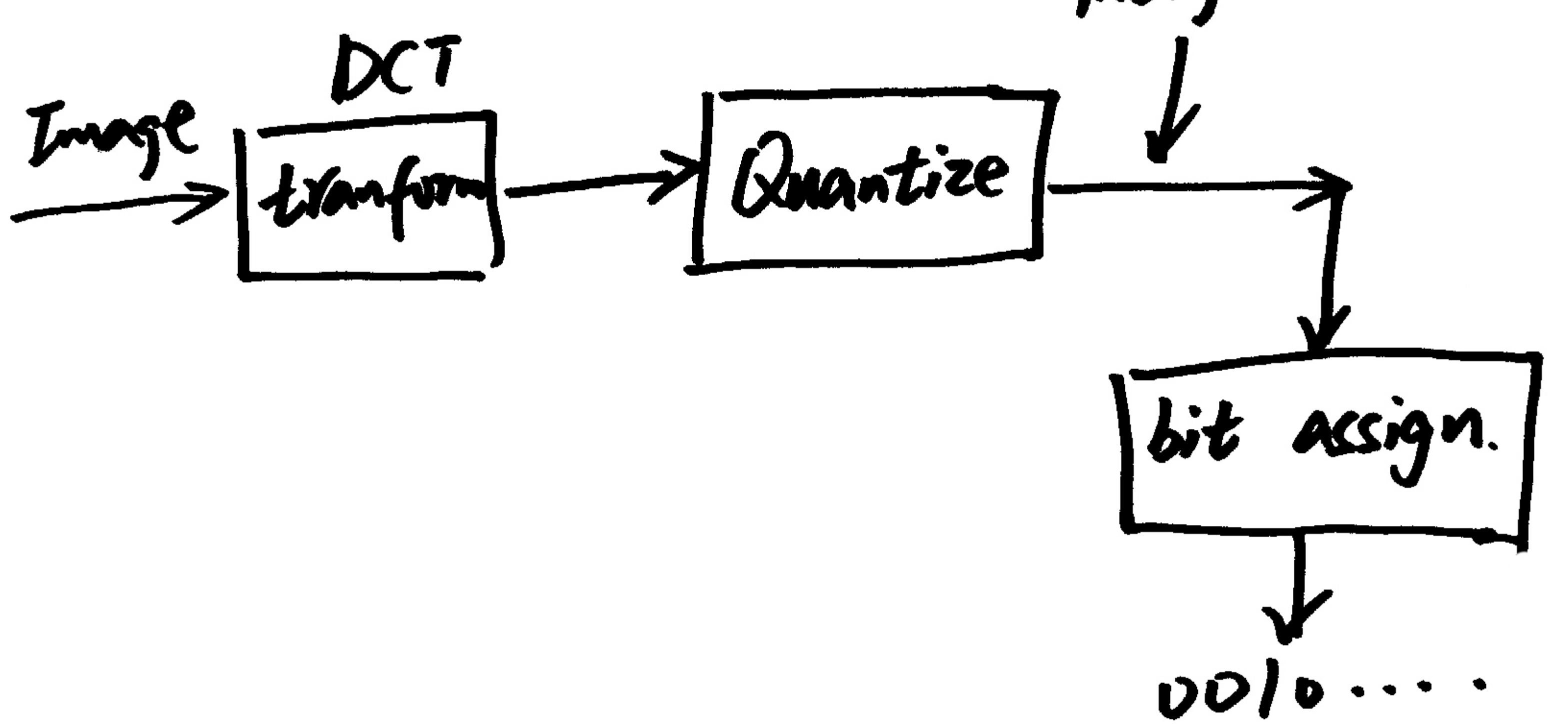
lloyd – Max algorithm.

Scalar case:



$$g = \int_{-\infty}^{f} P_f(x)\,dx - \frac{1}{2}.$$

Codeword Design.



	Prob.	
"A"	1/2	01
"B"	1/4	10
"C"	1/8	11
"D"	1/8	00

"D" 010

"AB"

2 bits/sym.

"A"	1/2	0
"B"	1/4	10
"C"	1/8	110
"D"	1/8	111

$1 \times 1/2 + 2 \times 1/4$

$+ 3 \times 1/8 + 3 \times 1/8$

$= 1.75$ bits/sym.

"

Overview.

entropy

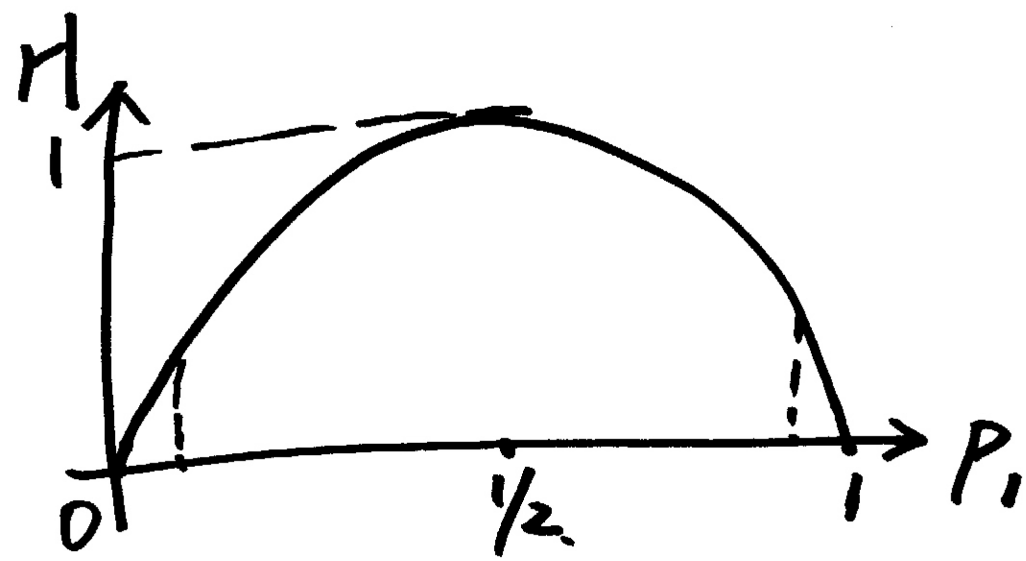
$$H = -E[\log_2 p_i] \qquad p_i = \text{Prob.}(\text{sym} = i).$$

$$= -\sum_{i=1}^{L} p_i \log_2 p_i$$

$$0 \le H \le \log_2 L.$$

- $L = 2$, $p_1 = 1$, $p_2 = 0 \Rightarrow H = 0$.

  $p_1 = 1/2$, $p_2 = 1/2 \Rightarrow H = 1$



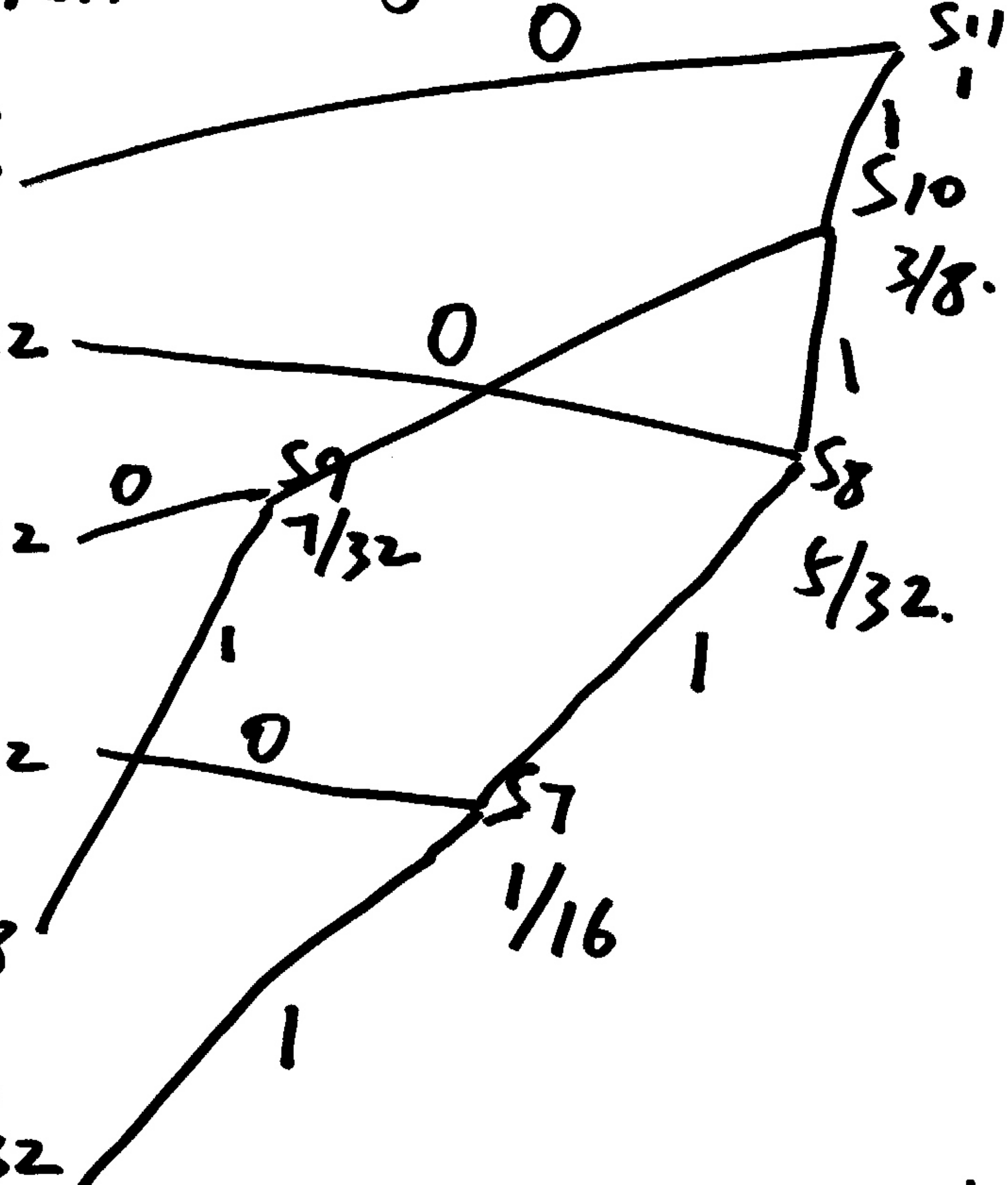
H : lower bound of bit rate

Ex: L symbols, power of 2.

$P_i = 1/L \rightarrow H = \log_2 L$.

Huffman Coding

	Code	Prob.
S_1	0.	5/8
S_2	110	3/32
S_3	100	3/32
S_4	1110	1/32
S_5	101	1/8
S_6.	1111	1/32



Performance:

Uniform: 3 bits/sym.

Huffman: $1 \times 5/8 + 3 \times 3/32 + \cdots +$.

$= 1.813$ bits/sym.

Entropy: $-\left(5/8 \cdot \log_2 5/8 + 3/32 \cdot \log_2 3/32 + \cdots\right)$.

$= 1.752$ bits/sym.

$$H(S) \le \bar{l} \le H(S) + 1.$$

tight upperbound.

$P_{max} < 0.5$. $\quad H(S) + P_{max}$.

$P_{max} \geq 0.5$ $\quad H(S) + P_{max} + 0.086$.

$\{a_1\ a_2\ a_3\}$ $\quad P_1 = 0.95,\ P_2 = 0.02,\ P_3 = 0.03$.

Huffman: 1.05 bits/sym. $\qquad$ iid source.

Entropy: 0.335 bits/sym.

Dictionary

Main idea: 1. build a list of patterns

2. index in the list.

static Dict.

Dynamic Dict.

Static Dictionary.

$A = \{a, b, c, d, r\}$.

ad.	ac.	ab.	r	d	c	b	a	Entry
111	110	101	100	011	010	001	000	Code.

Encode a b r a c a d a b r a.

1. ab → 101
2. ra X.
3. r 100.
4. ac. 110

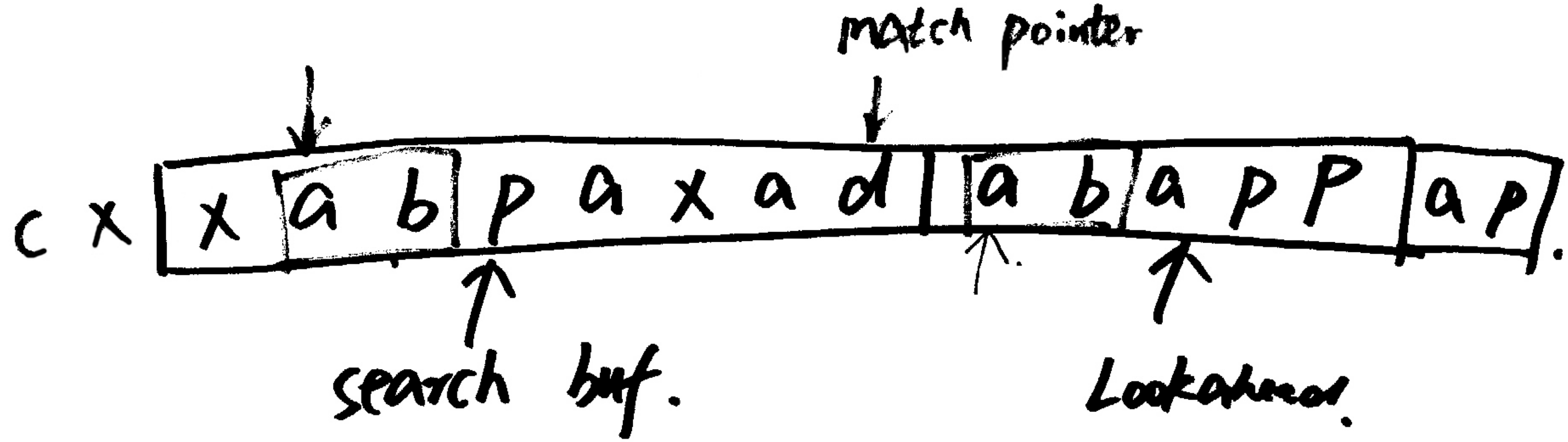
Adaptive Dictionary.

<u>LZ77</u> LZ78.

Ziv Lempel.

Basic idea: portion of previously encoded seq. as dictionary

Sliding win.: search buffer.
Lookahead buffer.



{0, l, C(next symbol)}

{2, 1, }

{4, 1, }

{7, 2, ~~Ca~~ C(a)}

{

no match.

{0, 0, C(sym)}.

S: len. of search buf.

W: len. of window (search buf. + LA buf).

A: size of symbol. sets

$$\lceil \log_2 S \rceil + \lceil \log_2 W \rceil + \lceil \log_2 A \rceil.$$

$$(W-S)$$

Example.

c a b r a c a d a b r a r r a r r a d

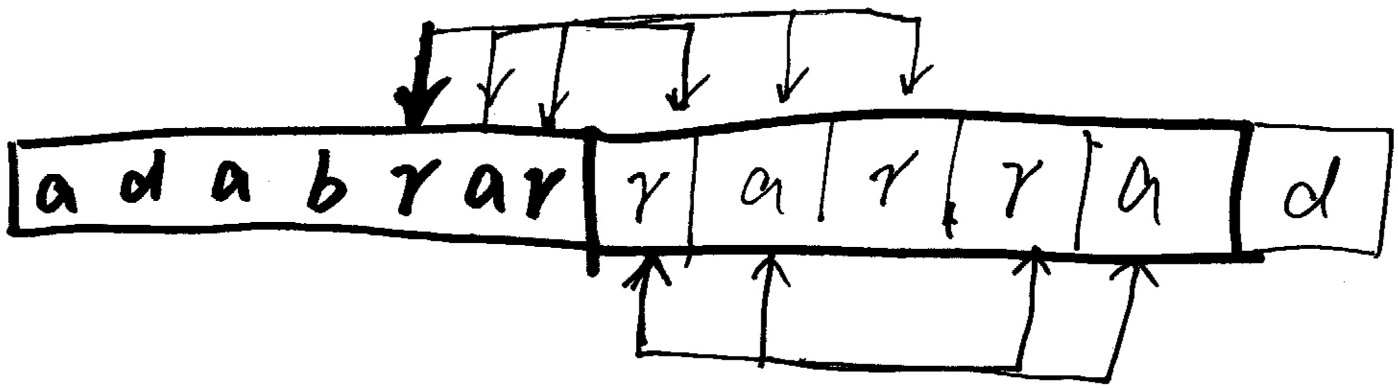
1. No match ~~to~~ d. <0, 0, C(d)>.

2. match for a: <7, 4, C(r)>.

3. match begins from Search buf.

   End at LA buf. <3, 5, C(d)>.

Decoder: $\langle 3, 5, c(d) \rangle$.



Arithmetic Coding.

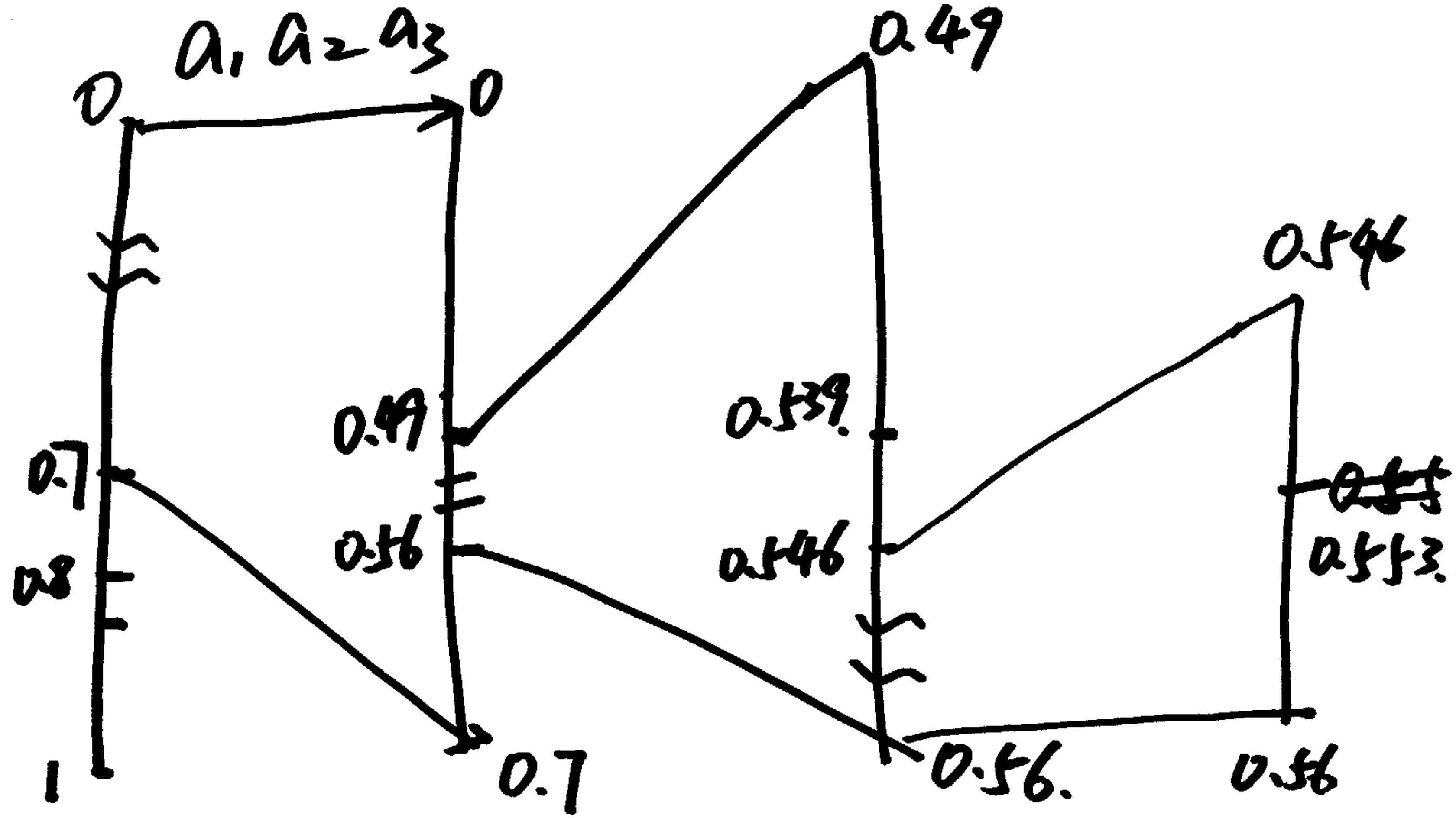
Two steps:

1) Generate a tag: real number.

2). binary code → tag.

$A = \{a_1, a_2, a_3\}$.

$P(a_1) = 0.7$, $P(a_2) = 0.1$, $P(a_3) = 0.2$.



Decoder. $\underline{\underline{0.553}}$

a_1 $\boxed{[0, 0.7]}$ $\longrightarrow$ a_1 $[0, 0.49]$

a_2 $[0.7, 0.8]$ $\quad$ a_2 $\underline{[0.49, 0.56]}$

a_3 $[0.8, 1]$ $\quad$ a_3 $[0.56, 0.7]$

$a_1 a_2 a_3$. $\longrightarrow$ a_1 $[\quad\quad]$

a_2 $[\quad\quad]$

a_3 $[\quad\quad]$

$0.553 \rightarrow a_1 a_2 a_3$.

$P(a_1 a_2 a_3) = P(a_1)P(a_2)P(a_3) = P(\vec{X})$.

$\underbrace{\left\lceil \log_2 \frac{1}{P(\vec{X})} + 1 \right\rceil \text{ bits}} \rightarrow 0.553$.

$P(\vec{X}) = 0.014$.

# of bits $= 8$ bits.

$0.553 = \frac{1}{2^1} + \frac{1}{2^5} + \frac{1}{2^6} + \frac{1}{2^8}$.

$= 0.10001101 \Rightarrow 10001101$

$$\underline{H(X)} \le l_A \le H(X) + \frac{2}{m}$$

len. of sym. seq.