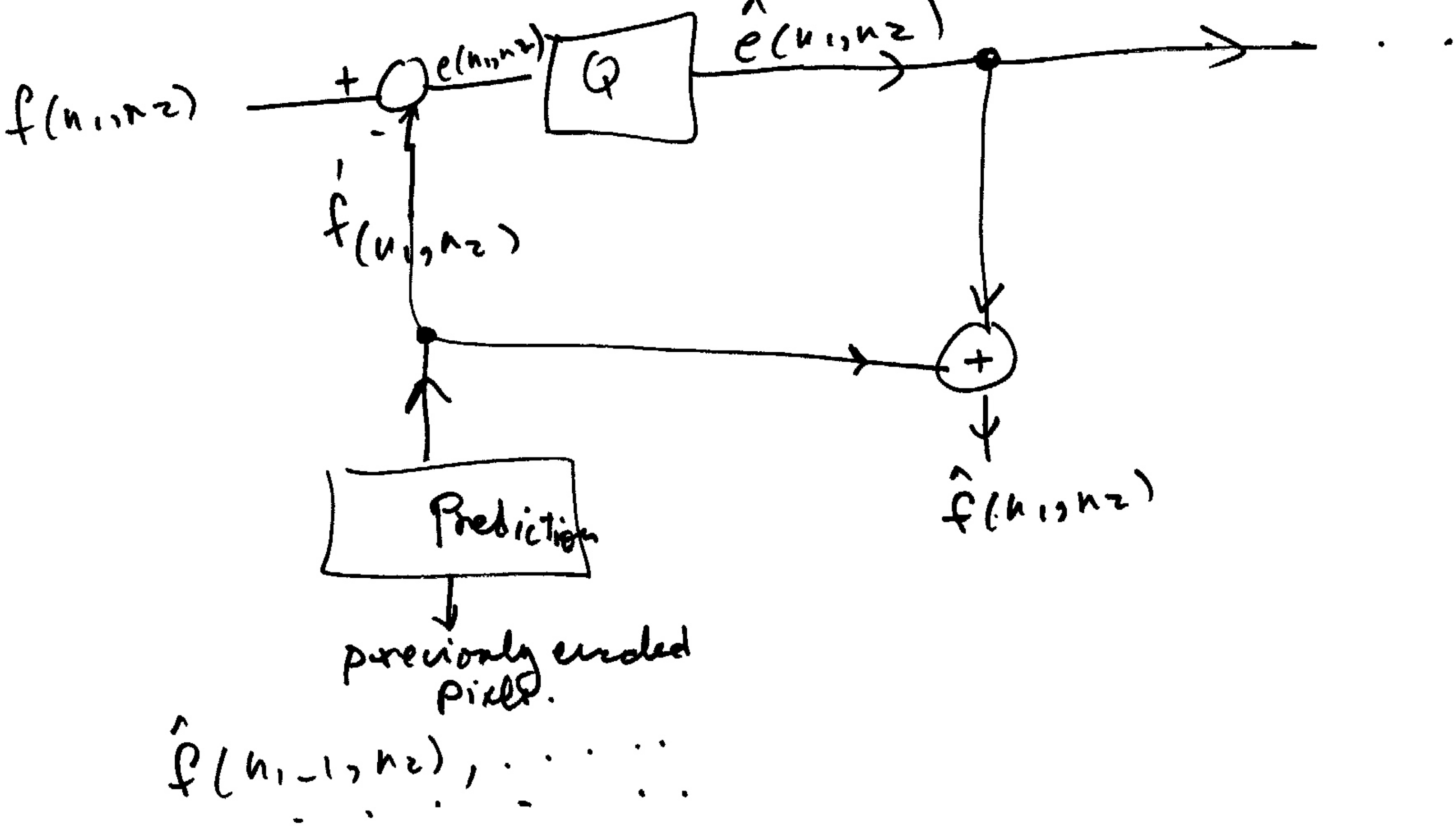
DPCM

April 21 06



1

How To predict

$$\hat{f}(n_1, n_2) = \sum_{(k_1,k_2) \in R_a}\sum a(k_1, k_2)\, \hat{f}(n_1 - k_1, n_2 - k_2)$$

How To choose a

Minimize $E[e^2(n_1, n_2)] =$

$$E[e^2(n_1,n_2)] = E\left[\left(f(n_1,n_2) - \sum_{k_1,k_2}\sum a(k_1,k_2)\,\hat{f}(n_1-k_1, n_2-k_2)\right)^2\right]$$

Ideally $f \simeq \hat{f}$

$$E[e^2(n_1,n_2)] = E\left[\left(f(n_1,n_2) - \sum_{k_1,k_2}\sum a(k_1,k_2) f(n_1-k_1, n_2-k_2)\right)^2\right]$$

Quadratic in $a(k_1, k_2)$

Minize w.r.t. To a.

take derivaten, set equal to zero $\Longrightarrow$ 2 linear sys of Eqn.

Soln:

$$R_f(l_1, l_2) = \sum\sum_{(k_1,k_2)\in R_a} a(k_1,k_2)\, R_f(l_1-k_1, l_2-k_2)$$

$R_f(n_1, n_2)$ is correlation fn for the stationary random process f.

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assume R_a, assume $R_f \Longrightarrow a(k_1,k_2)$

Fig 10.30 $\longrightarrow$ J lim $\quad a(1,0) = a(0,1) = 0.95$

$a(1,1) = -0.95$

Possible Prediction

$$\hat{f}(n_1,n_2) = 0.97\, f(n_1, n_2-1) \quad (1)$$

$$\hat{f}(n_1,n_2) = \frac{1}{2} f(n_1, n_2-1) + \frac{1}{2} f(n_1-1, n_2) \quad (2)$$

$$\hat{f}(n_1,n_2) = \frac{3}{4} f(n_1, n_2-1) + \frac{3}{4} f(n_1-1, n_2) - \frac{1}{2} f(n_1-1, n_2-1) \quad (3)$$

$$\hat{f}(n_1,n_2) = \begin{cases} 0.97\, f(n_1, n_2-1) & \text{if } \Delta h \leq \Delta V \\ 0.97\, f(n_1-1, n_2) & \text{otherwise} \end{cases} \quad (4)$$

$$\Delta h = \left| f(n_1-1, n_2) - f(n_1-1, n_2-1) \right| \rightarrow \text{horizontal gradient at } (n_1, n_2)$$

$$\Delta V = \left| f(n_1, n_2-1) - f(n_1-1, n_2-1) \right| \rightarrow \text{vertical gradient})$$

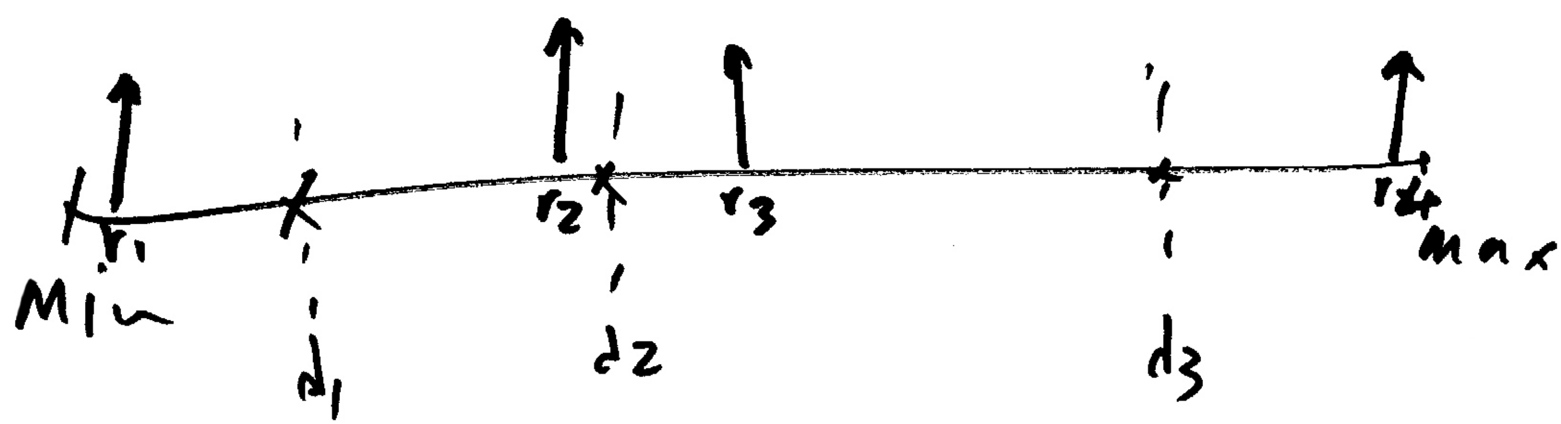
Show Fig 8.24 in G/W.

entropies
4.9, 3.7, 3.3, 4.1

Lyod max

- optimum nonuniform Q.
- Assume pdf . R.V.
- Want to quantize N levels.

What is decision boundaries?
Recon levels?



Assume pdf of prediction err
is Laplacian $\Longrightarrow$ Quantize it 2
4
8

Tables. given kind of pdf
- Gaussian
- Laplacian
- Uniform
- Rayleigh.

unit variance
$\Longrightarrow$ optimal d_i, r_i

Table 10.1 in J. Lim

DPCM adaptive

4x4 block.

variance of 4x4 block → var 1, var 2, var 3, var 4

4 scaled quantizers

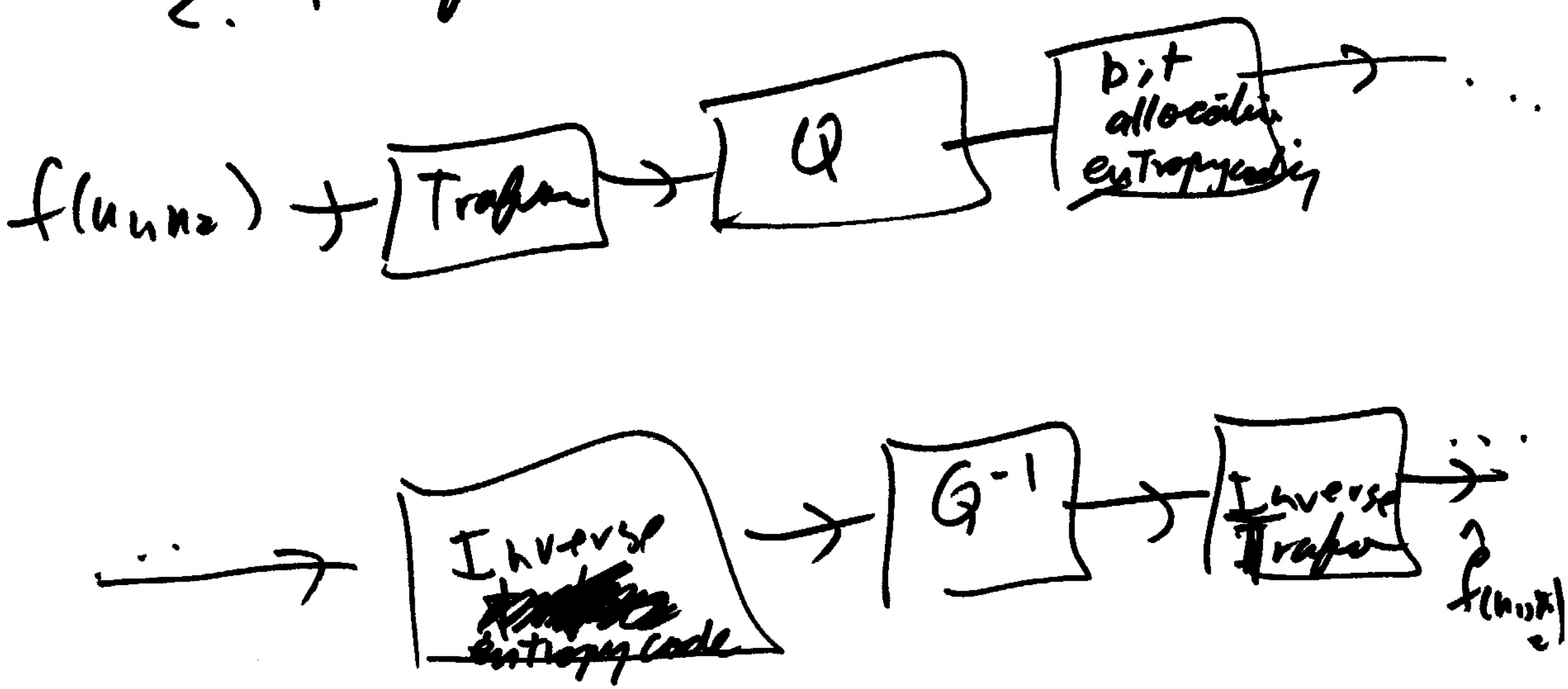
0.5
1.0
1.75
2.5

2 bits of overhead to do classification

2 bits / 16 pixels (4x4) → 0.125 bits overhead.

What To code

1. Waveform Coding
2. Transform Coding



Why Transform

(1) energy compaction in a small # of coeff.

(2) Reduce or eliminate correlation between Transform coefficient

No need for prediction.

ConsTraints

(1) ~~TE~~ Must be invertible

(2) Computationally Tractable

Optimum / statistical Transform

. KLT

Karhunen-Loeve.

1. Optimal in a sense that coeff. are completely uncorrelated.
2. On average, first M coefficients have more energy than any other Transform

$$\text{pixel } 1000 \longrightarrow \boxed{\text{KLT}} \longrightarrow \text{coeff. } 1000$$

c_1
c_2
$\vdots$
c_{1000}

$$\sum\sum f^2(n_1, n_2) = \sum_{k_1, k_2}\sum F^2(k_1, k_2)$$

— Assuming $f(n_1, n_2)$ stationary random process

- Covariance fn.

$$K_f(n_1, n_2 \,;\, l_1, l_2) \triangleq E\left[\left(x(n_1, n_2) - \bar{x}(n_1, n_2)\right)\left(x(l_1, l_2) - \bar{x}(l_1, l_2)\right)\right]$$

Then basis fn: $A(n_1, n_2 \,;\, k_1, k_2)$ satisfies eigen equation

$$\lambda(k_1, k_2)\, A(n_1, n_2 \,;\, k_1, k_2) = \sum_{k_1=0}^{N_1-1} \sum_{k_2=0}^{N_2-1} K_f(n_1, n_2 \,;\, k_1, k_2) . A(l_1, l_2 \,;\, k_1, k_2)$$

$$F_{KLT}(k_1, k_2) = \sum_{n_1} \sum_{n_2} f(n_1, n_2)\, A(n_1, n_2 \,;\, k_1, k_2)$$

//

KLT in practice $\longrightarrow$ difficult

Image Markov model.

$$E[f(n_1, n_2)\, f(n_1 - i, n_2 - j)] =$$

$$\sigma^2 \rho_v^i \rho_h^j$$

ρ_h = horizontal correlation coeff ")) = ρ

ρ_v = vertical " "

Then can show as $\rho \longrightarrow 1$ Three

$$KLT \longrightarrow DCT$$ = Discrete cosine Transform.

4 Types of DCT : Type II

$$S(k_1,k_2) = \sqrt{\frac{4}{N^2}}\, C(k_1)\, C(k_2) \sum_{n_1, n_2 = 0}^{N-1} s(n_1,n_2) \cos\left(\frac{\pi(2n_1+1)k_1}{2N}\right) \cos\left(\frac{\pi(2n_2+1)k_2}{2N}\right)$$

Forward DCT Transform

$$C(k) \triangleq \begin{cases} \frac{1}{\sqrt{2}} & k = 0 \\ 1 & \text{otherwise} \end{cases}$$