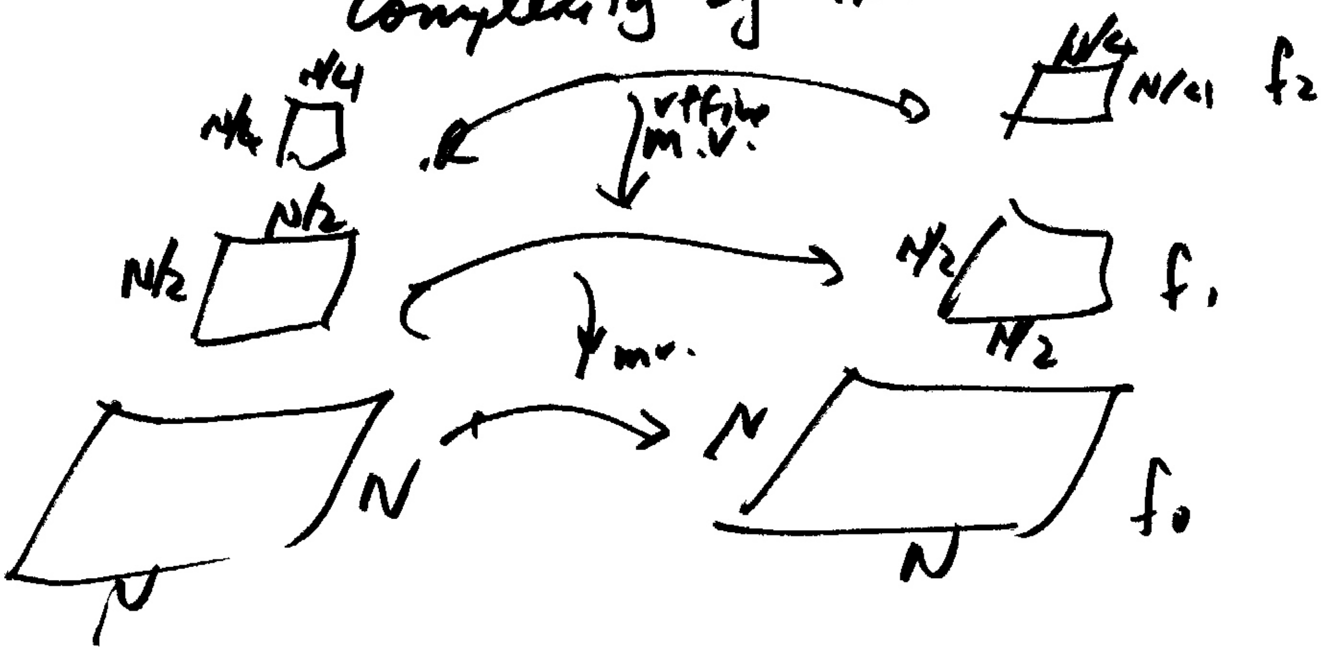
May 3, 2006

MultiResolution – Subband Pyramid – Wavelet

- Multiresolution introduced : Burt + Adelson 1982

Reviewer of 1982 paper → very negative

Initial motivation ~~how~~ → reduce complexity of motion estimation



How To Build a pyramid?

- Basic idea $\longrightarrow$ successive low pass filtering + subsampling.

$$f_i(n_1,n_2) \rightarrow \boxed{\text{LPF}} \rightarrow \boxed{\text{subsample}} \rightarrow f_{i+1}(n_1,n_2)$$

Process of generating (i+1) st level from i th level.

* How to filter?

$$f_i^L(n_1,n_2) = f_i(n_1,n_2) * h(n_1,n_2)$$

determines The kind of pyramid

* Subsampling: $$f_{i+1}(n_1,n_2) = \begin{cases} f_i^L(n_1,n_2) & 0 \le n_1,n_2 \le 2^{N-1} \\ 0 & \text{otherwise.} \end{cases}$$

* Gaussian pyramid:

$$h(n_1, n_2) = h(n_1)\, h(n_2).$$

$$h(n) = \begin{cases} a & n = 0 \\ 1/4 & n = \pm 1 \\ \frac{1}{4} - \frac{a}{2} & n = \pm 2 \end{cases}$$

usually $0.3 < a < 0.6$

show fig 10.34 → 10.36
J. Lim

Pyramid Coding

- Basic idea: Code successive images down the pyramid from the ones above it.

- Interpolate $f_{i+1}(n_1, n_2)$ to get prediction $f_i(n_1, n_2)$

prediction for f_i $\hat{f}_i(n_1, n_2) = I\left[f_{i+1}(n_1, n_2) \right]$

- Code prediction error.

  $e_i(n_1, n_2) = f_i(n_1, n_2) - \hat{f}_i(n_1, n_2)$

- Repeat unitil the bottom level image is reconstructed $\rightarrow$ i.e original.

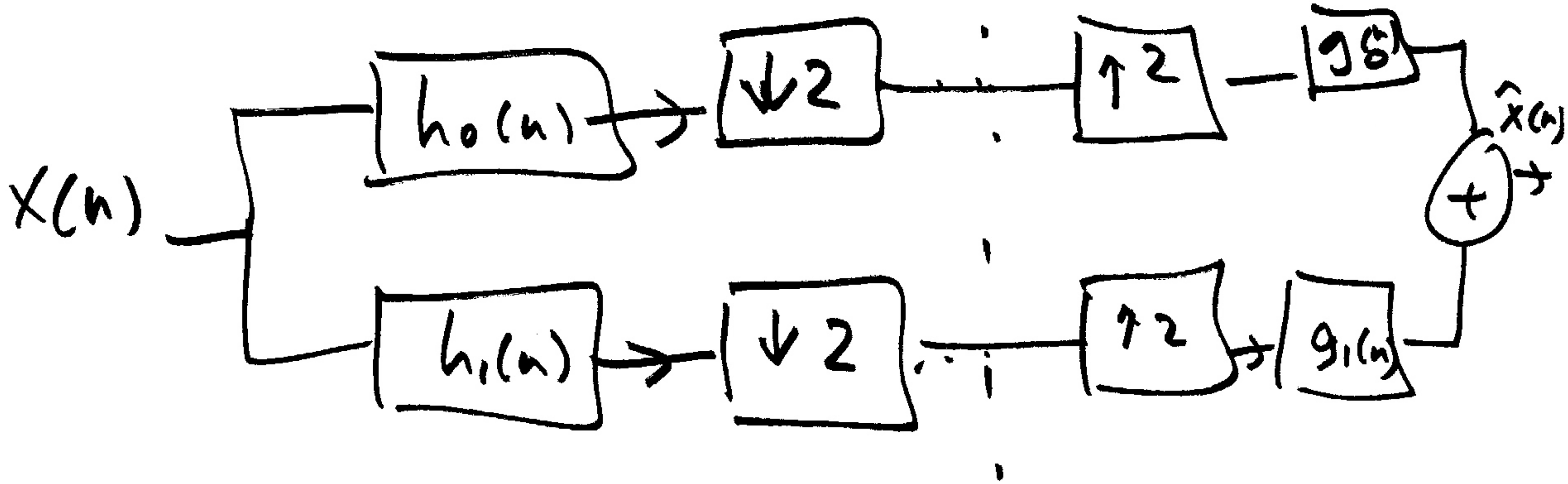
- Seq $f_i \rightarrow$ Gaussian Pyramid $e_i(n_1, n_2) \rightarrow$ Laplacian pyramid.

Fig 10.38 + Fig 10.37 J. S. Lim
Fig 7.1 → 7.3 G/W.

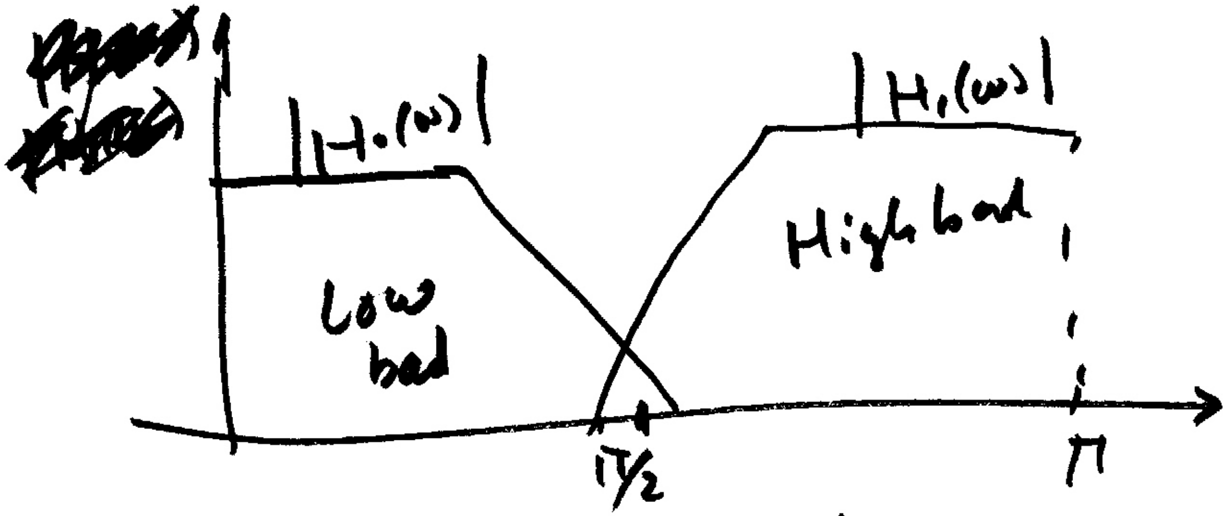
Pyramid Coding: expansion of # of samples over transf. coding.

Subband Coding

Speech coding → Croiser, Esteban, Galand. 1976



Subband: No expansion on # of sample



$$\hat{X}(z) = \frac{1}{2}\left[H_0(z)\,G_0(z) + H_1(z)\,G_1(z) \right] X(z) + \frac{1}{2}\left[H_0(-z)\,G_0(z) + H_1(-z)\,G_1(z) \right] X(-z)$$

Aliasing Term

Cancel Aliasing, we must have.

$$H_0(z)\, G_0(z) + H_1(-z)\, G_1(z) = 0$$

To get perfect reconstruction, we must have:

$$H_0(z)\, G_0(z) + H_1(z)\, G_1(z) = 2$$

Define $H_m(z) = \begin{bmatrix} H_0(z) & H_0(-z) \\ H_1(z) & H_1(-z) \end{bmatrix}$

to cancel aliasing + achieve PR. we must have

(*) $\begin{bmatrix} G_0(z) & G_1(z) \end{bmatrix} H_m(z) = \begin{bmatrix} 2 & 0 \end{bmatrix}$

$H_0, H_1 \longrightarrow$ Analysis filters

$G_0, G_1 \longrightarrow$ Synthesis filters.

Can show if $(*)$ is satisfied then analysis + synthesis filters are "bi-orthogonal"

$$\langle h_i(2n-k), g_j(k) \rangle = \delta(i-j)\,\delta(n)$$

$$i, j = \{0, 1\}$$

$$\downarrow$$

$$\sum_k h_i(2n-k)\, g_j(k)$$

Example 2 channel filter Bank with perfect Reconstruct.

Analysis

Low pass: $\left(-\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, \frac{1}{2}, -\frac{1}{4}\right)$

high pass: $\left(-\frac{1}{4}, -\frac{1}{2}, \frac{1}{4}\right)$

Synthesis

Low pass $\left(\frac{1}{4}, \frac{1}{2}, \frac{1}{4}\right)$

high pass $\left(\frac{1}{4}, \frac{1}{2}, -\frac{3}{2}, \frac{1}{2}, \frac{1}{4}\right)$

Bi-orthogonal 5/3 filters

Le Gall filters

Show Freq Respon in B.G. slides

3 main classes of Perfect Reconstruction filter families

① QMF = Quadrature. Mirror filters:

Aliasing cancellation:

$$H_1(z) = H_0(-z) = -G_1(z) = G_0(-z)$$

← Estoban 1976.

- High pass band is the mirror of the low pass band in freq. domain.
- ~~For~~ Only design 1 filter for Aliasing Cancell.
- To achieve PR, design "1" filter:

$$H_0^2(z) - H_0^2(-z) = 2$$

② CQF = Conjugate Quadrature filters

Achieve aliasing

$$H_0(z) = G_0(z^{-1}) \triangleq f(z)$$

$$H_1(z) = G_1(z^{-1}) = z f(-z^{-1})$$

Smith, Barnwell 1986

time domain

$$h_0(k) = g_0(-k) = f(k)$$

$$h_1(k) = g_1(-k) = (-1)^{k+1} f(-(k+1))$$

<u>PR</u> :

$$H_0(z) H_0(z^{-1}) + H_0^2(z) H_0(-z^{-1}) = 2$$

Freq. Domain:

$$|H_0(\omega)|^2 + |H_0(\omega \pm \pi)|^2 = 2$$

3) OrThonormal:

$$\begin{cases} H_0(z) = G_0(z^{-1}) \\ H_1(z) = G_1(z^{-1}) \\ G_1(z) = -z^{-2K+1} G_0(-z^{-1}) \\ G_0(z)\, G_0(z^{-1}) + G_0(-z)\, G_0(-z^{-1}) = 2 \end{cases}$$

Aliast PR.

$2K$ = # of filter Taps in each filter.

Time domain: $\langle g_i(n), g_j(n+2m) \rangle = \delta(i-j)\,\delta(m)$

$g_1(n) = (-1)^n g_0(2K-1-n)$

$h_i(n) = g_i(2K-1-n) \qquad i = 0, 1$