

Systems

system Transform input into output

$$T \left[\underset{\substack{\text{input} \\ \nearrow}}{x(n_1, n_2)} \right] = \underset{\text{output}}{y(n_1, n_2)}$$

- Linear : $T \left\{ a x_1(n_1, n_2) + b x_2(n_1, n_2) \right\}$
 $= a T \left\{ x_1(n_1, n_2) \right\} + b T \left\{ x_2(n_1, n_2) \right\}$

Shift Invariance:

$$T \left\{ x(n_1, n_2) \right\} = y(n_1, n_2)$$

$$\text{Then } T \left[x(n_1 - k_1, n_2 - k_2) \right] = y(n_1 - k_1, n_2 - k_2)$$

2D-LSI system.

LSI \longrightarrow Impulse response.

$$h(n_1, n_2)$$

- enables to entirely characterize

an LSI system.

- tells you output given input.

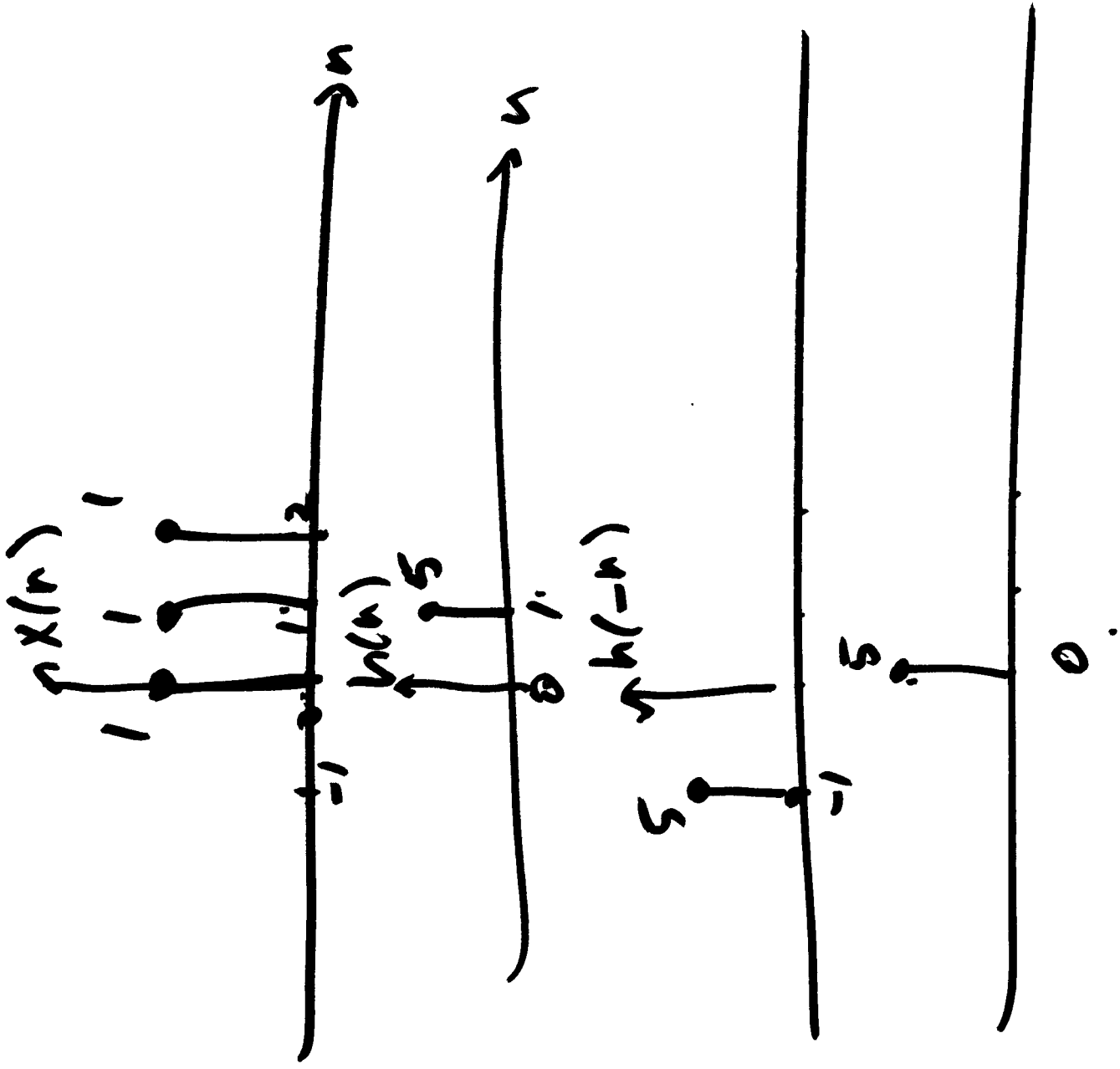
$$Y(n_1, n_2) = X * h \longrightarrow \text{convolution}$$

$$y(n_1, n_2) = \sum_{k_1} \sum_{k_2} x(k_1, k_2) h(n_1 - k_1, n_2 - k_2)$$

$$Y(u) = \sum_k X(k) h(u - k) \quad 1D$$

$$y(1) = 1$$

$$y(0) = 0$$



Separable LSI systems

$$h(n_1, n_2) = h_1(n_1) h_2(n_2)$$



$N \times N$
 $M \times M$

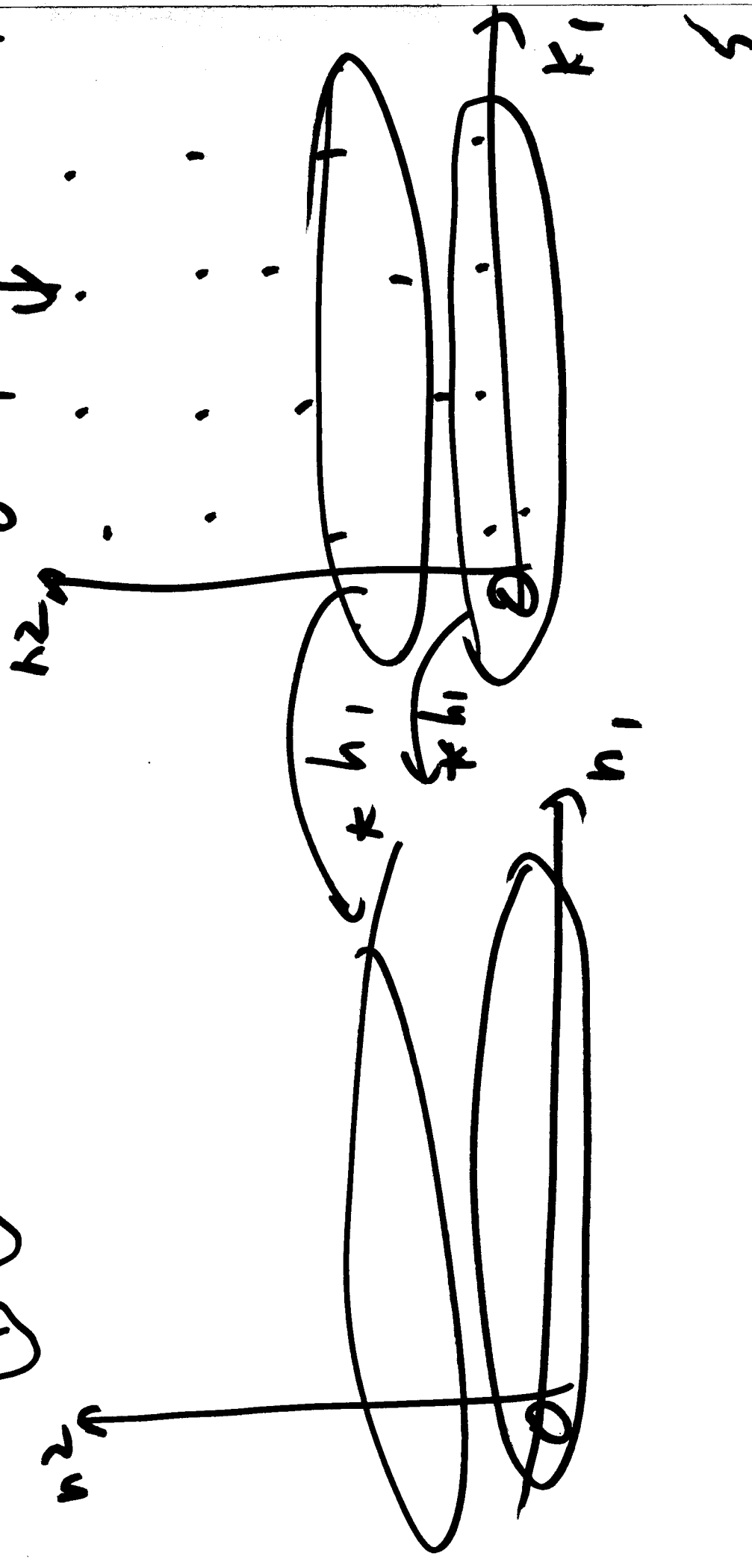
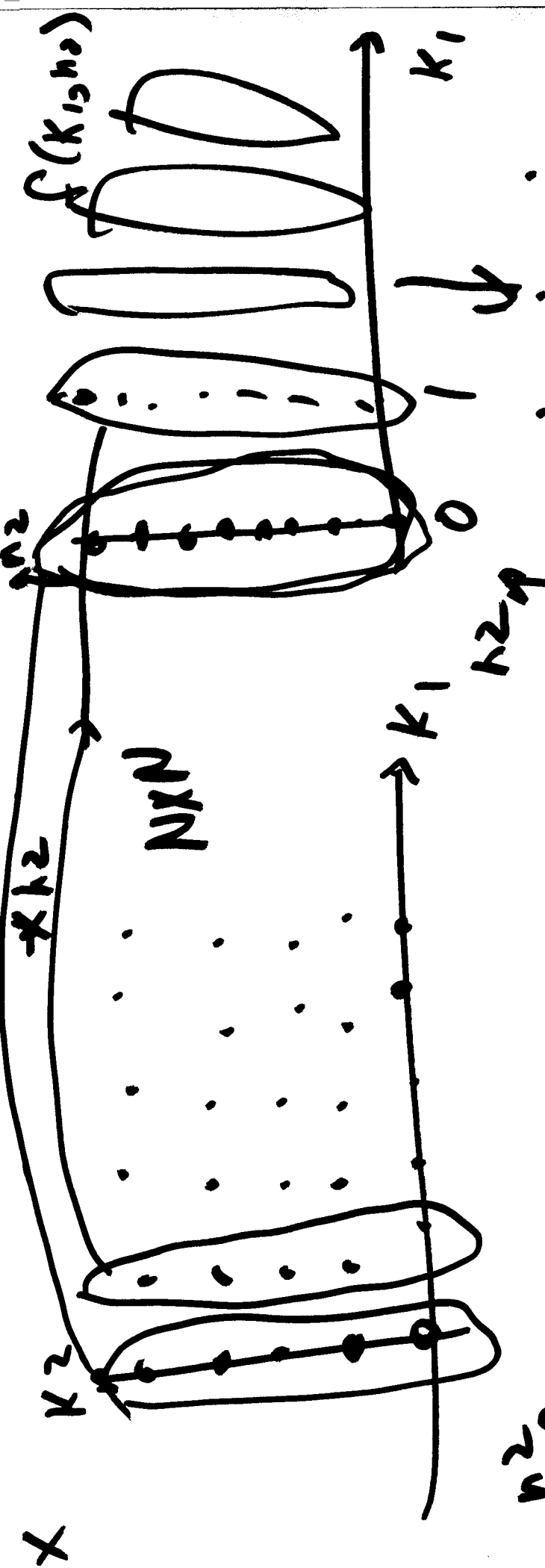
general:

$$y(n_1, n_2) = \sum_{k_1} \sum_{k_2} x(k_1, k_2) h_1(n_1 - k_1) h_2(n_2 - k_2)$$

sep. sys.

$$= \sum_{k_1} h_1(n_1 - k_1) \underbrace{\sum_{k_2} x(k_1, k_2) h_2(n_2 - k_2)}_{f(k_1, n_2)}$$

$$x(k_1) * h_2$$



Summary

1. Series of 1D convolutor To get.

$f(k, n_2)$:

N columns \rightarrow

N convolutor
NM oper / convol

$N^2 M$ op To build

② Series of 1-D conv To get $g(n, n_1)$

N rows \rightarrow

N convolutor
NM op / convol

$N^2 M$

Total $2 \cdot N^2 M$.

LSI system

* impulse response. $\rightarrow h(n_1, n_2)$

* stability of LSI system.

* BIBO stability \rightarrow

\rightarrow bounded input, bounded output.

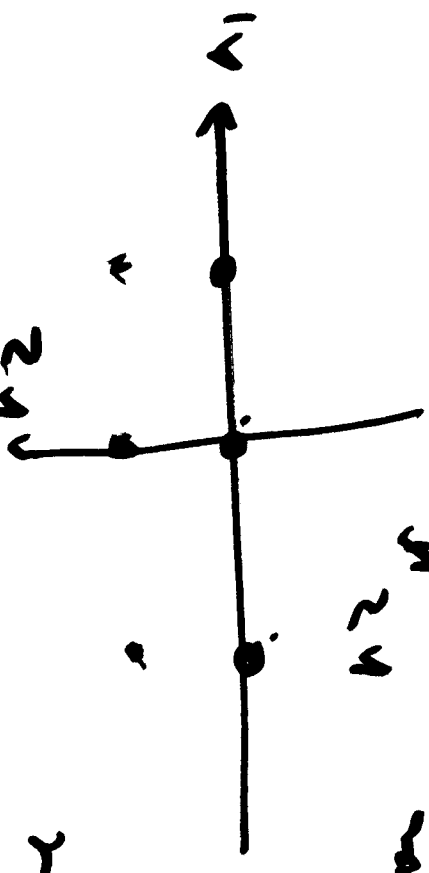
conv. cond.

* Can show, necessary + sufficient stable.

$$h(n_1, n_2) \text{ to be BIBO stable.}$$
$$\sum_{n_1=-\infty}^{+\infty} \sum_{n_2=-\infty}^{+\infty} |h(n_1, n_2)| < \infty$$

Finite impulse response

(FIR) \Rightarrow Always stable.



Infinite impulse response

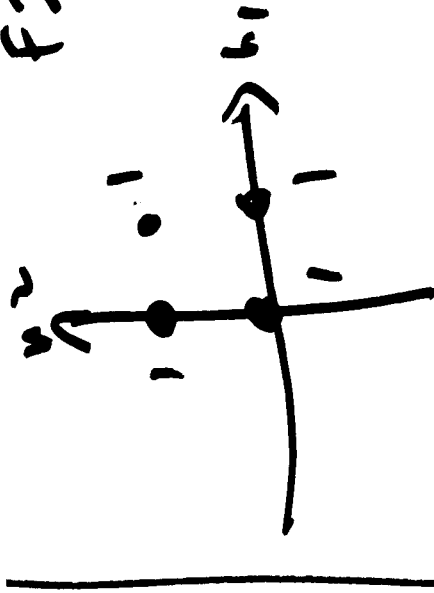
(IIR) some stable, some unstable.



$\sum_{n=-\infty}^{\infty} \frac{1}{n}$ blows up $\sum_{n=1}^{\infty} \frac{1}{n^2}$ does not blow up.

$$\begin{aligned}
 & \sum_{n_1, n_2} h(n_1, n_2) \xrightarrow{\text{Z.T.}} \sum_{n_1, n_2} h(n_1, n_2) z_1^{-n_1} z_2^{-n_2} \\
 & \text{FIR} \Rightarrow = H(z_1, z_2) = \sum_{n_1, n_2} z_1^{-n_1} z_2^{-n_2}
 \end{aligned}$$

FIR



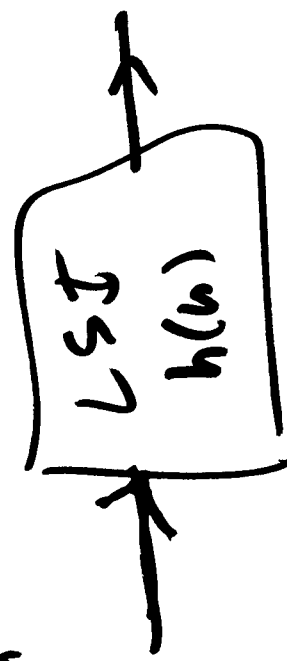
$$H(z_1, z_2) = 1 + z_1^{-1} + z_2^{-1} + z_1^{-1} z_2^{-1}$$

2D polynomial in z_1^{-1} and in z_2^{-1}

2.P.F.I.T.

$$H(\omega_1, \omega_2) = \sum_{n_1} \sum_{n_2} h(n_1, n_2) e^{-j\omega_1 n_1 - j\omega_2 n_2}$$

$e^{j\omega n}$



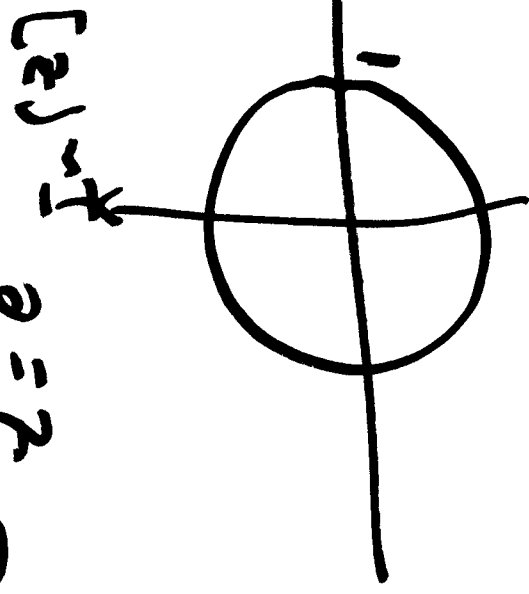
NOT a fr of $n \cdot j\omega n$

$$H(\omega_0) = [H(\omega)]_{\omega=\omega_0} = \sum_{n=-\infty}^{+\infty} h(n) e^{-j\omega n}$$

$$H(\omega) = \sum_{n=-\infty}^{+\infty} h(n) e^{-j\omega n}$$

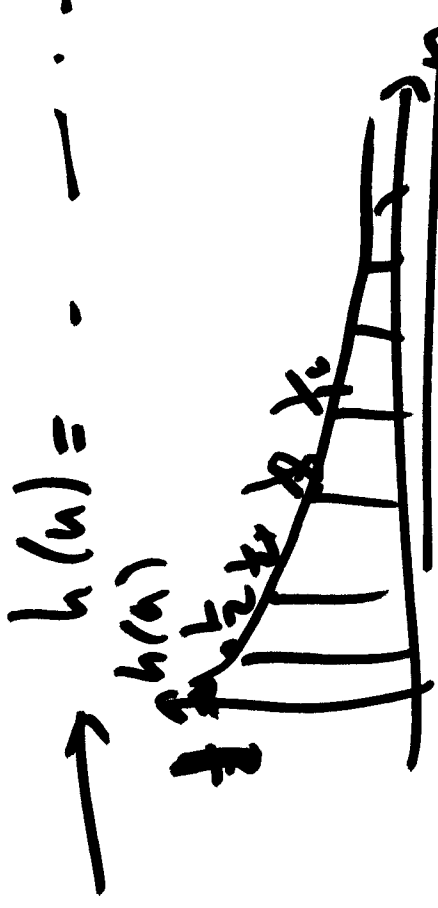
$$H(z) = \sum_{n=-\infty}^{+\infty} h(n) z^{-n}$$

$$H(j\omega) = [H(z)]_{z=e^{j\omega}}$$



$$H(z) = \frac{1}{1 - \frac{1}{2}z^{-1}}$$

ROC: outside circle



$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty$$

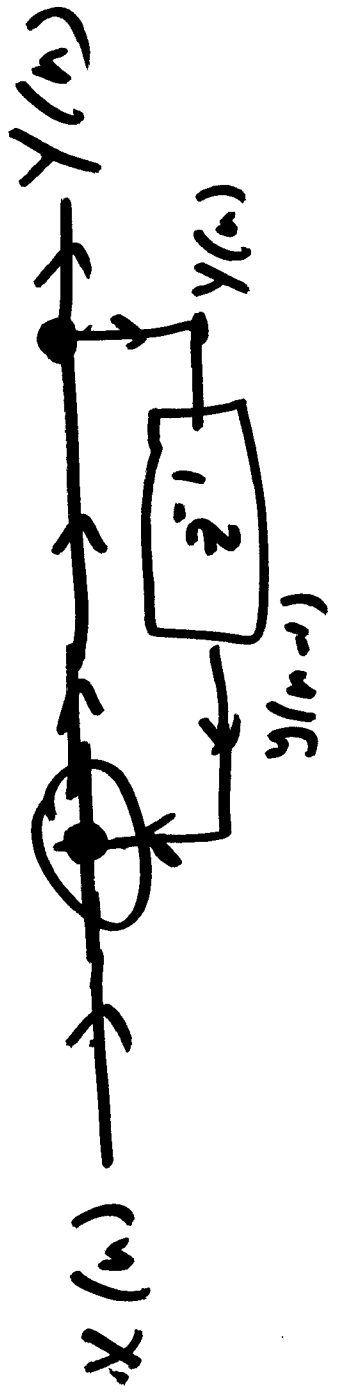
BIBO

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 - \frac{1}{2}z^{-1}}$$

$$Y(z) \left[1 - \frac{1}{2}z^{-1} \right] = X(z)$$

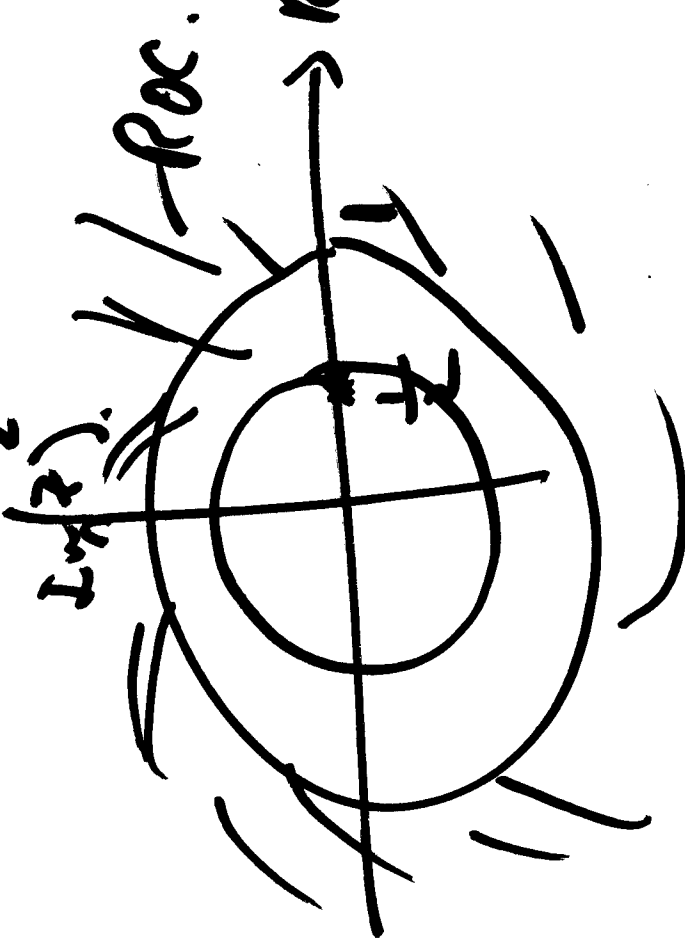
~~$$Y(n) - \frac{1}{2}Y(n-1) = X(n)$$~~

$$Y(n) = X(n) + \frac{1}{2}Y(n-1)$$



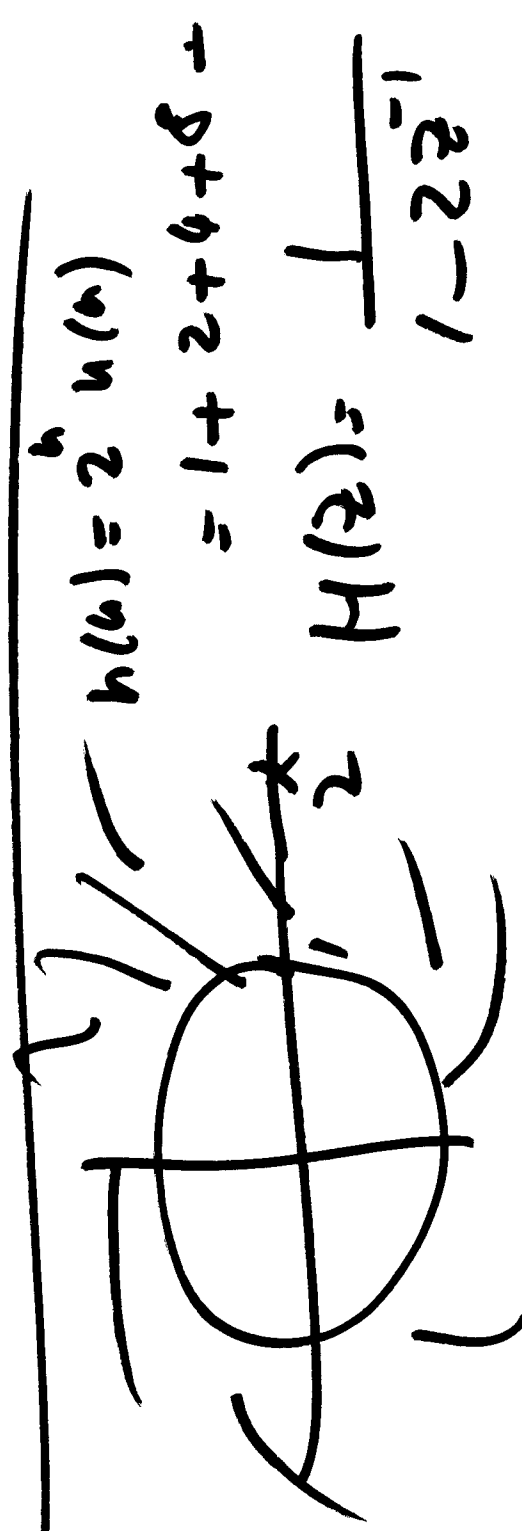
pole $1 - \frac{1}{2}z^{-1} = 0 \Rightarrow z = \frac{1}{2}$

$z = \frac{1}{2} \Rightarrow z = \frac{1}{2}$
 ROC.



$|z| > \frac{1}{2}$
 $h(n) = \frac{1}{2}^n u(n)$

Unstable



$h(n) = 2^n u(n)$
 $= 1 + 2 + 4 + 8 + \dots$
 $H(z) = \frac{1}{1 - 2z^{-1}}$

$h(z) :=$

$$\frac{1}{1 + 5z^{-1} + 3z^{-2} + 9z^{-3} + 12z^{-4} + 18z^{-5}}$$

2-D polynomial can always
BS factored.

→ Fundamental Thm of Algebra.

Any 1-D polynomial ~~can~~ of deg. n
can be factored into n
polynomials of deg. 1.

$$\prod_{i=1}^n (1 - \alpha_i z^{-1})$$

Story in 2D

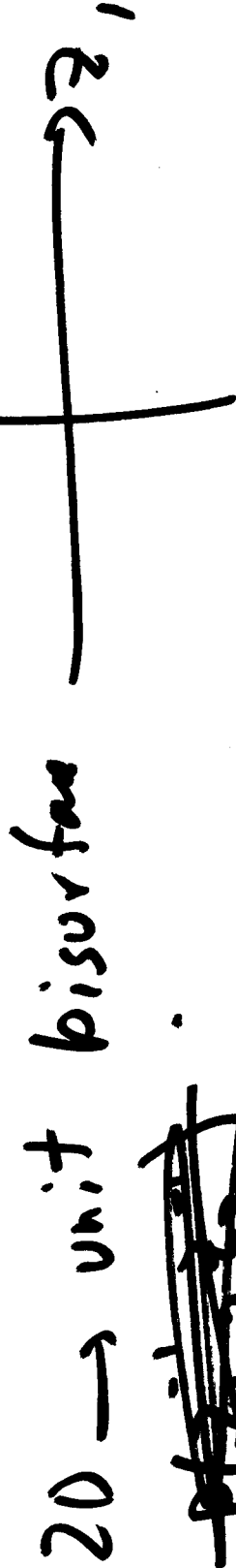
$$\text{IIR: } H(z_1, z_2) = \frac{1}{P(z_1, z_2)}$$

$$P(z_1, z_2) = (1 - z_1^{-1})(1 - z_2^{-1})$$

Thm: Hayn

1980's: \leftarrow

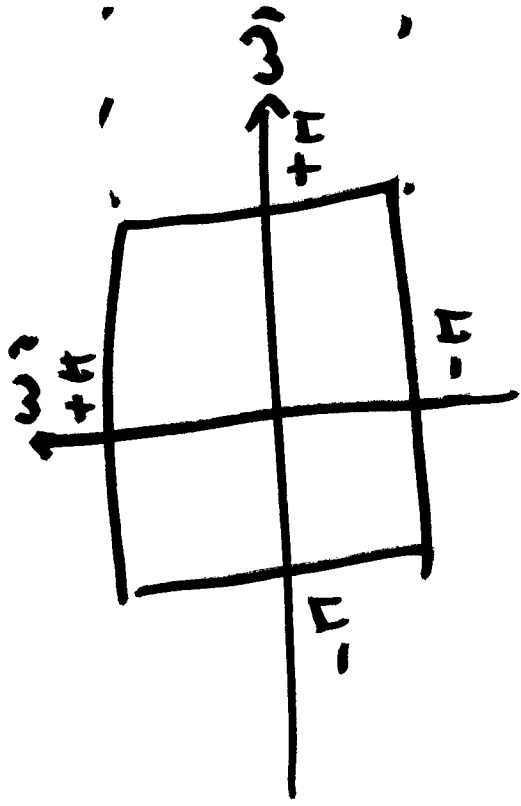
Set of reducible 2D polynomials is set of nonzero 0 in the set of all 2D polynomials.



~~$P(z_1, z_2)$~~

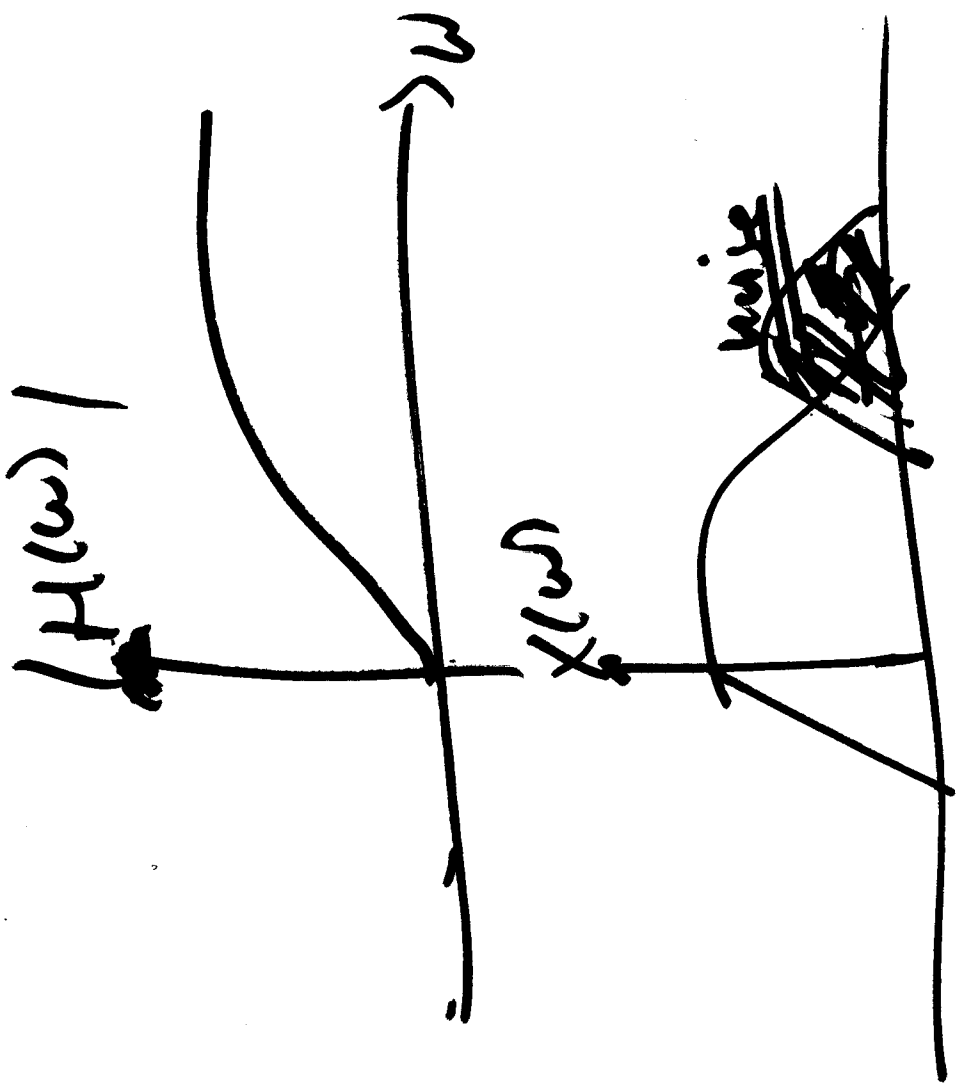
Do the pde surface cross the
unit Bi-surface? \Rightarrow Some things
covered later.

Proving stability in 2D is hard

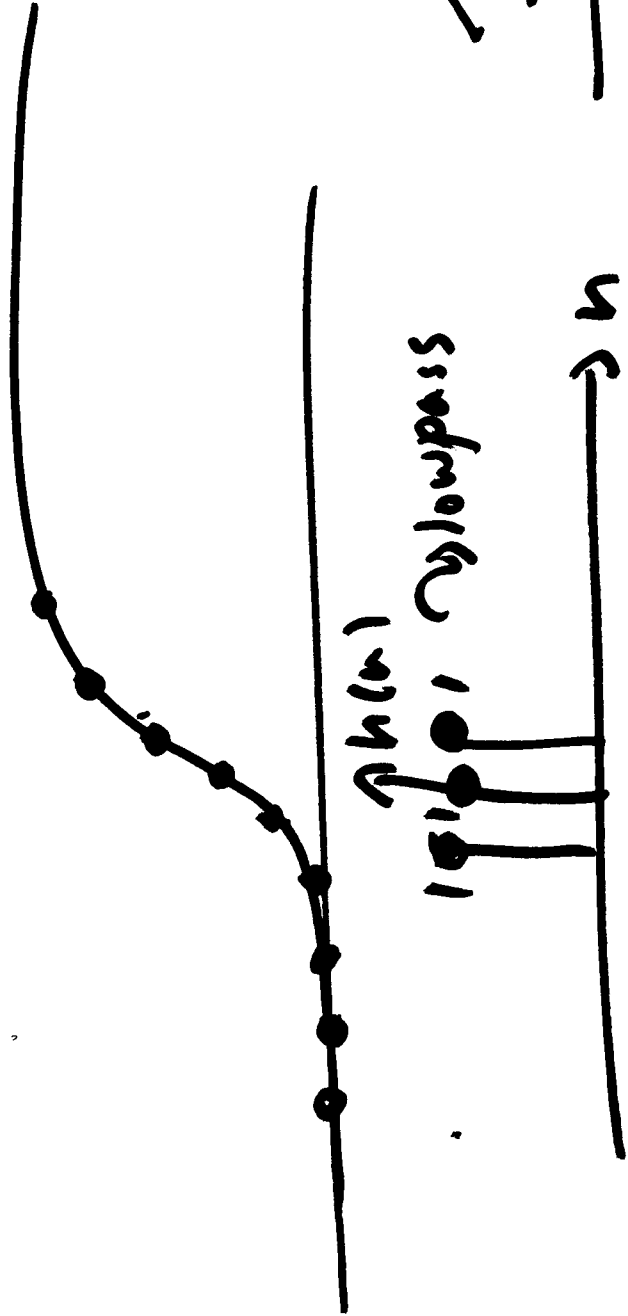


$X(n_1, n_2)$ → discrete integers

$X(\omega_1, \omega_2)$ → Real variables.



Signal.



low pass

