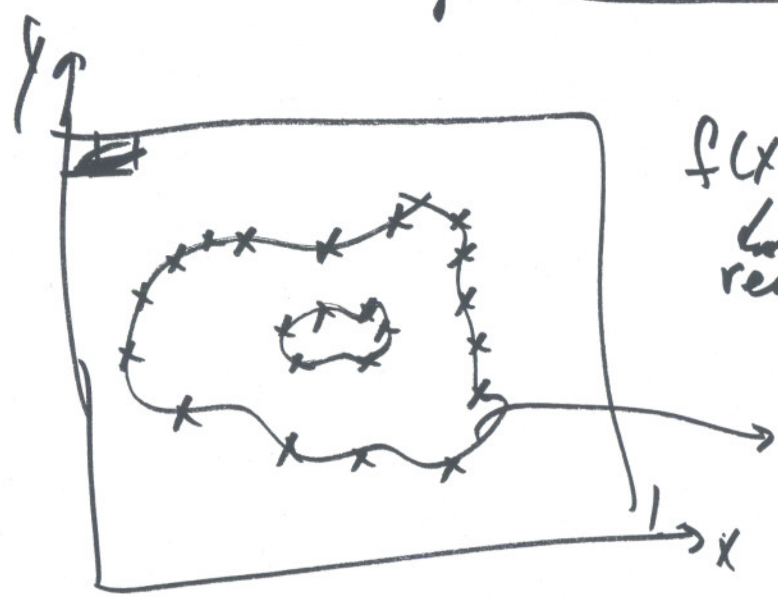
Recon of M-D. Signals from Zero Crossing

Feb 31, 06

Last lecture.. 1 bit of Phase of F.T. of signal $\longrightarrow$ reconstruct it

Curtis & Oppenheim ~ 1983-5



$f(x,y)$ = continuous space 2-D signal

real variables.

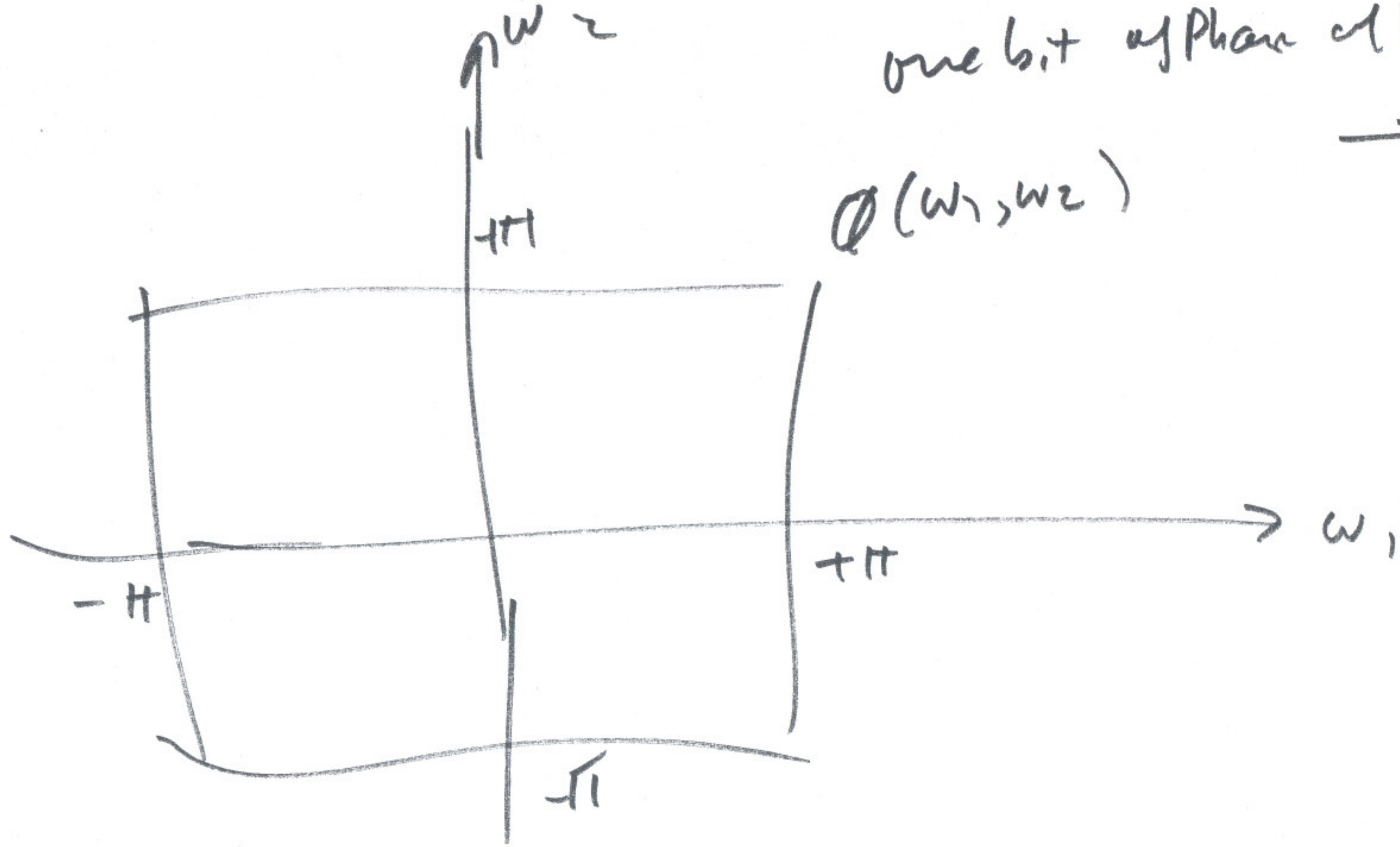
contour represents "Level crossing" of signal

$\longrightarrow$ dual of the problem

$$\underbrace{X(\omega_1, \omega_2)}_{\mathcal{F}_x(n_1,n_2)} = |\quad| \, e^{j\phi(\omega_1,\omega_2)}$$

one bit of Phase of F.T.

→ Van Hove.



$$f(x,y) = \sum_{k_1=-N}^{+N} \sum_{k_2=-N}^{+N} F(k_1,k_2)\, e^{j2\pi k_1 x}\, e^{j2\pi k_2 y}$$

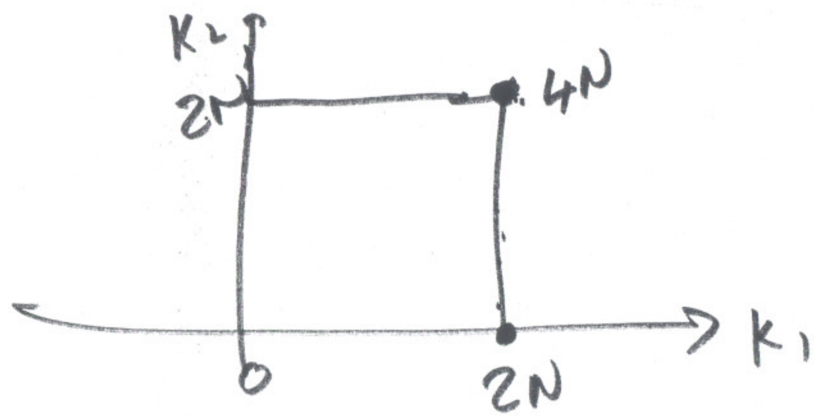
continuous space signal is BLP = Bandlimited & Periodic

Fourier series coefficients.

change of variables $e^{j2\pi x} = W \quad e^{j2\pi y} = Z$

$$f(x,y) = \sum_{k_1=-N}^{+N} \sum_{k_2=-N}^{+N} F(k_1,k_2)\, W^{k_1} Z^{k_2}$$

polynomial of deg $2N$ in W
and deg $2N$ in Z



<u>Bezout's thm</u>: If 2 2D irreducible polynomials of (max deg in w, z) <u>degree</u> r and s, then at most they have rs common zeros.

$\Longrightarrow$ $16N^2$ or more zeros of $f(x,y)$ $\Longrightarrow$ Bezout $\longrightarrow$ uniquely reconstruct $\underline{\underline{it}}$

$\Longrightarrow$ Can $F(K_1, K_2)$ uniquely determined.

Sample $f(x,u)$ at its $16N^2$ zero crossings.

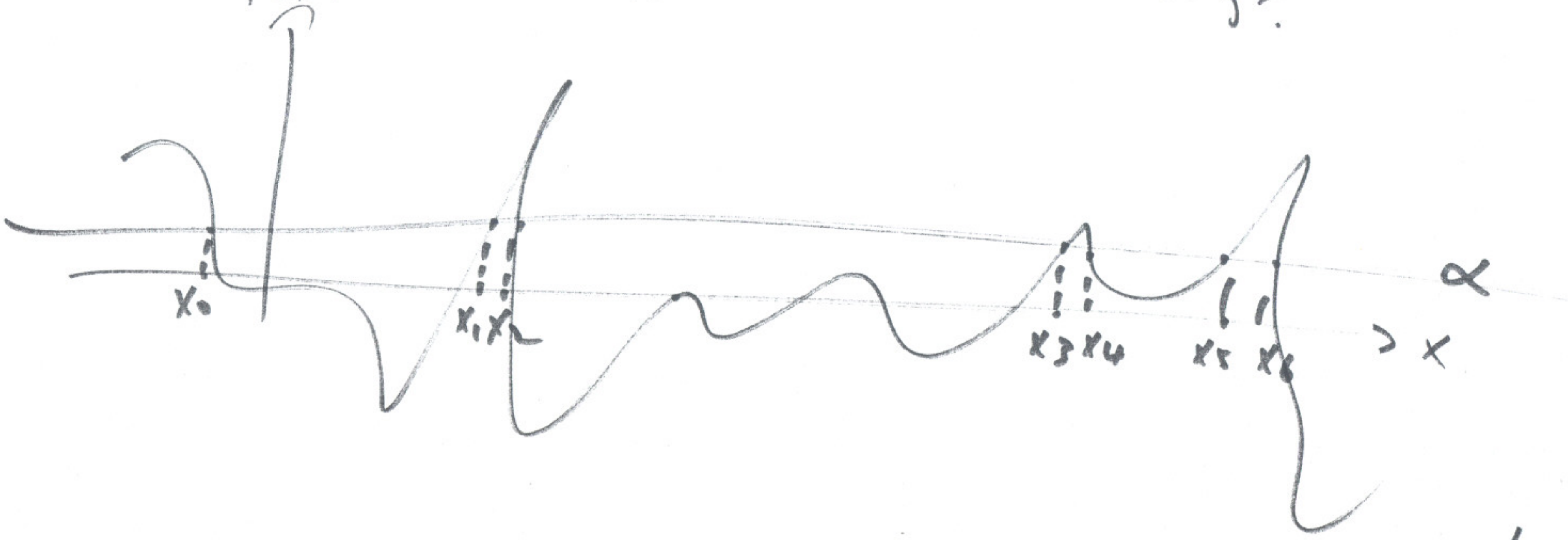
(x_0, y_0)
(x_1, y_1)
(x_2, y_2)
$\vdots$
(x_{16N^2}, y_{16N^2})

level crossing $= \alpha$

$$\underbrace{\begin{bmatrix} \alpha \\ \alpha \\ \alpha \\ \vdots \\ \alpha \end{bmatrix}}_{16N^2} = \begin{array}{c} (x_0,y_0) \rightarrow \\ (x_1,y_1) \rightarrow \\ \\ \\ \end{array} \underbrace{\begin{bmatrix} e^{j2\pi 0 x_0} & e^{j2\pi 0 \cdot y_0} & e^{j2\pi} & \cdots \\ e^{j2\pi 0 \cdot x_1} & e^{j2\pi 0 y_1} & \cdots & \\ \vdots & & & \\ \vdots & & \cdots & \end{bmatrix}}_{4N^2} \begin{bmatrix} F(0,0) \\ F(0,1) \\ \vdots \\ F(2N,2N) \end{bmatrix}$$

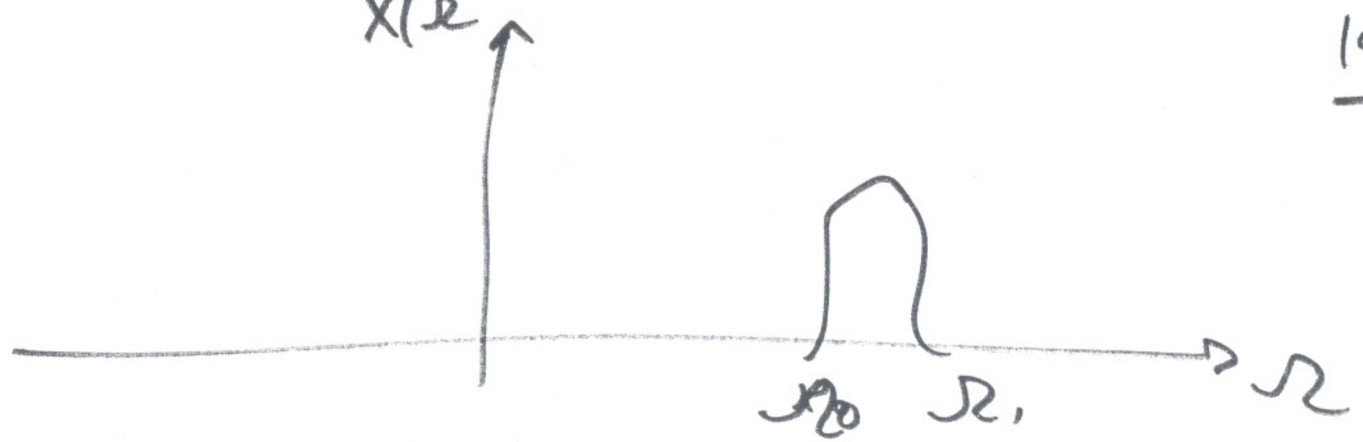
To get reasonable recon $\longrightarrow$
need to know the location of zero crossings
To very high precision
or else $\longrightarrow$ garbage out.
$\Longrightarrow$ Problem is ill conditioned

<u>1D case</u> : Can you recon 2D signal from one level crossing?



__Logan thm__: Band pass 1-D signals that are Band limited under certain condition can be reconstructed from level crossings

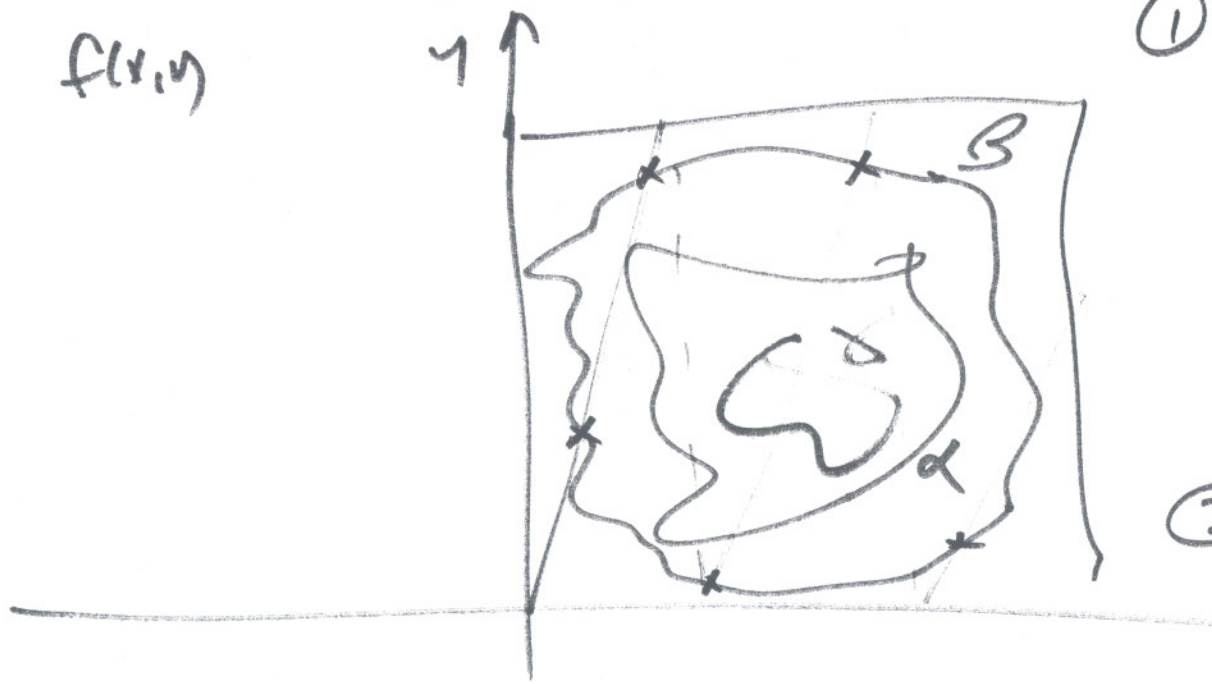
__1950–1960.__



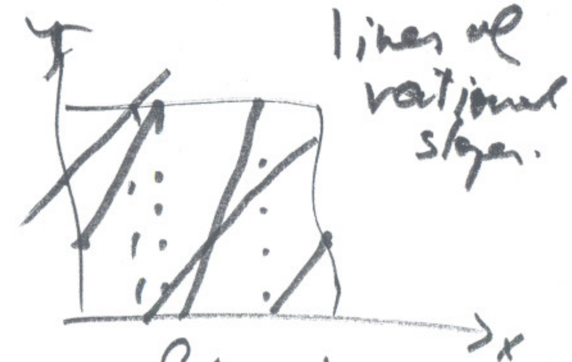
Multiple level crossing

$\Longrightarrow$ relax the precision requirements for the location of the sample

$f(x,y)$



① Semi-implicit.



$f(x,y)$

② Implicit ← Algebraic geometry

Spectrum of sampling techniques

Lots of position precision
No amplitude precision

Tradeoffs
position accuracy
against amplitude accuracy

No position precision
lots of amplitude

Zero crossing
– one-level crossing.

N levels
$\Longrightarrow \log_2 N$ bits
of amplitude.

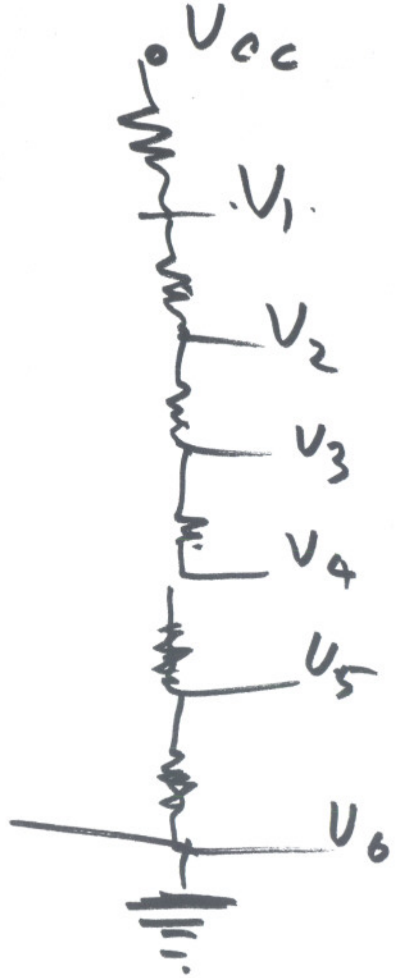
Nyquist

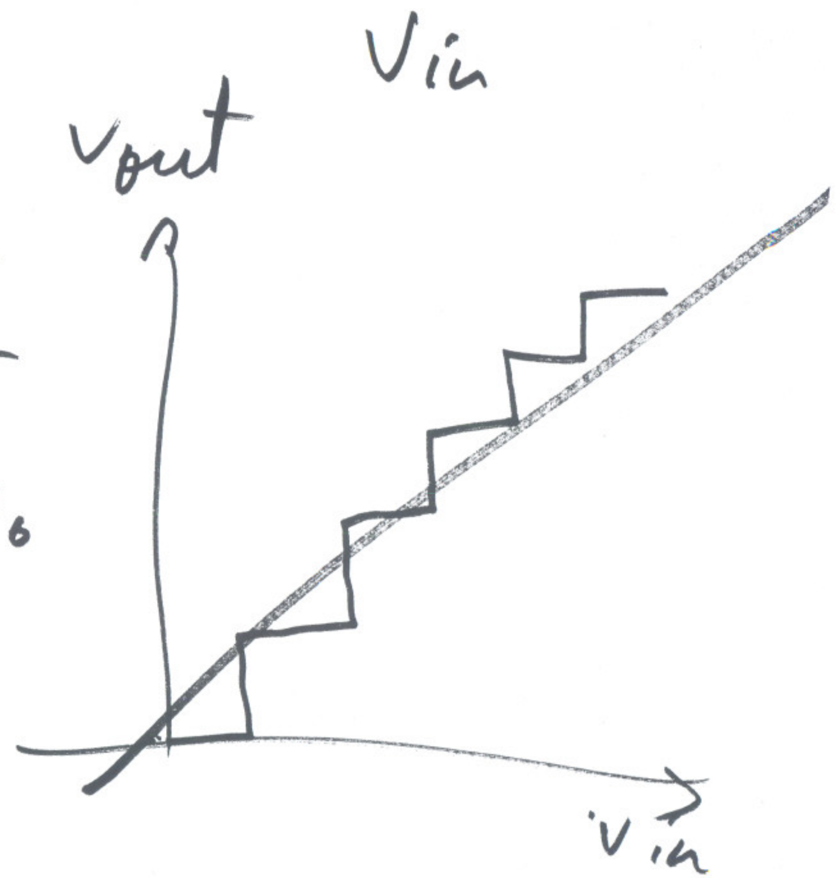
A/D

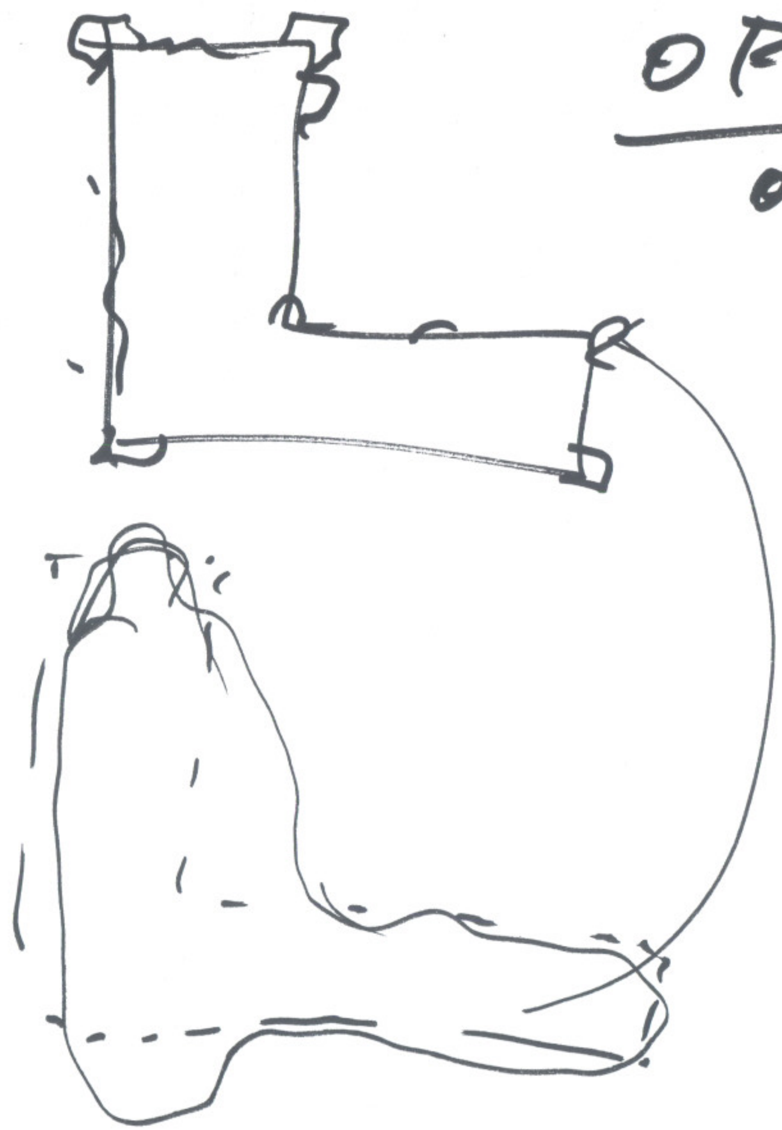
$\Sigma\Delta$:

Tradeoff:
sample rate
against
amplitude

amp : 16 b.
oversample : larger.







ORC
optional pipe
connection

2D Z. Transform

$$X(z_1, z_2) = \sum_{n_1 = -\infty}^{+\infty} \sum_{n_2} x(n_1, n_2)\, z_1^{-n_1}\, z_2^{-n_2}$$

$$\text{2D. DTFT}\{x(n_1, n_2)\} = X(\omega_1, \omega_2) = \Big[X(z_1, z_2) \Big]_{\substack{z_1 = e^{j\omega_1} \\ z_2 = e^{j\omega_2}}}$$

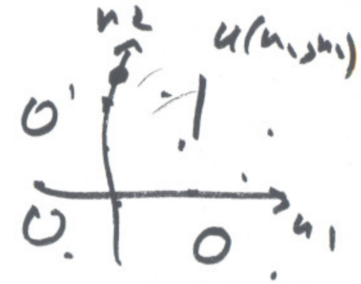
unit bi-circle.

$z = re^{j\omega}$ $\longrightarrow$ In 1-D ROC only depends on r, not ω.

in 2D ROC depends r_1, r_2.

Ex

$x(n_1, n_2) = a^{n_1} \, b^{n_2} \, \underline{u(n_1, n_2)}$



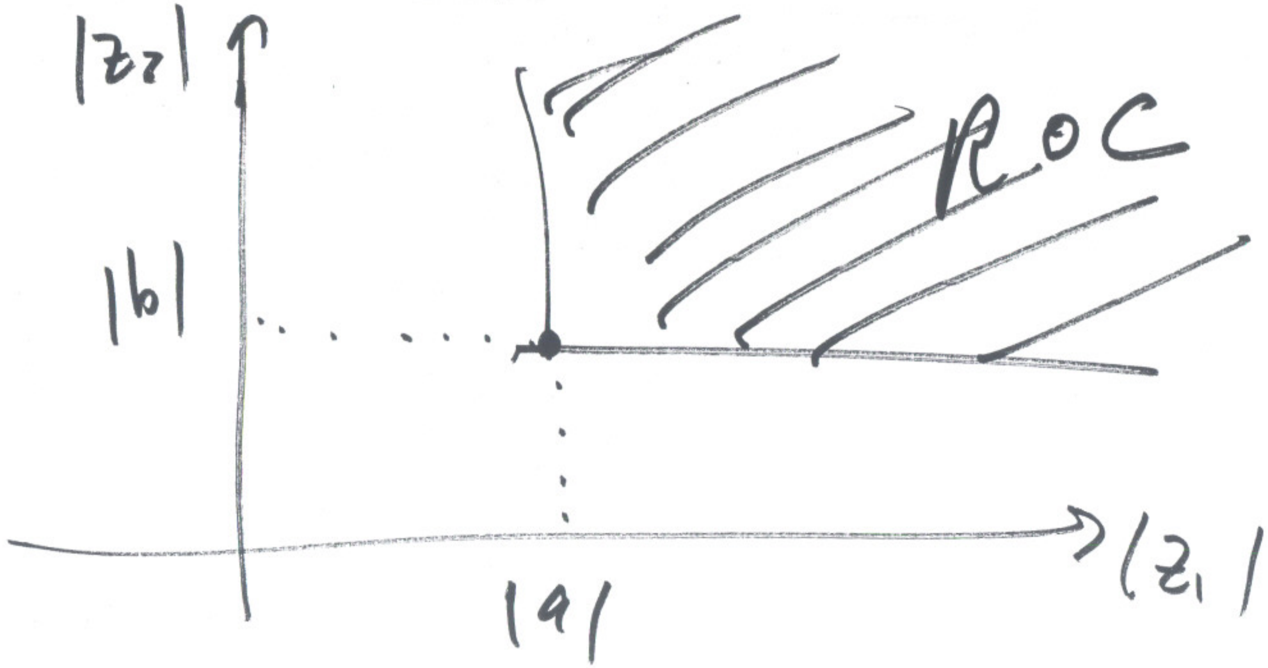
$$X(z_1, z_2) = \sum_{n_1} \sum_{n_2} a^{n_1} \, b^{n_2} \, z_1^{-n_1} \, z_2^{-n_2}$$

$$X(z_1, z_2) = \frac{1}{1 - b z_2^{-1}} \quad \frac{1}{1 - a z_1^{-1}}$$

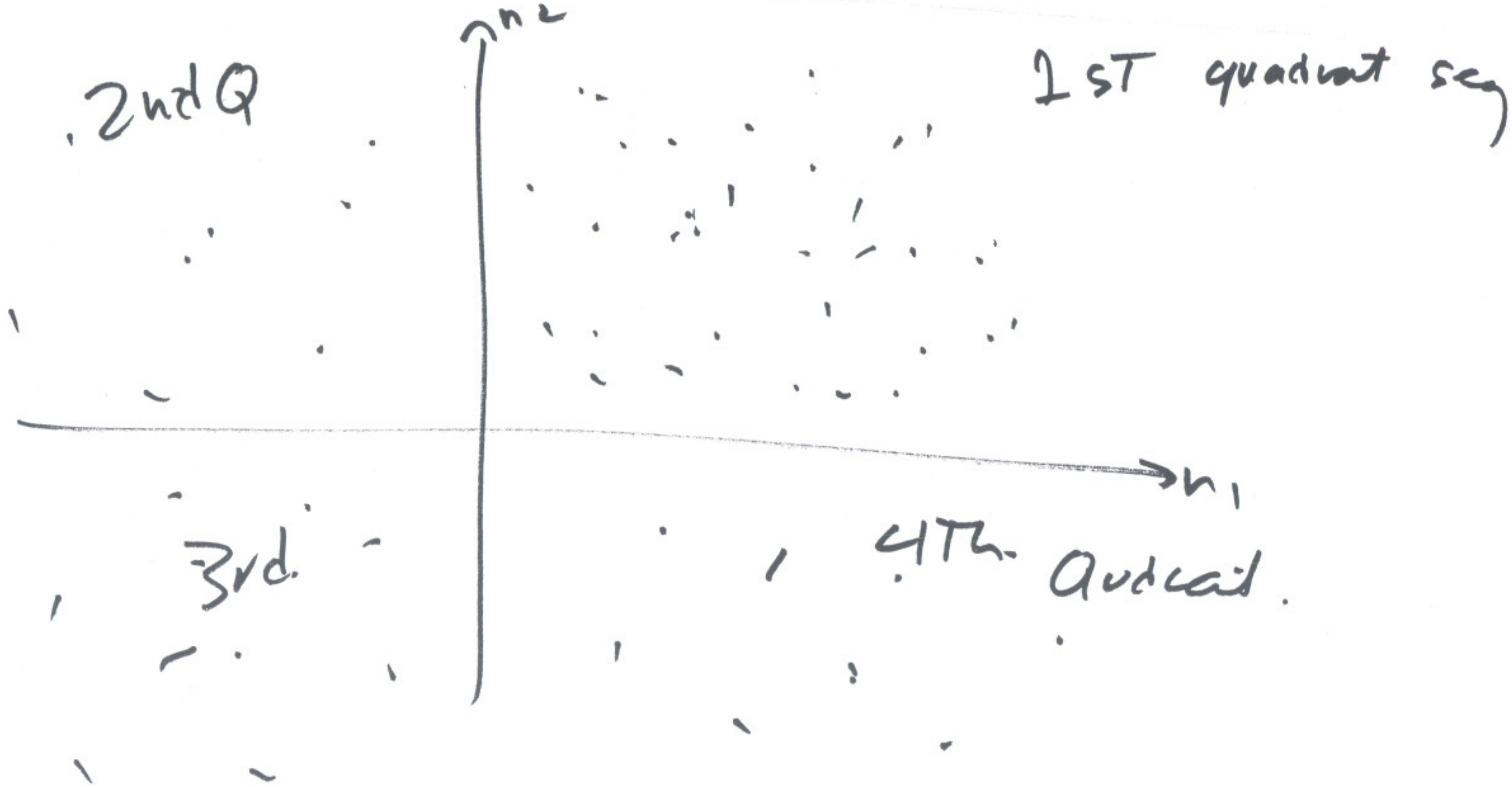
ROC: $\boxed{|z_2| > |b|}$ $\quad \boxed{|z_1| > |a|}$

$$\underbrace{\left(\sum_{n_2=0}^{\infty} b^{n_2} z_2^{-n_2} \right)} \left(\sum_{n_1=0}^{\infty} a^{n_1} z_1^{-n_1} \right)$$

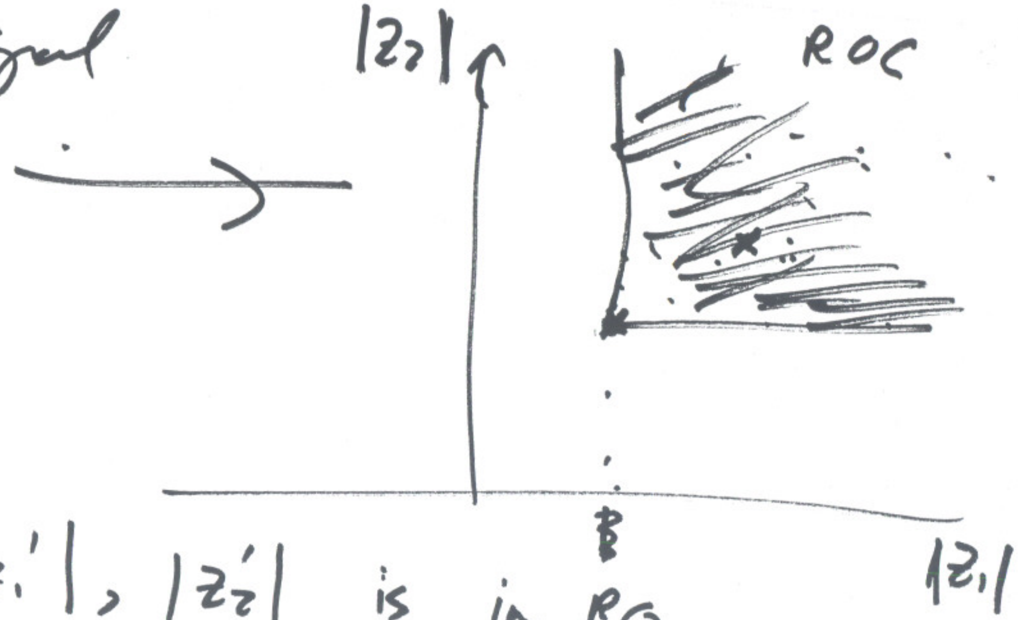
Region converge.



|z2|
|b|
ROC
|a|
|z1|


n2
2nd Q
1ST quadrant
n1
3rd
4Th

~~2~~ If 1st quad signal

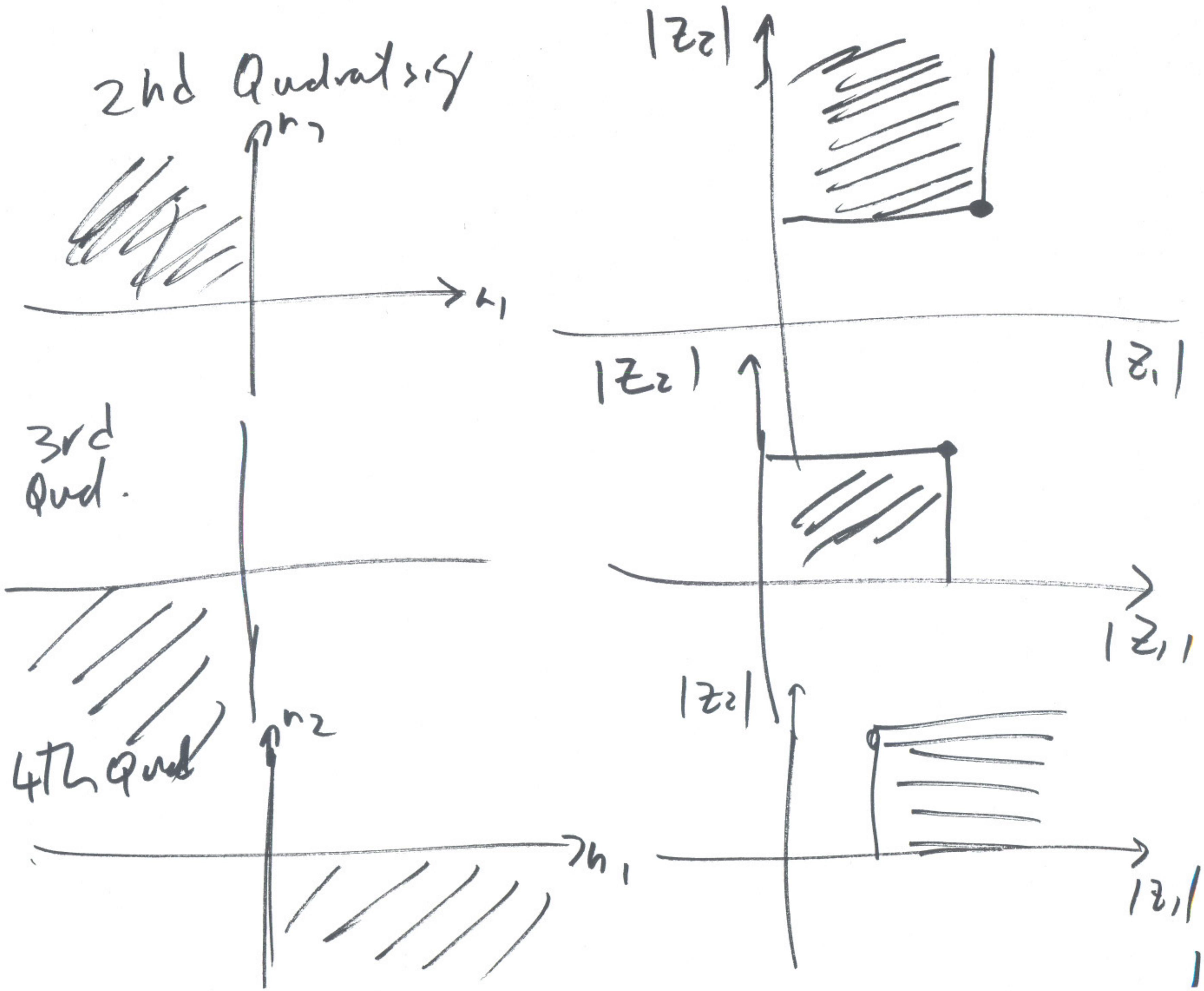


if point $|z_1'|, |z_2'|$ is in RO

Then $\forall$ $|z_1| > |z_1'|$ and

$|z_2| > |z_2'|$ $\Longrightarrow$ will also be in ROC

Ditto: for 2nd, 3rd, 4th Quadrant seq.


2nd
n_2
n_1
3rd
Qud.
4th Qud
n_2
n_1
$|z_2|$
$|z_1|$
$|z_2|$
$|z_1|$
$|z_2|$
$|z_1|$

Ex $x(n_1, n_2) = -a^{n_1} b^{n_2} u(-n_1-1, n_2-1)$