Feb. 20, 2008

Detour into Numerical Analysis

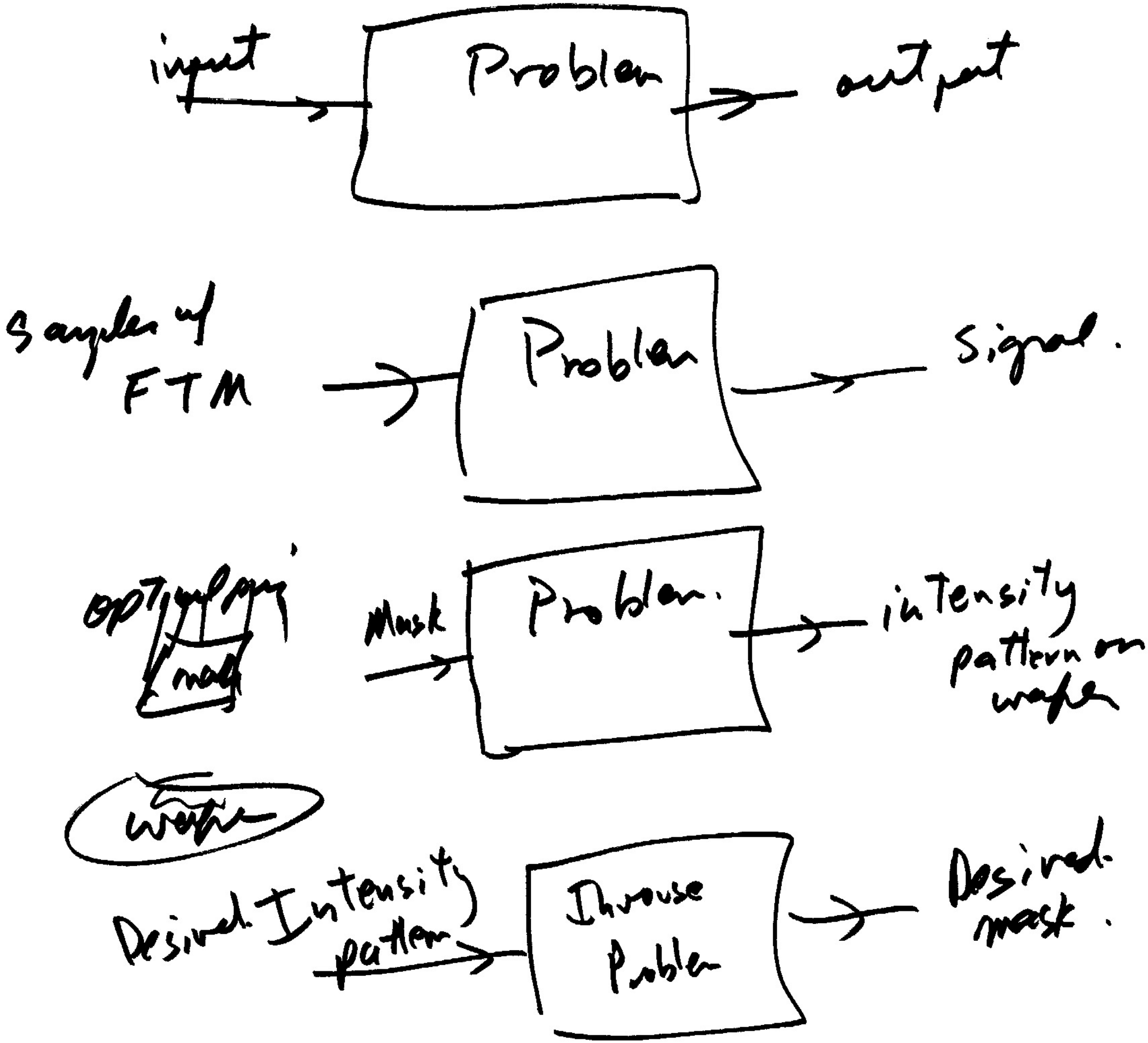
Problems → well Conditioned

Problems → ill Conditioned.

Algorithms applied to problem → Well Condi

Algorithms applied to problem → Ill Conditioned.

Condition # = measure of how "well conditioned" either alg or a problem is.


input
Problem
output
samples of FTM
Problem
Signal.
Optical proj
mask
Mask
Problem.
intensity pattern on wafer
wafer
Desired Intensity pattern
Inverse Problem
Desired mask.

matrix A
vector b $\longrightarrow$ $\boxed{\text{Solving Linear sys of Eqn}}$ $\longrightarrow$ $\vec{X}$ Unknown

$Ax = b$

Initial Condition $\longrightarrow$ $\boxed{\text{weather}}$ $\longrightarrow$ Tomorrow Temp, Fair, ⋮

Prev Temp
sattelite imagery

a, b, c, d $\longrightarrow$ $\boxed{\text{find roots of poly}}$ $\longrightarrow$ roots

$ax^3 + bx^2 + cx + d.$

Algorithm: Method to solve a particular problem.

Ex $A\vec{x} = \vec{b}$

List of possible Algorithm

1. Gaussian Elimination. Ill cond Alg.
2. Compute Inverse Terrible.
3. Gauss Elimination with pivoting good
4. SVD excellent
5. QR Decomposition. excellent
6. Iterative alg. large problem

Condition of Problem

Condition # = $\dfrac{\text{Perturbation in output}}{\text{perturbation in input}}$.

$A, \vec{b}$ $\quad A\vec{x} = \vec{b}$ $\quad \vec{x}$

input $\longrightarrow$ $\boxed{\text{Problem}}$ $\longrightarrow$ output.

Condition # = $\dfrac{\text{relative chg output}}{\text{relative chg input}}$

If condition # is large ($10^6, 10^{10}, 10^{12}$) $\longrightarrow$ problem is ill condit.

" " " " small (~ 1) $\longrightarrow$ problem is well condit.

$$Ax = b$$

$$A \longrightarrow \text{Condition } \# = \frac{\text{largest singular value}}{\text{smallest singular value}}$$

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Condition # of problem depends on the "Intrisic" aspects of the problem itself.

$$\begin{tikzcd}
 & \text{Well condition} \arrow[r, "\text{well Alg}"] \arrow[dr, "\text{Ill}"', "\text{Alg.}"] & \text{good answer} \\
\text{Problem} \arrow[ur] \arrow[dr] & & \text{messed up answer.} \\
 & \text{Ill condition} \arrow[r] & \text{out of luck.}
\end{tikzcd}$$

Condition # of Alg

$$\text{input} \longrightarrow \boxed{\text{Alg}} \longrightarrow \text{output}$$

$$\text{Condition \#} = \frac{\text{relative chnge in output}}{\text{relative chng in input.}}$$

$Ax = b$ $\qquad$ $A =$ very large Condit.

Recon from FTM

- Even Though Theoretically Hayes/McClellan showed uniqueness in practice $\Longrightarrow$ ill conditioned
- 2D polynomials can almost closely be approximated by factors

- Close form $\rightarrow$ Izraelivitz + Lim
- Iterative

Oppenheim + Mersereau $\longrightarrow$ 1972 Proceedings of IEEE

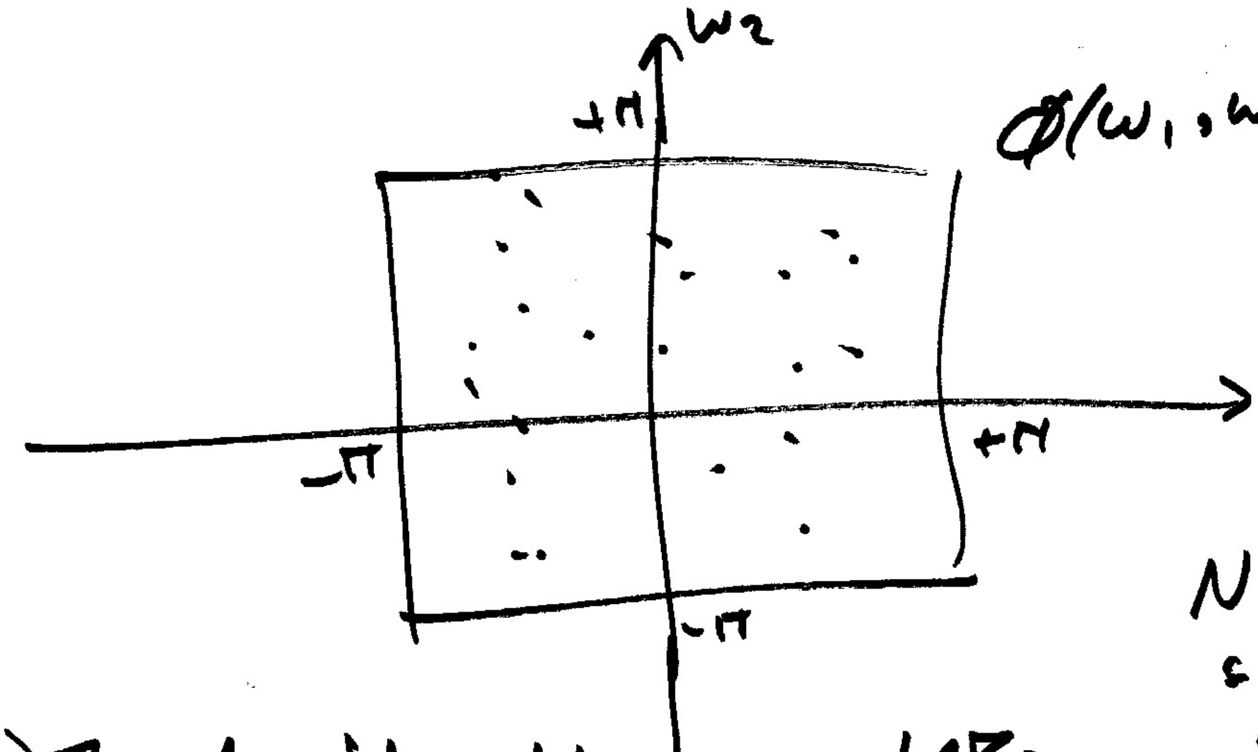
P.S.T.

Recon from F.T. Phase

$x(n_1, n_2) \xrightarrow{\text{D.T.F.T}} X(\omega_1, \omega_2) = | \quad | e^{j \measuredangle \phi}$

$N \times N$

$$X(\omega_1, \omega_2) = \sum_{n_1} \sum_{n_2} x(n_1, n_2) e^{-j\omega_1 n_1} e^{-j\omega_2 n_2}$$



$\omega_1 \rightarrow$ continuous valued variables

Need to have samples of ϕ at more than N^2

Patrick Van Hove $\approx$ 1982

Two Alg. → iterative.
→ direct.

~~the~~ 1982 → Even quantizing phase of F.T. to <u>one</u> bit & yet reconst. signal successful.

Iterative

x = original sigh

$N \times N$ signal.

Know ROS : $N \times N$.

→ samples at $2N \times 2N$ of phase of F.T.

ϕ_x