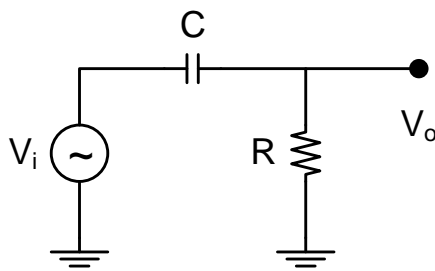


EECS247 Fall 2010 HW1 Due 9/9/10

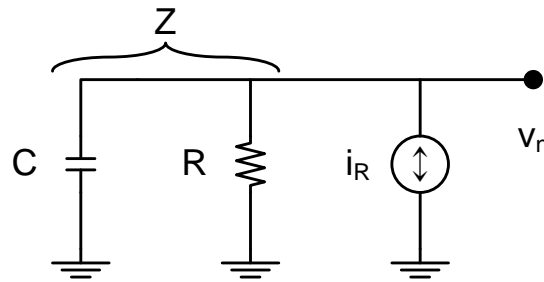
Jaehwa Kwak

P1)

1. 1st order RC highpass filter and its noise model are shown below:



<RC highpass filter>



<Noise model, R is noise source>

Using KCL,

$$Z = R \parallel \frac{1}{sC} = \frac{R}{1 + sRC}$$

$$i_R^2 = 4kT \frac{1}{R} \Delta f$$

$$v_n = i_R Z$$

$$v_n^2 = i_R^2 Z^2 = 4kT \frac{1}{R} \int_0^\infty \left| \frac{R}{1 + sRC} \right|^2 df = 4kT \frac{1}{R} \frac{R^2}{4RC} = \frac{kT}{C}$$

2. Two constraints:

$$\omega_p = \frac{1}{RC} = 2\pi \times 7kHz$$

$$v_n = \sqrt{\frac{kT}{C}} = 15\mu V_{rms}$$

With simple math, we can easily get $kT = 4.14 \times 10^{-21} J$ at 27°C. Therefore,

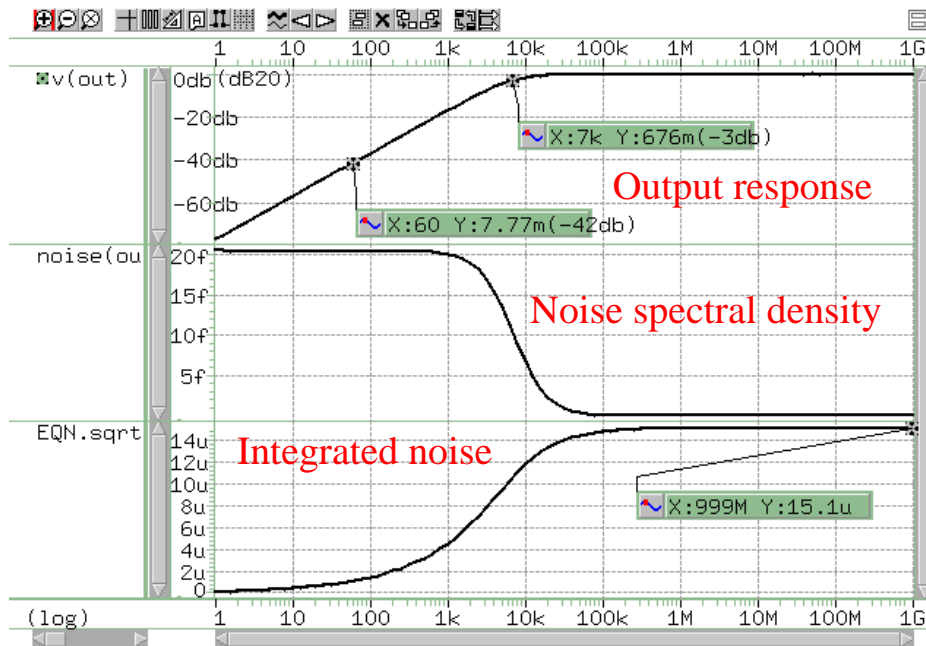
$$v_n = 15\mu V_{rms} = \sqrt{\frac{kT}{C}} = \sqrt{\frac{4.14 \times 10^{-21} J}{C}}$$

$$\therefore C = 18.4 pF$$

$$\omega_p = 2\pi \times 7kHz = \frac{1}{RC} = \frac{1}{R \times 18.4 pF}$$

$$\therefore R = 1.236 M\Omega$$

The plots for the magnitude and the noise are:



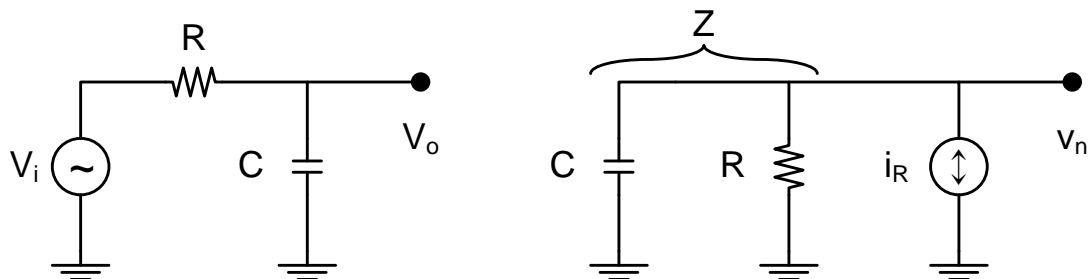
a. The corner frequency is 7kHz as we expect. The attenuation level at 60Hz is -42dB which is not bad. This kind of filter is widely used in many applications such as audio equalizer, DC cancellation, and image processing. I used HPF when I wanted to boost the high pitch sound of my guitar playing.

b. Noise spectral density and the total noise are shown above. The total noise at 1GHz is 15.1uV which is almost same with the constraint. Output logfile also proves that the total noise is about 15uV.

**** total output noise voltage = 15.0728u volts

**** total equivalent input noise = 1.0153m

3. . 1st order RC lowpass filter and its noise model are shown below:



Note that the noise model is same with that of HPF. The filter constraints are:

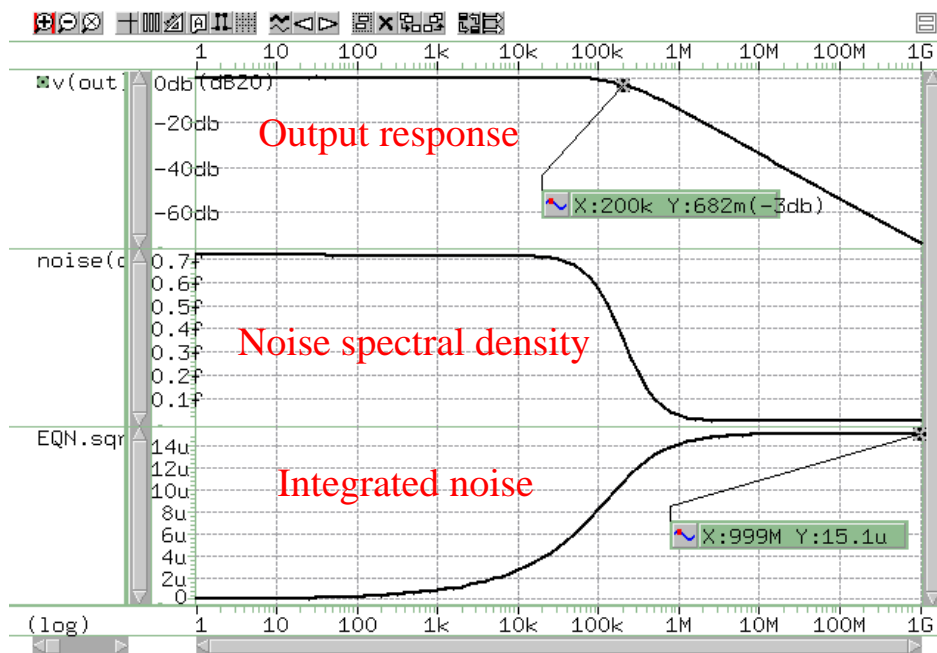
$$v_n = 15\mu V_{rms} = \sqrt{\frac{kT}{C}} = \sqrt{\frac{4.14 \times 10^{-21} J}{C}}$$

$$\therefore C = 18.4 pF$$

$$\omega_p = 2\pi \times 200 kHz = \frac{1}{RC} = \frac{1}{R \times 18.4 pF}$$

$$\therefore R = 43.249 k\Omega$$

The plots for the magnitude and the noise are:

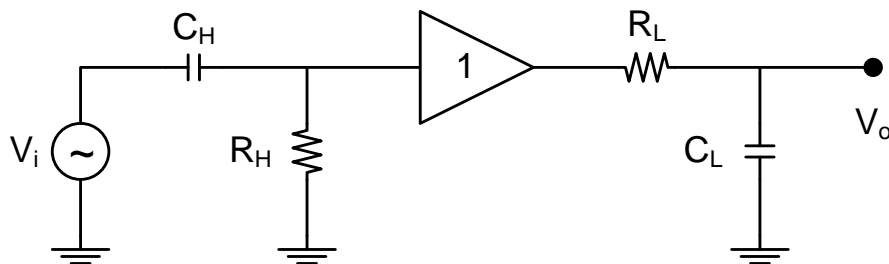


**** total output noise voltage = 15.0725u volts

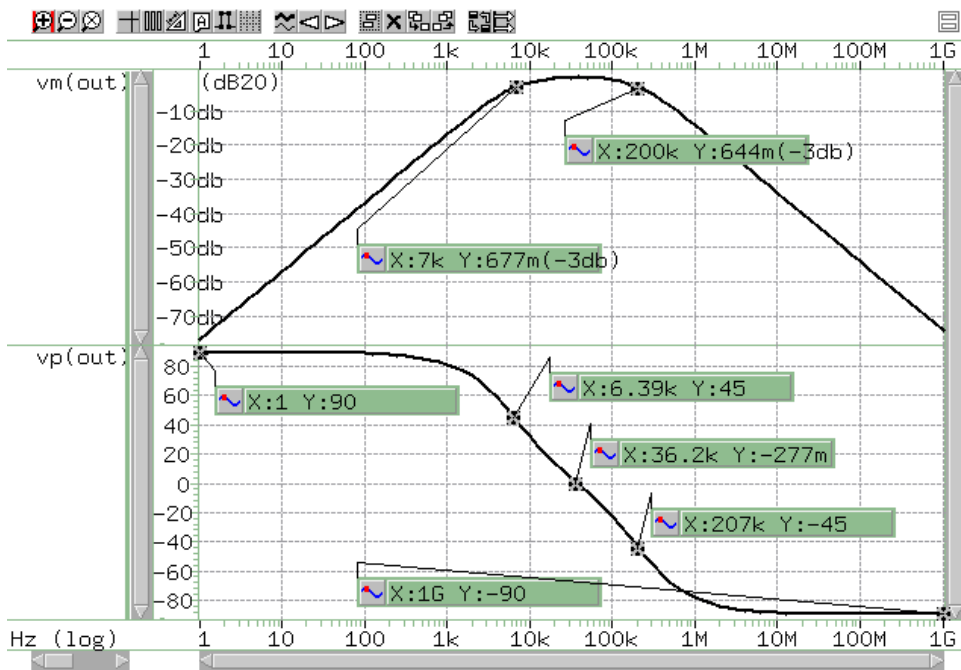
**** total equivalent input noise = 846.6905u

All constraints are met.

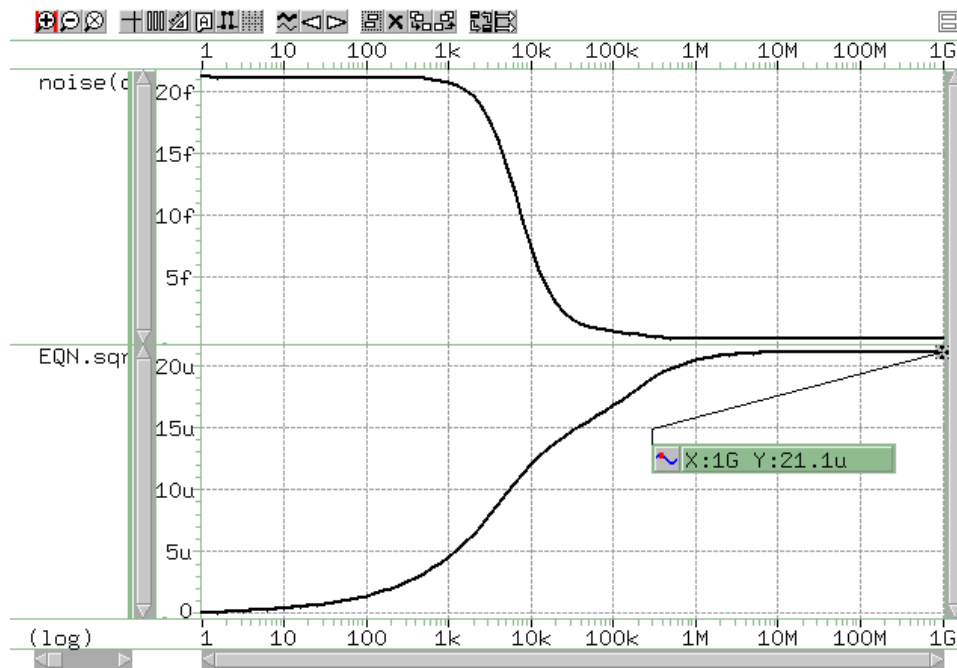
4. The schematic for the combined filter:



The magnitude and phase response plots are shown below. The plots show that the combined filter is a Bandpass filter. The corner frequencies are almost same, but the 45 degree frequencies are slightly moved because two phase responses are affected each other.



5. Noise plots are shown below. The total noise increases because the number of the noise sources increases from 1 to 2.

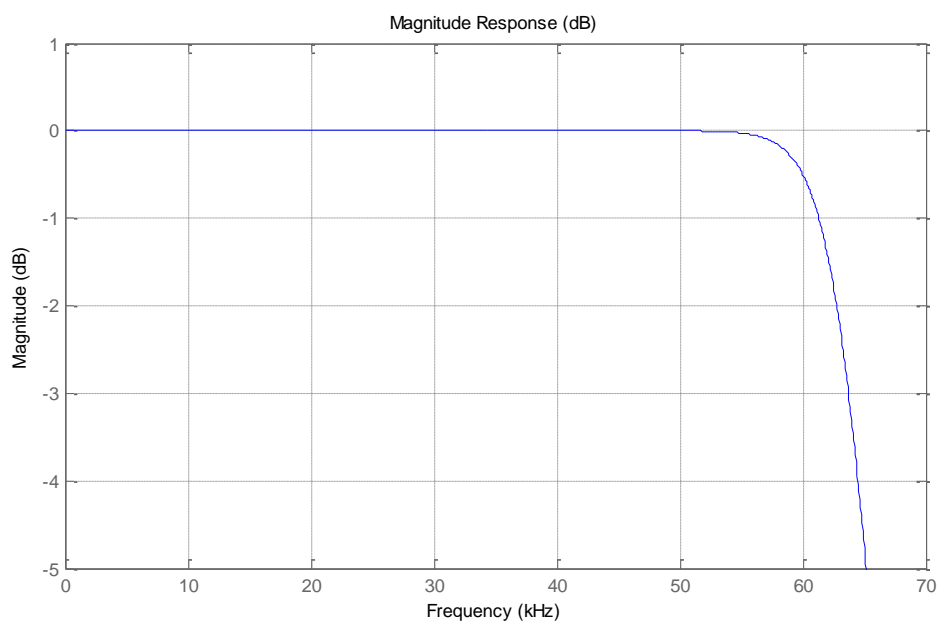
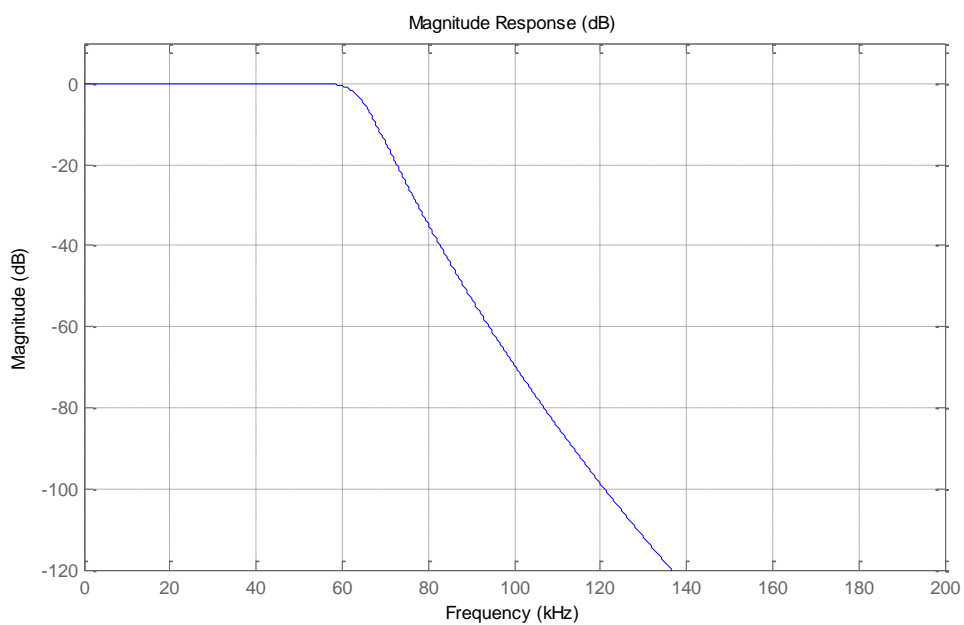


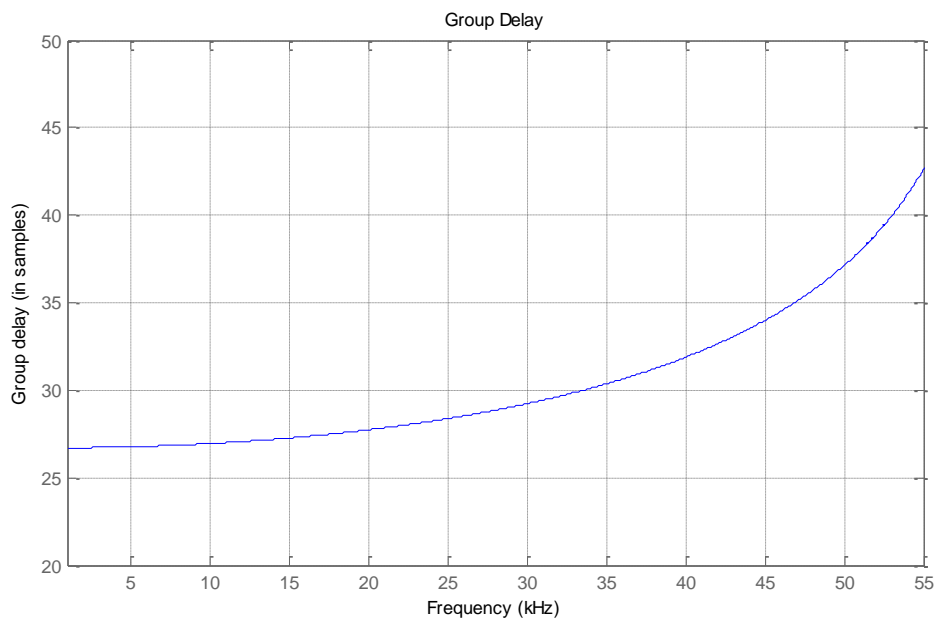
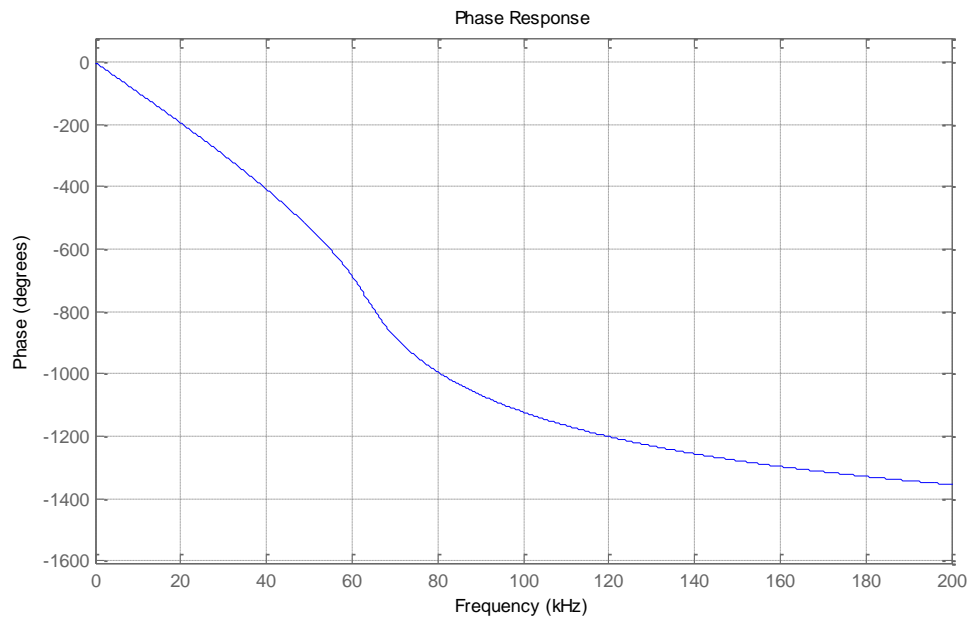
Note added by H.K.: The total noise for the bandpass is $v_n^2 = v_{n1}^2 + v_{n2}^2$ and thus $v_n = 15 \times \sqrt{2}$

P2)

1. Filter plots:

a. Butterworth filter





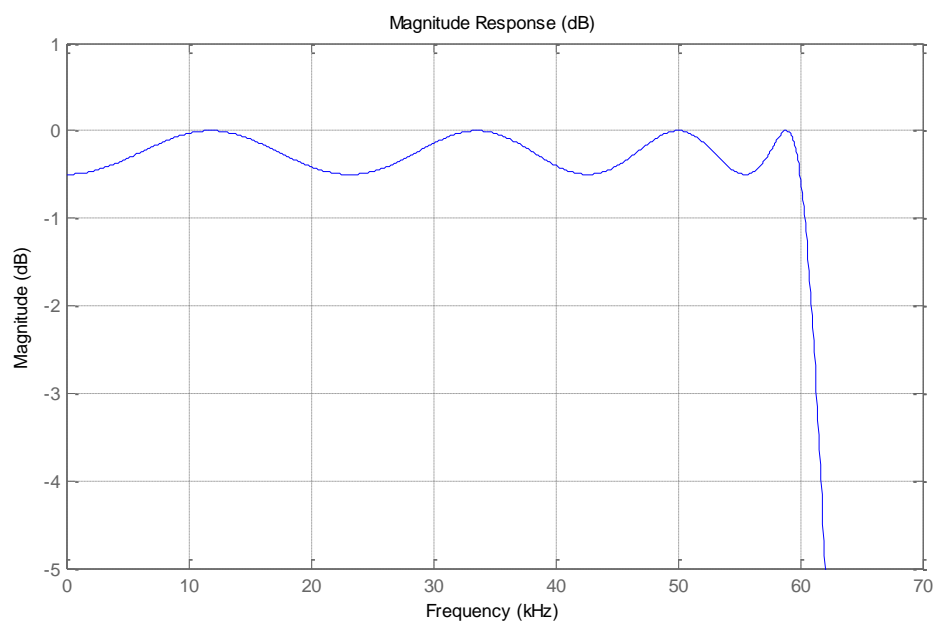
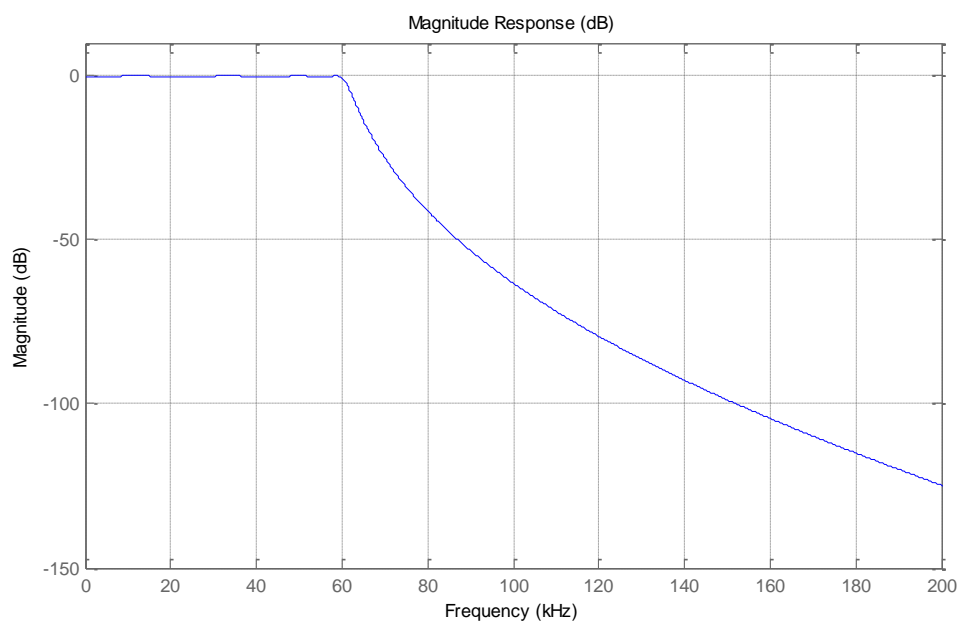
Filter order: 17

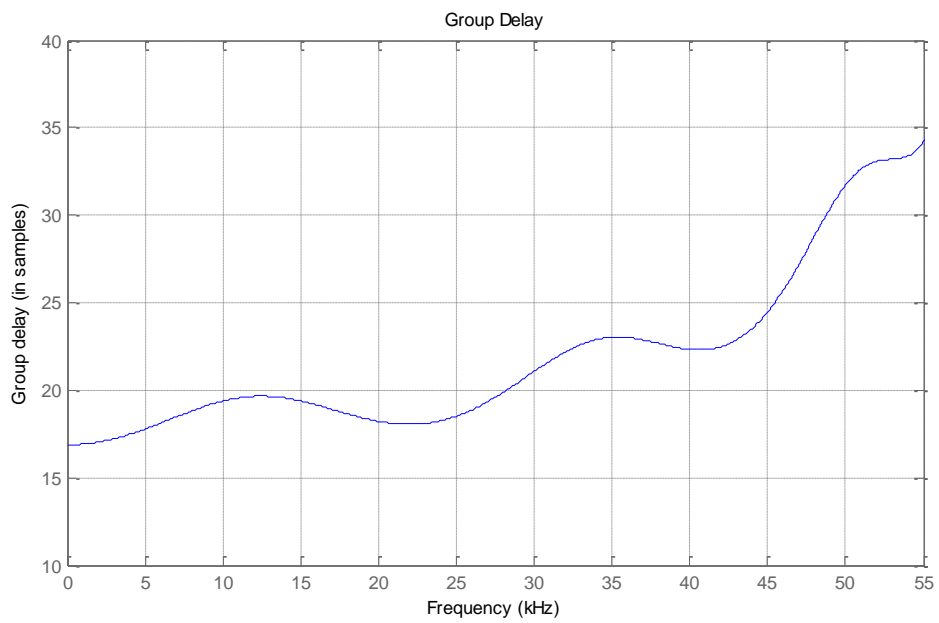
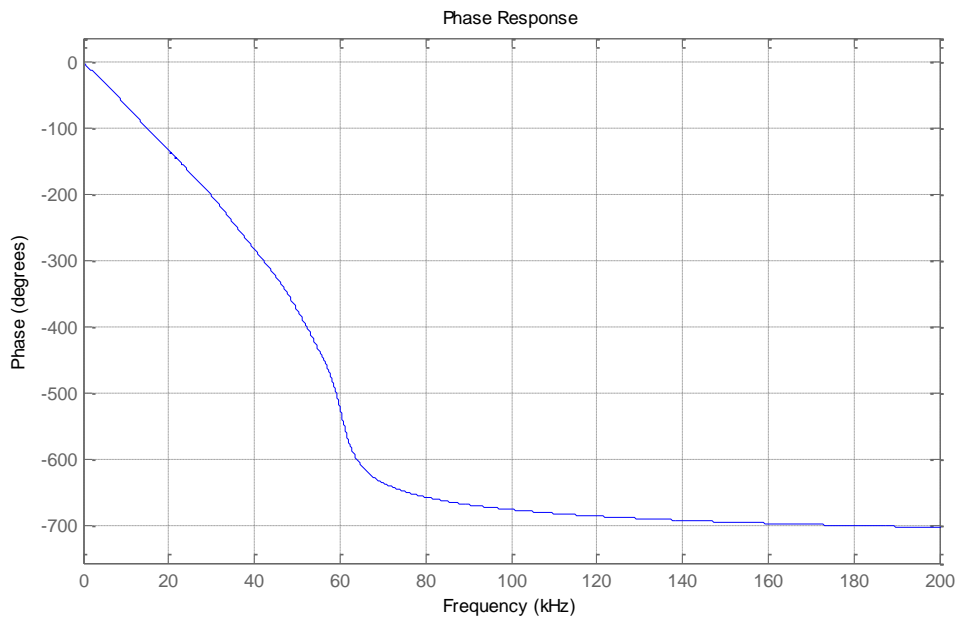
GD_{\min} : 26.705us (@1kHz)

GD_{\max} : 42.614us (@55kHz)

GD_{diff} : 15.909us

b. Chebyshev I filter





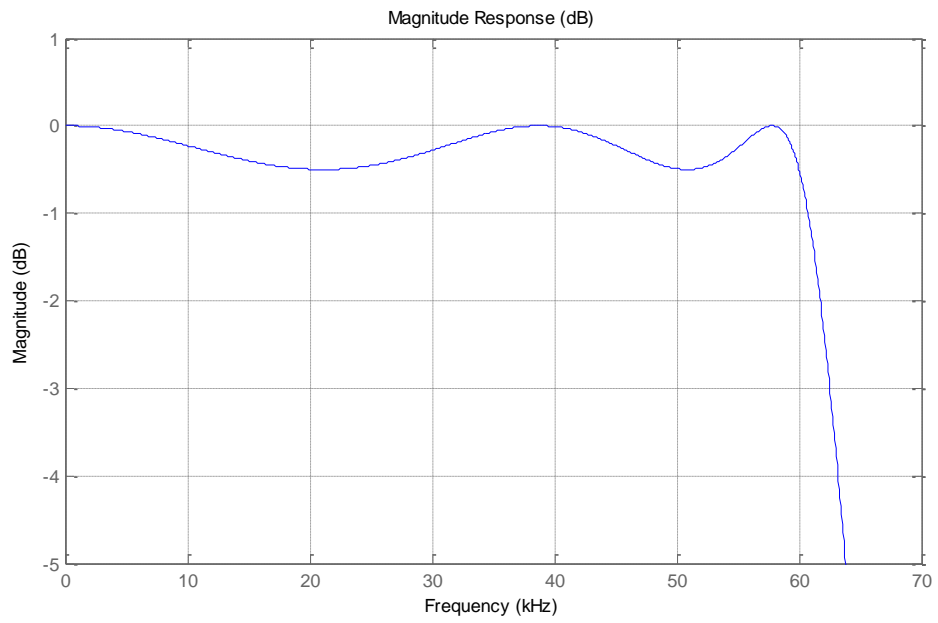
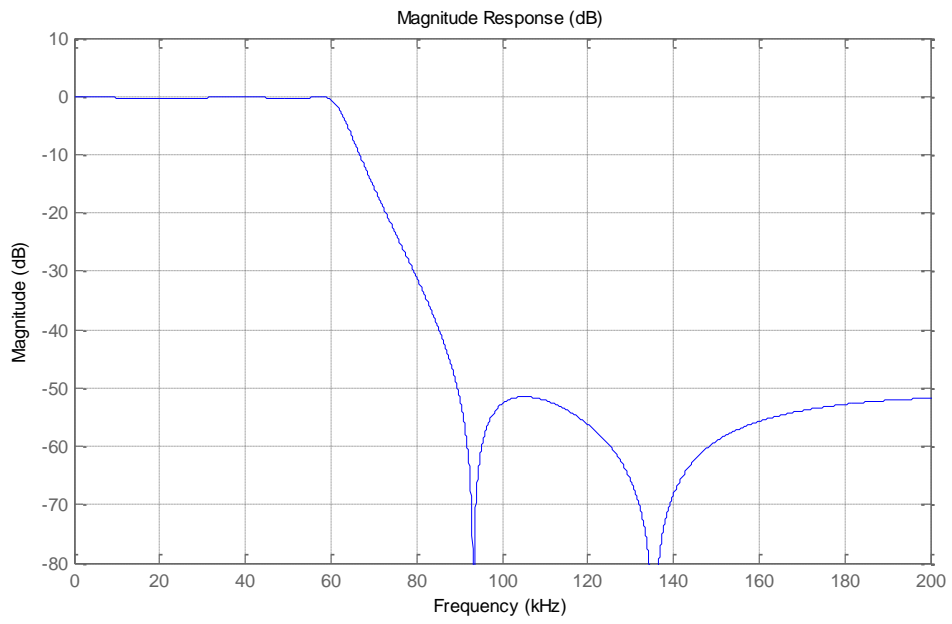
Filter order: 8

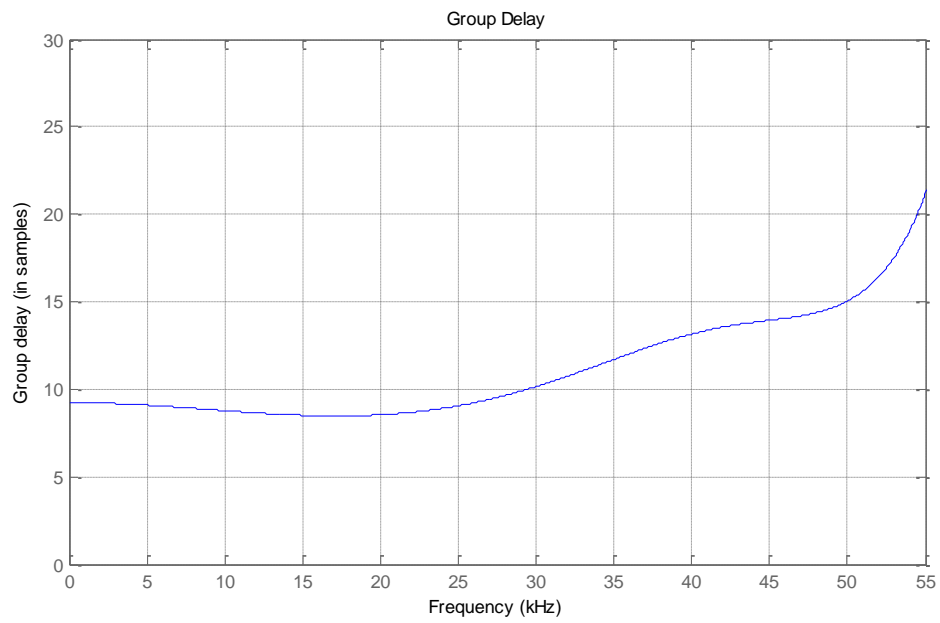
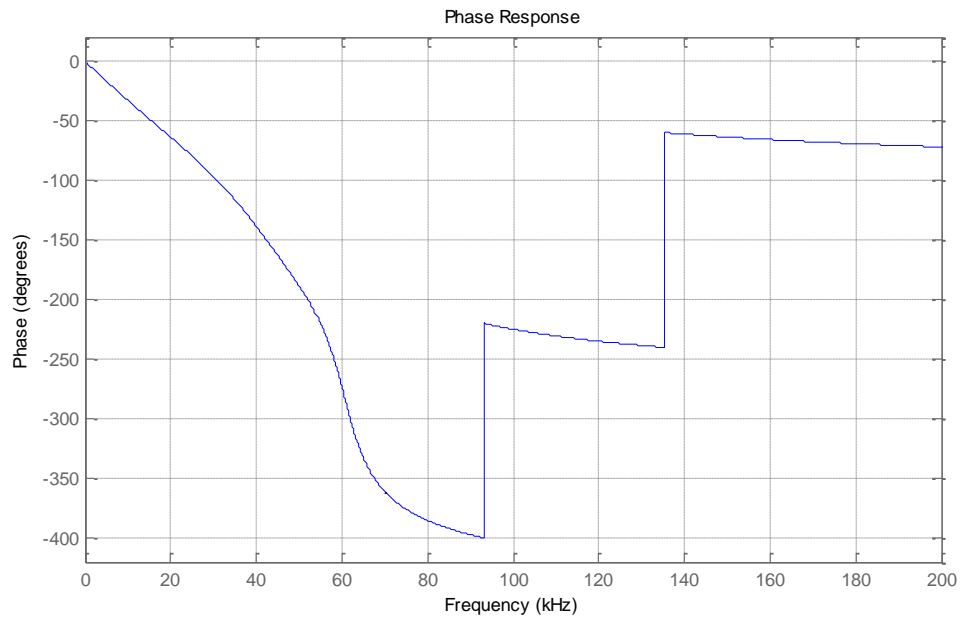
GD_{\min} : 16.916us (@1kHz)

GD_{\max} : 34.205us (@55kHz)

GD_{diff} : 17.289us

c. Elliptic filter





Filter order: 5

GD_{\min} : 8.477us (@17.22kHz)

GD_{\max} : 21.165us (@55kHz)

GD_{diff} : 12.688us

2. Filter summary table

| Filter Type | Filter Order | Group Delay |
|-------------|--------------|-------------|
| Butterworth | 17 | 15.909us |
| Chevyshev I | 8 | 17.289us |
| Elliptic | 5 | 12.688us |

3. Since all the group delay of the filters meet the constraints, I'd like to select the Elliptic filter because it has the smallest filter order.