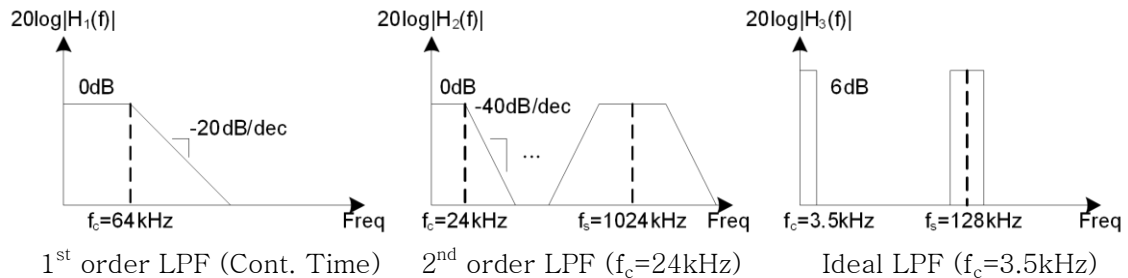


## EECS247 Fall 2010 HW3 Solution

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P1) In this problem, we use that  $|H(f)| = (f_c / f)^n = 20n \log \frac{f_c}{f}$  for  $f > f_c$ . First we draw the magnitude responses for the filters.



Let's calculate the magnitude and the frequency of each signal component after passing the filter.

a) Full configuration

	Signal 1 (wanted)	Signal 2 (unwanted)	Signal 3 (unwanted)	Signal 4 (unwanted)	Signal 5 (unwanted)
$V_i$	$A_1, f_1=2\text{kHz}$	$A_2, f_2=129\text{kHz}$	$A_3, f_3=122\text{kHz}$	$A_4, f_4=893\text{kHz}$	$A_5, f_5=1156\text{kHz}$
X	$A_{1X} = A_1$ $f_1 = 2\text{kHz}$	$A_{2X} = (64/129)A_2$ $= 0.496A_2$ $f_2 = 129\text{kHz}$	$A_{3X} = (64/122)A_3$ $= 0.525A_3$ $f_3 = 122\text{kHz}$	$A_{4X} = (64/893)A_4$ $= 0.072A_4$ $f_4 = 893\text{kHz}$	$A_{5X} = (64/1156)A_5$ $= 0.055A_5$ $f_5 = 1156\text{kHz}$
Y	$A_{1Y} = A_1$ $f_1 = 2\text{kHz}$	$A_{2Y} = (24/129)^2 A_{2X}$ $= 0.0172A_2$ $f_2 = 129\text{kHz}$	$A_{3Y} = (24/122)^2 A_{3X}$ $= 0.0203A_3$ $f_3 = 122\text{kHz}$	$A_{4Y} = (24/131)^2 A_{4X}$ $= 0.0024A_4$ $f_4 = 131\text{kHz}$	$A_{5Y} = (24/132)^2 A_{5X}$ $= 0.0018A_5$ $f_5 = 132\text{kHz}$
Z	$A_{1Z} = 2A_1$ $f_1 = 2\text{kHz}$	$A_{2Z} = 2A_{2Y}$ $= 0.0343A_2$ $f_2 = 1\text{kHz}$	$A_{3Z} = 0$ $f_3 = \text{N/A}$	$A_{4Z} = 2A_{4Y}$ $= 0.0048A_4$ $f_4 = 1\text{kHz}$	$A_{5Z} = 0$ $f_5 = \text{N/A}$

We can find that signal 3 and 5 are eliminated after passing the ideal LPF.

1. The input of the ADC has three components:  $2A_1$ ,  $0.0343A_2$ , and  $0.0048A_4$ . The frequencies are shown above.

2. The peak-to-peak value of the wanted signal is  $2 \times 2A_1 = 0.5 \times 0.512V$ . Therefore,  $A_1$  is 0.128V.

3. The peak-to-peak value of the unwanted signal 2 is  $2 \times 0.0343A_2 = 0.5 \times 0.512V / 256$ . Therefore,  $A_2$  is 0.0146V.

The peak-to-peak value of the unwanted signal 4 is  $2 \times 0.0048A_4 = 0.5 \times 0.512V / 256$ . Therefore,  $A_4$  is 0.1042V.

b) No 1<sup>st</sup> stage configuration

	Signal 1 (wanted)	Signal 2 (unwanted)	Signal 3 (unwanted)	Signal 4 (unwanted)	Signal 5 (unwanted)
Vi	$A_1, f_1=2\text{kHz}$	$A_2, f_2=129\text{kHz}$	$A_3, f_3=122\text{kHz}$	$A_4, f_4=893\text{kHz}$	$A_5, f_5=1156\text{kHz}$
Y	$A_{1Y} = A_1$ $f_1 = 2\text{kHz}$	$A_{2Y} = (24/129)^2 A_2$ $= 0.0346 A_2$ $f_2 = 129\text{kHz}$	$A_{3Y} = (24/122)^2 A_3$ $= 0.0387 A_3$ $f_3 = 122\text{kHz}$	$A_{4Y} = (24/131)^2 A_{4X}$ $= 0.0335 A_2$ $f_4 = 131\text{kHz}$	$A_{5Y} = (24/132)^2 A_{5X}$ $= 0.0331 A_2$ $f_5 = 132\text{kHz}$
Z	$A_{1Z} = 2A_1$ $f_1 = 2\text{kHz}$	$A_{2Z} = 2A_{2Y}$ $= 0.0692 A_2$ $f_2 = 1\text{kHz}$	$A_{3Z} = 0$ $f_3 = \text{N/A}$	$A_{4Z} = 2A_{4Y}$ $= 0.0671 A_4$ $f_2 = 1\text{kHz}$	$A_{5Z} = 0$ $f_5 = \text{N/A}$

We can find that signal 3 and 5 are eliminated after passing the ideal LPF.

1. The input of the ADC has three components:  $2A_1$ ,  $0.0343A_2$ , and  $0.0048A_4$ . The frequencies are shown above.
2. The peak-to-peak value of the wanted signal is  $2 \times 2A_1 = 0.5 \times 0.512V$ . Therefore,  $A_1$  is  $0.128V$ .
3. The peak-to-peak value of the unwanted signal 2 is  $2 \times 0.0692A_2 = 0.5 \times 0.512V / 256$ . Therefore, **A2 is 0.0072V**.  
The peak-to-peak value of the unwanted signal 4 is  $2 \times 0.0671A_2 = 0.5 \times 0.512V / 256$ . Therefore, **A4 is 0.0075V**.

c) No 1<sup>st</sup> stage & 2<sup>nd</sup> stage configuration

	Signal 1 (wanted)	Signal 2 (unwanted)	Signal 3 (unwanted)	Signal 4 (unwanted)	Signal 5 (unwanted)
Vi	$A_1, f_1=2\text{kHz}$	$A_2, f_2=129\text{kHz}$	$A_3, f_3=122\text{kHz}$	$A_4, f_4=893\text{kHz}$	$A_5, f_5=1156\text{kHz}$
Z	$A_{1Z} = 2A_1$ $f_1 = 2\text{kHz}$	$A_{2Z} = 2A_2$ $f_2 = 1\text{kHz}$	$A_{3Z} = 0$ $f_3 = \text{N/A}$	$A_{4Z} = 2A_4$ $f_2 = 1\text{kHz}$	$A_{5Z} = 0$ $f_5 = \text{N/A}$

We can find that signal 3 and 5 are eliminated after passing the ideal LPF.

1. The input of the ADC has three components:  $2A_1$ ,  $0.0343A_2$ , and  $0.0048A_4$ . The frequencies are shown above.
2. The peak-to-peak value of the wanted signal is  $2 \times 2A_1 = 0.5 \times 0.512V$ . Therefore,  $A_1$  is  $0.128V$ .
3. The peak-to-peak value of the unwanted signal 2 is  $2 \times 0.0343A_2 = 0.5 \times 0.512V / 256$ . Therefore, **A2 is 0.25mV**.  
The peak-to-peak value of the unwanted signal 4 is  $2 \times 0.0048A_2 = 0.5 \times 0.512V / 256$ . Therefore, **A4 is 0.25mV**.

d) The wanted signal is same in all case. However, the unwanted signal constraints are different as we remove the pre-LPF before the main LPF. With pre-LPF, the unwanted signals are attenuated. (That is, the peak amplitude for the unwanted signals can be bigger.) Thus we can increase the dynamic range by using the LPF before the main LPF.

P2) Let's say that  $\frac{V_{in}}{2V_T} = x$  for convenience. Then, we can expand,

$$\Delta I = I_{Tail} \times \tanh \frac{V_{in}}{2V_T} = I_{Tail} \times \tanh x = I_{Tail} \times \left(x - \frac{x^3}{3}\right)$$

Since HD3 and IM3 are the function of the ratio of the 1<sup>st</sup> and 3<sup>rd</sup> amplitude, we can use I instead of V.

a. For HD3,

$$HD3 = \frac{1}{4} \left| \frac{-\frac{1}{3} I_{Tail} x^3}{I_{Tail} x} \right| = \frac{1}{12} x^2 = 2\% = 0.02 \rightarrow x^2 = 0.24 \rightarrow x = \frac{V_{in}}{2V_T} = 0.4899$$

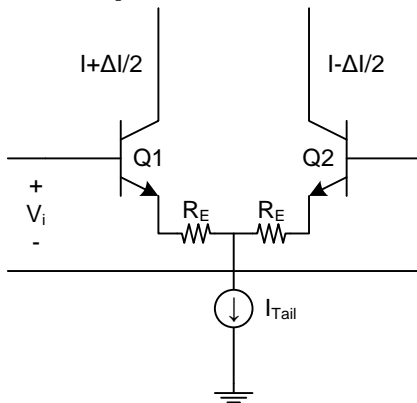
Therefore,  $V_{in} = 25.475mV$ .

b. For IM3,

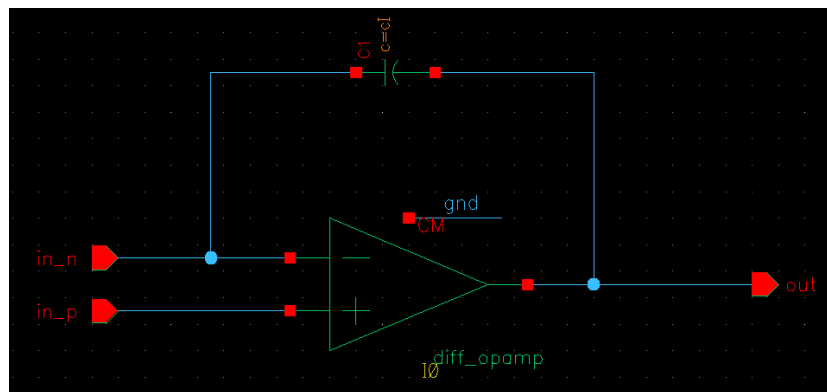
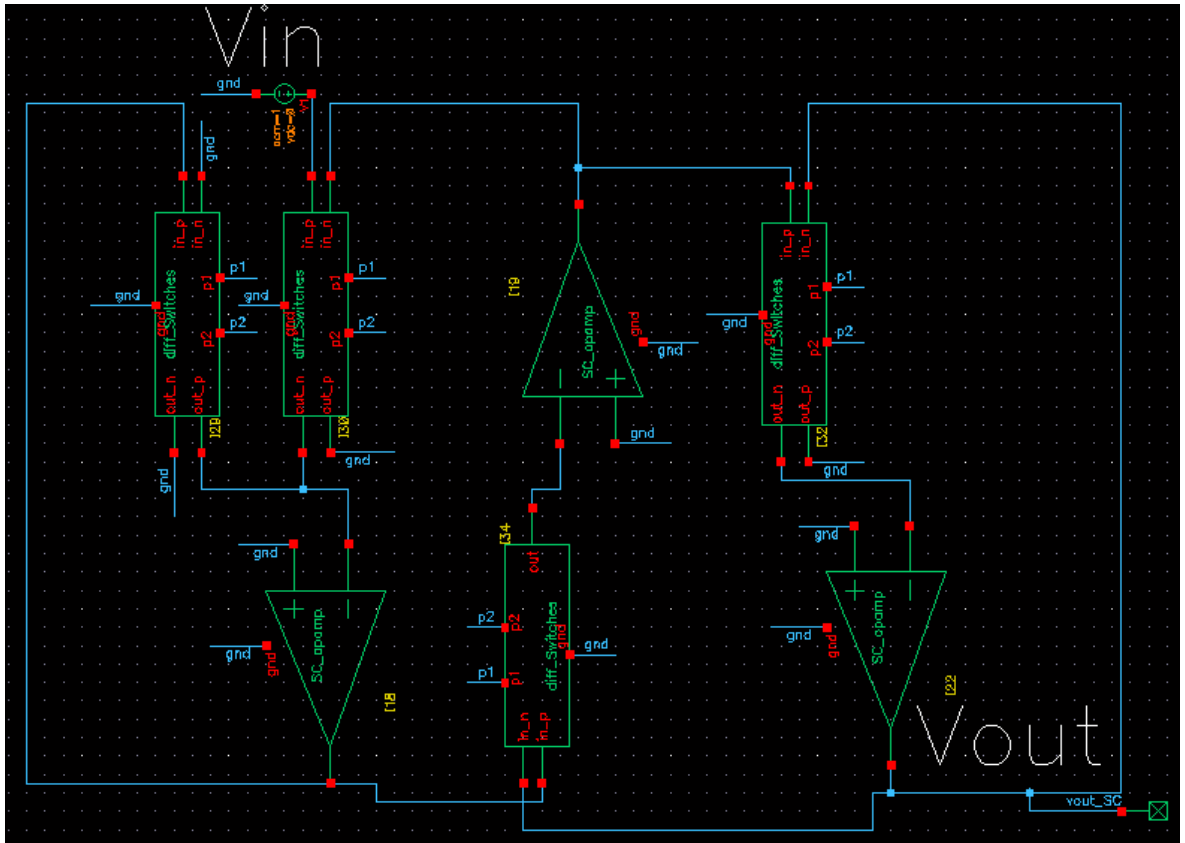
$$IM3 = \frac{3}{4} \left| \frac{-\frac{1}{3} I_{Tail} x^3}{I_{Tail} x} \right| = \frac{3}{4} x^2 = 1\% = 0.01 \rightarrow x^2 = 0.04 \rightarrow x = \frac{V_{in}}{2V_T} = 0.2$$

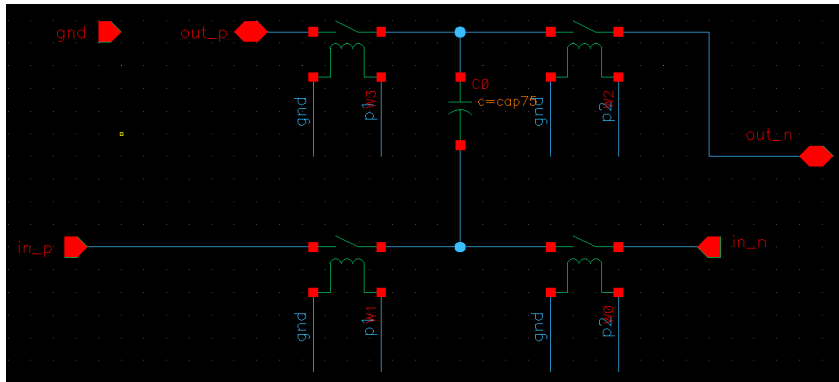
Therefore,  $V_{in} = 10.4mV$ .

c. If we add the emitter-degeneration resistors, the input swing can be bigger with same output current difference.



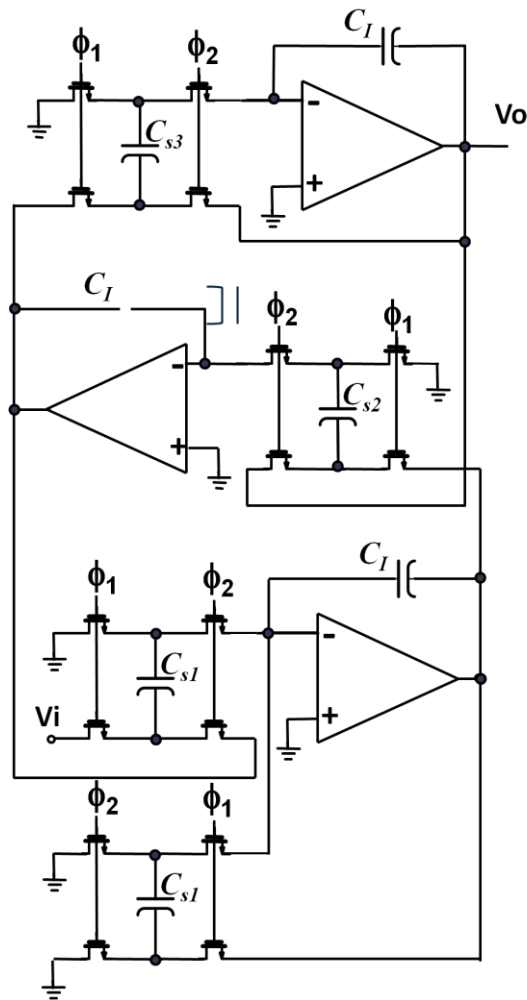
- a) For the schematic, I attach below the schematic of the overall filter and the detail for the two building blocks: the opamp with integrating capacitance and the input switches that behave like a SC resistor. Note that **p1** and **p2** in the schematic refer to phase 1 and phase 2 respectively.





Added by H.K.

The schematic is:



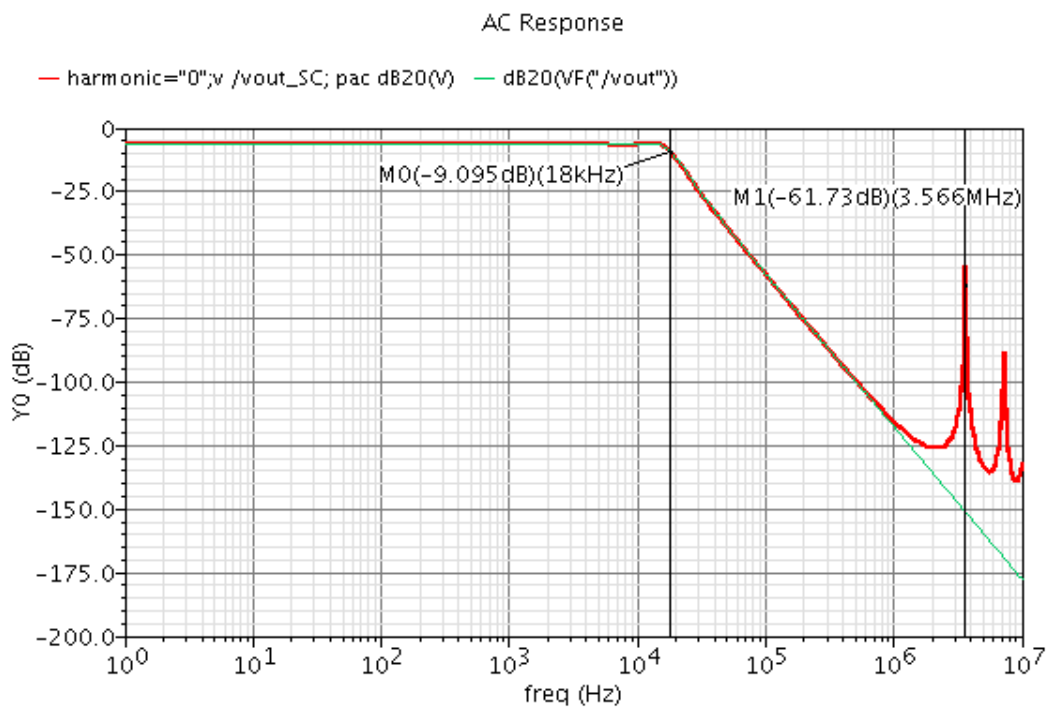
End added by H.K.

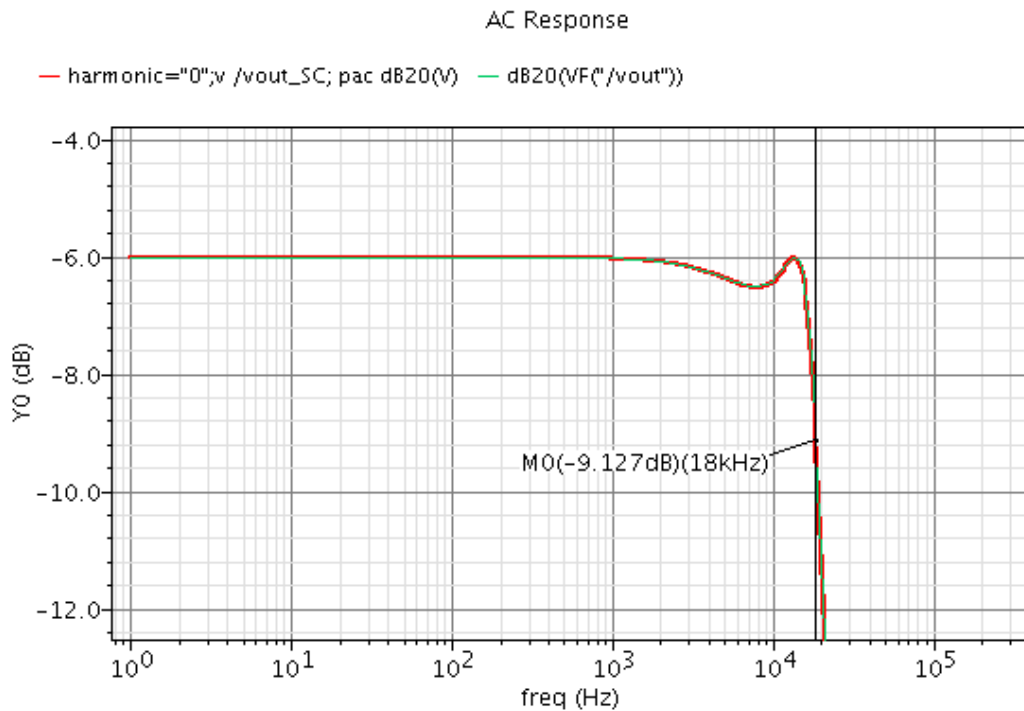
- b) Given  $C_I = 5pF$ , and  $f_s = 3.6MHz$ , we can determine the values of the switching capacitors as follows:

$$\tau_1 = \tau_3 = 16.48\mu s = R_1 C_I = \frac{1}{f_s C_1} C_I \Rightarrow C_1 = \frac{C_I}{\tau_1 f_s} = 84.28 fF$$

$$\tau_2 = 11.32\mu s = R_2 C_I = \frac{1}{f_s C_2} C_I \Rightarrow C_2 = \frac{C_I}{\tau_2 f_s} = 122.7 fF$$

- c) The two loops have an LDI behavior, as it can be inferred by inspection of the schematic. Moreover, the two switches that implement the termination resistors are switched on opposite phases, to implement the complex conjugate termination technique discussed in class. In the following I report the result of the PAC analysis and the detail of the pass band. Note that in the detailed plot, also the continuous time response is reported (I implemented the filter also in the CT version): the two TF matches perfectly.





- d) The main advantage of SC technique is the fact that both the value of  $f_{-3dB}$  and the values of all integrator  $\tau$  depend on the ratio of two components. While absolute values in ICs are subject to high tolerances, mismatch errors can be limited by a careful layout, so a high accuracy can be achieved. In comparison, in CT filters  $f_{-3dB}$  is determined by the absolute values of the capacitors and resistors, thus tuning is required. SC filters can also achieve narrow BWs in a small area.

SC circuits cannot be implemented for free. First of all, from a system level perspective, they require the generation and distribution of a two-phase clock, where the two phases cannot overlap. Second, switches introduce several non-idealities, since real switches don't behave like ideal ones. For example, MOS switches introduce non-linearities, and, even more, they inject charge on the capacitors through their parasitic capacitors, when they are turned on and off: the effects of MOS parasitics become more and more relevant while increasing the frequency of operation.

- e) To compute the minimum required SR for the final stage, I use the formula given on the slides. Note that the approximated formula can be used, since  $f_s \gg f_o$

$$HD_3 = \frac{8\pi V_o f_o^2}{15 * SR * f_s} = 1\% \Rightarrow SR = \frac{8\pi V_o f_o^2}{15 * 1\% * f_s} = 1163 \frac{V}{s}$$