

EE247

Lecture 19

ADC Converters

- ADC architectures and design (continued)
 - Flash ADC and its sources of error (continued): sparkle code & meta-stability
- Comparator design
 - Single-stage open-loop amplifier
 - Cascade of open-loop amplifiers
 - Problem associated with DC offset
 - Cascaded output series cancellation
 - Input series cancellation
 - Offset cancellation through additional input pair plus offset storage capacitors
 - Latched comparators
 - Comparator examples

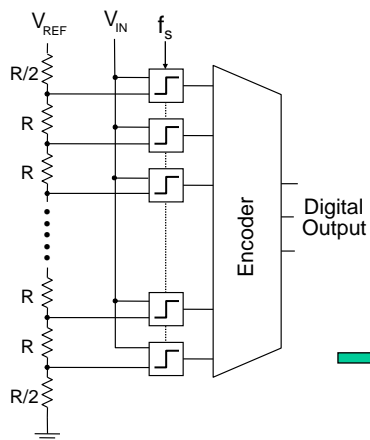
Term project

- Design & simulate an ADC with the following specs.
 - 6bit resolution
 - Signal bandwidth 0 to 150MHz
 - ENOB~6bit for $f_{\text{signal}} < 10\text{MHz}$
 - Architecture of your choice targeted for minimum power dissipation
 - Detailed project description posted in the homework section
 - Teams of two preferred
 - Report due date: Nov. 30th or earlier
 - Dec. 1st: Visit with the instructor by appointment to discuss the implementation
 - Dec. 2nd & Dec. 7th: PowerPoint presentation
~10min/student in class

ADC Architectures

- Slope type converters
- Successive approximation
- • Flash
- Time-interleaved / parallel converter
- Folding
- Residue type ADCs
 - Two-step
 - Pipeline
 - ...
- Oversampled ADCs

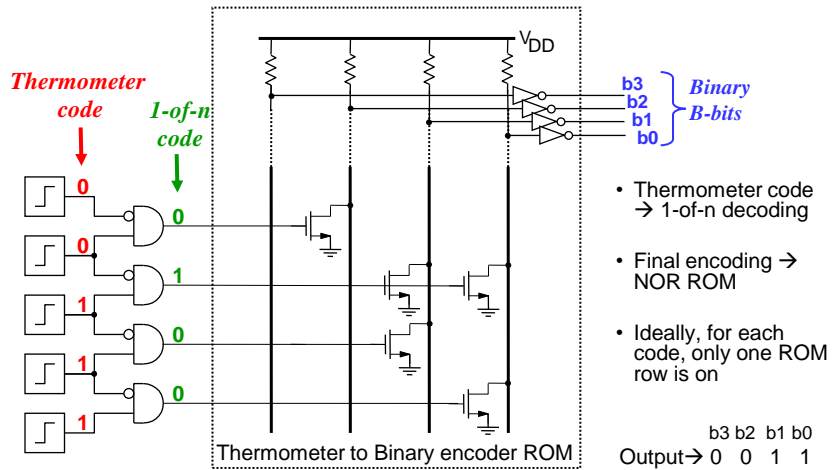
Flash Converter Sources of Error



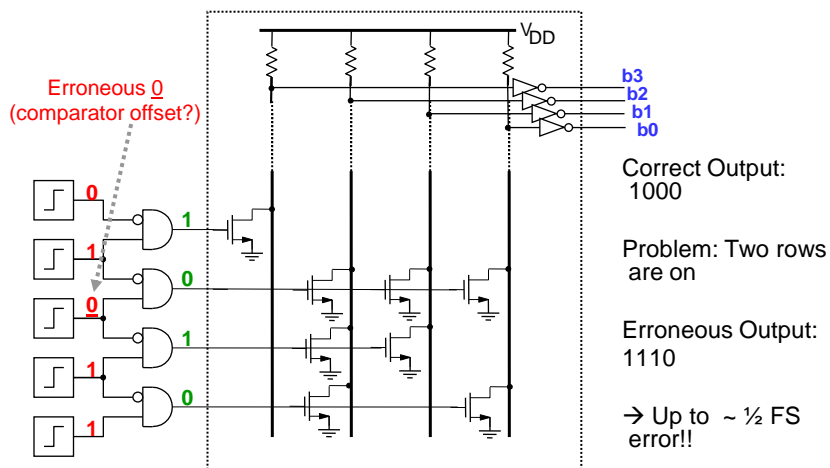
- Comparator input:
 - DC offset
 - Nonlinear input capacitance
 - Feedthrough of input signal to reference ladder
 - Kickback noise (disturbs reference)
 - Signal dependent sampling time

- • Comparator output:
 - Sparkle codes (... 0001101111)
 - Meta-stability

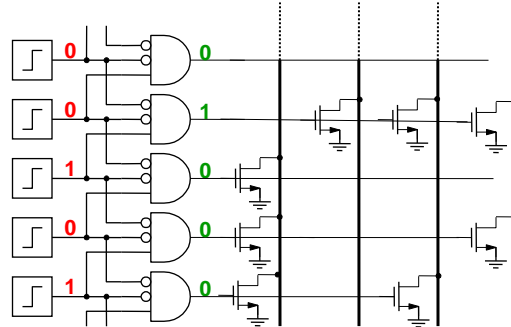
Typical Flash Output Encoder



Flash ADC Sparkle Code Problem



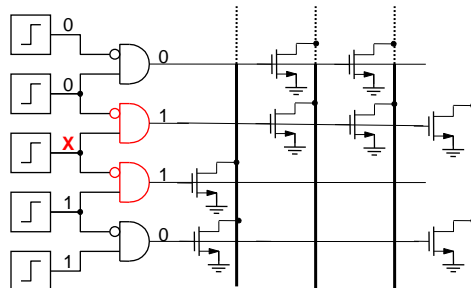
Sparkle Tolerant Encoder



- Protects against a *single* sparkle.
- Possible to improve level of sparkle protection by increasing # of NAND gate inputs

Ref: C. Mangelsdorf et al, "A 400-MHz Flash Converter with Error Correction," JSSC February 1990, pp. 997-1002

Flash ADC Comparator Meta-Stability Issue



Different gates interpret meta-stable output X differently

Correct output: 1000

Erroneous output: 0000

Solutions:

- Add latches to comparator outputs (high power)
- Gray encoding

Ref: C. Portmann and T. Meng, "Power-Efficient Metastability Error Reduction in CMOS Flash A/D Converters," JSSC August 1996, pp. 1132-40

Gray Encoding Example: 3bit ADC

Thermometer Code							Gray			Binary		
T ₇	T ₆	T ₅	T ₄	T ₃	T ₂	T ₁	G ₃	G ₂	G ₁	B ₃	B ₂	B ₁
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	0	0	1	0	0	1
0	0	0	0	0	1	1	0	1	1	0	1	0
0	0	0	0	1	1	1	0	1	0	0	1	1
0	0	0	1	1	1	1	1	1	0	1	0	0
0	0	1	1	1	1	1	1	1	1	1	0	1
0	1	1	1	1	1	1	1	0	1	1	1	0
1	1	1	1	1	1	1	1	0	0	1	1	1

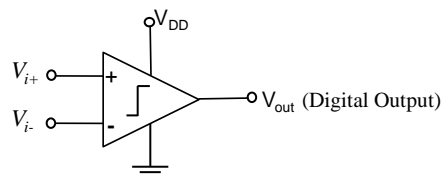
$$G_1 = T_1 \overline{T_3} + T_5 \overline{T_7}$$

$$G_2 = T_2 \overline{T_6}$$

$$G_3 = T_4$$

- Each T_i affects only one G_i
→ Avoids disagreement of interpretation by multiple gates
- To a certain extent, protects also against sparkles & meta-stability issue
- Follow Gray encoder by (latch and) binary encoder

Voltage Comparators



Play an important role in majority of ADCs

Function: Compare the instantaneous value of two analog signals & generate a digital output voltage based on the sign of the difference:

$$\text{If } V_{i+} - V_{i-} > 0 \rightarrow V_{out} = "1"$$

$$\text{If } V_{i+} - V_{i-} < 0 \rightarrow V_{out} = "0"$$

Voltage Comparator Architectures

- High gain amplifier with differential analog input & single-ended large swing output
 - Output swing has to be compatible with driving digital logic circuits
 - Open-loop amplification → no frequency compensation required
 - Precise gain not required
- Latched comparators: In response to a strobe (clock edge), input stage disabled & digital output stored in a latch till next strobe
 - Two options for implementation :
 - Latch-only comparator
 - Low-gain preamplifier + high-sensitivity latch
- Sampled-data comparators
 - T/H input
 - Offset cancellation

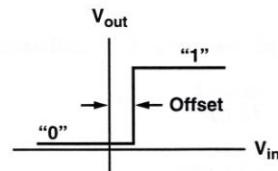
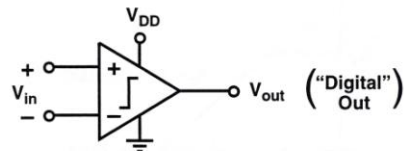
Comparator Built with High-Gain Amplifier

Amplify $V_{in(min)}$ to V_{DD}
 → $V_{in(min)}$ determined by ADC resolution

Example: 12-bit ADC with:
 - $V_{FS} = 1.5V \rightarrow 1LSB = 0.36mV$
 - $V_{DD} = 1.8V$

→ For 1.8V output & 0.5LSB precision:

$$A_v^{Min} = \frac{1.8V}{0.18mV} \approx 10,000$$



Comparator Design 1-Single-Stage Amplification

- Amplifier maximum Gain-Bandwidth product (f_u) for a given technology, typically a function of maximum device f_t

$$f_u = \text{unity-gain frequency, } f_o = -3\text{dB frequency} \quad f_o = \frac{f_u}{A_v}$$

Example: $f_u = 10\text{GHz}$ & $A_v = 10,000$

$$f_o = \frac{10\text{GHz}}{10,000} \approx 1\text{MHz}$$

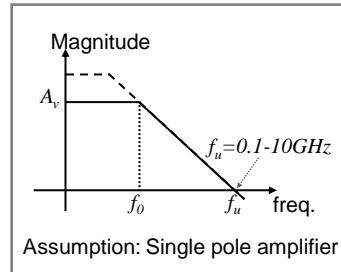
$$\tau_{\text{settling}} = \frac{1}{2\pi f_o} = 0.16\mu\text{sec}$$

Allow a few τ for output to settle

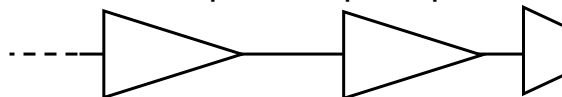
$$f_{\text{Clock}}^{\text{Max.}} \rightarrow \frac{1}{5\tau_{\text{settling}}} \approx 1.26\text{MHz}$$

Too slow for majority of applications!

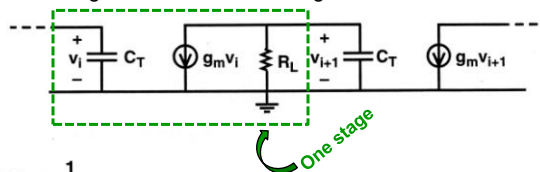
→ Try cascade of lower gain stages to broaden frequency of operation



Comparator Design 2- Cascade of Open Loop Amplifiers



The stages identical → small-signal model for the cascades:



One stage:

$$|A_v(0)| = g_m R_L$$

$$\omega_o = -3\text{dB frequency} = \frac{1}{R_L C_T}$$

$$\omega_u = -\text{unity gain frequency} = G \times \text{BW} = \frac{g_m}{C_T}$$

$$\therefore \omega_o = \frac{\omega_u}{|A_v(0)|}$$

Open Loop Cascade of Amplifiers

For an N-stage cascade:

$$A_T(j\omega) = [A_V(j\omega)]^N = \frac{[A_V(0)]^N}{\left(1 + j\frac{\omega}{\omega_p}\right)^N}$$

Define

$$\omega_{oN} \equiv \text{-3dB frequency of the N-stage cascade}$$

Then

$$|A_T(j\omega_{oN})| = \frac{|A_V(0)|^N}{\sqrt{2}}$$

and

$$\omega_{oN} = \omega_p \sqrt{2^{1/N} - 1} = \frac{\omega_u}{|A_V(0)|} \sqrt{2^{1/N} - 1}$$

∴ For a specified $|A_T(0)|$

$$|A_V(0)| = |A_T(0)|^{1/N}$$

$$\Rightarrow \omega_{oN} = \frac{\omega_u}{|A_T(0)|^{1/N}} \sqrt{2^{1/N} - 1}$$

Thus,

$$\frac{\omega_{oN}}{\omega_{o1}} = \left[\frac{\omega_u}{|A_T(0)|^{1/N}} \sqrt{2^{1/N} - 1} \right] \bigg/ \left[\frac{\omega_u}{|A_T(0)|} \right]$$

$$= |A_T(0)|^{\left(\frac{N-1}{N}\right)} \sqrt{2^{1/N} - 1}$$

Example: $N=4, A_T=10000 \rightarrow \omega_{oN}=430\omega_{o1}$

Open Loop Cascade of Amplifiers

For $|A_T(\text{DC})|=10,000$

N	ω_{oN}/ω_{o1}	$ A_V(0) $
1	1	10,000
2	64	100
3	236	21.5
4	435	10
5	611	6.3
10	1067	2.5
20	1185	1.6

Example:

$$N=3, f_u=10\text{GHz} \ \& \ |A_T(0)|=10000$$

$$f_{oN} = \frac{10\text{GHz}}{(10,000)^{1/3}} \sqrt{2^{1/3}-1} \approx 237\text{MHz}$$

$$\tau_{\text{settling}} = \frac{1}{2\pi f_o} = 0.7\text{nsec}$$

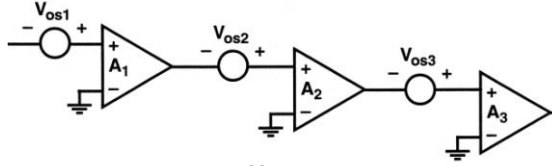
Allow a few τ for output to settle

$$f_{\text{Clock}}^{\text{Max}} \rightarrow \frac{1}{5\tau_{\text{settling}}} \approx 290\text{MHz}$$

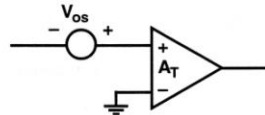
f_{max} improved from 1.26MHz to 290MHz $\rightarrow X236$

Open Loop Cascade of Amplifiers Offset Voltage

- From offset point of view: high gain/stage is preferred



- Choice of # of stages
→ bandwidth vs offset tradeoff

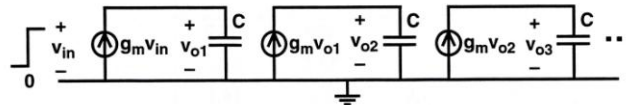


$$A_T = A_1 \cdot A_2 \cdot A_3$$

$$\text{Input-referred offset} \rightarrow V_{os} = V_{os1} + \frac{V_{os2}}{A_1} + \frac{V_{os3}}{A_1 \cdot A_2}$$

Open Loop Cascade of Amplifiers Step Response

- Assuming linear behavior (not slew limited)



$$v_{o1} = \frac{1}{C} \int_0^t g_m v_{in} dt = \frac{g_m}{C} v_{in} t$$

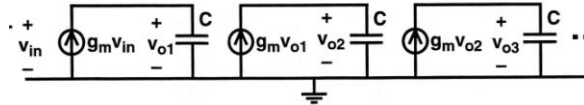
$$v_{o2} = \frac{1}{C} \int_0^t g_m v_{o1} dt = \frac{g_m}{C} \int_0^t \frac{g_m}{C} v_{in} t dt = \frac{1}{2} \left(\frac{g_m}{C} \right)^2 v_{in} t^2$$

$$v_{o3} = \frac{1}{C} \int_0^t g_m v_{o2} dt = \frac{g_m}{C} \int_0^t \left[\frac{1}{2} \left(\frac{g_m}{C} \right)^2 v_{in} t^2 \right] dt$$

$$= \frac{1}{3} \left(\frac{g_m}{C} \right)^3 v_{in} t^3$$

Open Loop Cascade of Amplifiers Step Response

• Assuming linear behavior for cascade of N stages:



N Stages

$$v_{oN} = \left(\frac{g_m}{C}\right)^N \left(\frac{t^N}{N!}\right) v_{in}$$

For the output to reach a specified v_{out} (i.e., $v_{oN} = v_{out}$) the delay is

$$\tau_D = \left(\frac{C}{g_m}\right) \left[(N!) \left(\frac{V_{out}}{V_{in}}\right) \right]^{1/N}$$

Open Loop Cascade of Amplifiers Delay/(C/g_m)

$$\tau_D = \left(\frac{C}{g_m}\right) \left[(N!) \left(\frac{V_{out}}{V_{in}}\right) \right]^{1/N}$$

- Minimum total delay broad function of N
- Relationship between # of stages resulting in minimize delay (N_{op}) and gain (V_{out}/V_{in}) approximately:

$$N_{opt} \approx 1 + \log_2 A_T \quad \text{for } A < 1000$$

$$N_{opt} \approx 1.2 \ln A_T \quad \text{for } A \geq 1000$$

		Delay/(C/g _m)			
		10	100	1000	10K
V_{out}/V_{in}	N				
1	1	10	100	1000	10K
2	2	4.5	14.1	44.7	141
3	3	3.9	8.4	18.2	39.1
4	4	3.9	7.0	12.4	22.1
5	5	4.1	6.5	10.4	16.4
6	6	4.4	6.4	9.5	13.9
7	7	4.7	6.5	9.1	12.6
8	8	5.0	6.7	8.9	11.9
9	9	5.4	6.9	8.9	11.5
10	10	5.7	7.2	9.0	11.4
11	11	6.1	7.5	9.2	11.3
12	12	6.4	7.8	9.4	11.4
20	20	9.3	10.5	11.7	13.2

Ref: J.T. Wu, et al., "A 100-MHz pipelined CMOS comparator" *IEEE Journal of Solid-State Circuits*, vol. 23, pp. 1379 - 1385, December 1988.

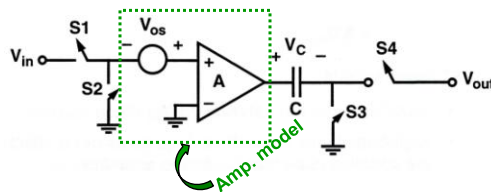
Offset Cancellation

- Offset cancellation for sampled-data cascade of amplifiers can be performed by:
 1. Store offset on ac-coupling caps in series with amp stages: Offset associated with a specific amp can be cancelled by storing it in series with either the input or the output of that stage
 2. Offset can be cancelled by adding a pair of auxiliary inputs to the amplifier and storing the offset on capacitors connected to the aux. inputs during offset cancellation phase

Ref: J.T. Wu, et al., "A 100-MHz pipelined CMOS comparator" *IEEE Journal of Solid-State Circuits*, vol. 23, pp. 1379 - 1385, December 1988.

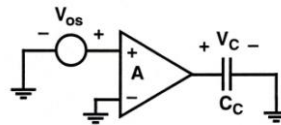
Offset Cancellation Output Series Cancellation

- Amp modeled as ideal + V_{os} (input referred)



1- Store offset:

- S1, S4 → open
 - S2, S3 → closed
- $V_C = A \cdot V_{os}$



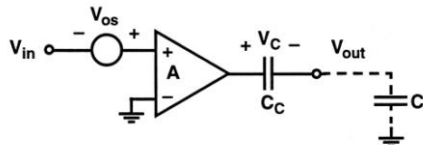
$$V_C = A \cdot V_{os}$$

Ref: J.T. Wu, et al., "A 100-MHz pipelined CMOS comparator" *IEEE Journal of Solid-State Circuits*, vol. 23, pp. 1379 - 1385, December 1988.

Offset Cancellation Output Series Cancellation

2- Amplification phase:

- S2, S3 → open
- S1, S4 → closed
- $V_C = A \times V_{OS}$

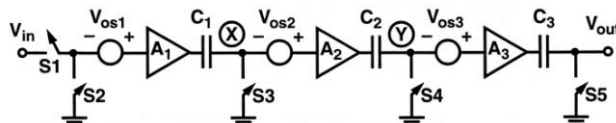


Circuit requirements:

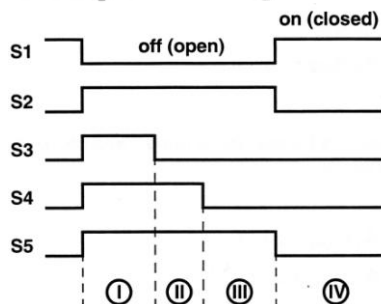
- Amp not saturate during offset storage
- High-impedance (C) load → C_c not discharged
- $C_c \gg C_L$ to avoid attenuation
- $C_c \gg C_{switch}$ avoid excessive additional offset due to charge injection

$$\begin{aligned} V_{out} &= A(V_{in} + V_{os}) - V_C \\ &= A(V_{in} + V_{os}) - A \cdot V_{os} \\ &= A \cdot V_{in} \end{aligned}$$

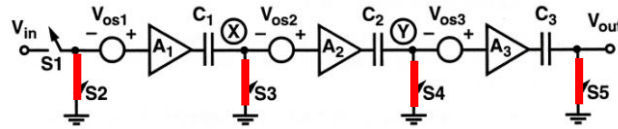
Offset Cancellation Cascaded Output Series Cancellation



Note: Offset storage capacitors in series with the amplifier outputs



Offset Cancellation Cascaded Output Series Cancellation

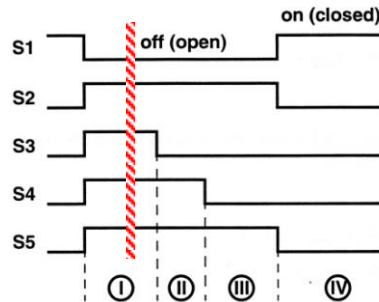


1- S1 → open, S2,3,4,5 closed

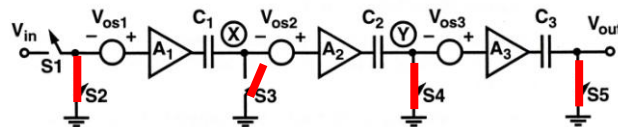
$$V_{C1} = A_1 \times V_{os1}$$

$$V_{C2} = A_2 \times V_{os2}$$

$$V_{C3} = A_3 \times V_{os3}$$



Offset Cancellation Cascaded Output Series Cancellation



2- S3 → open first

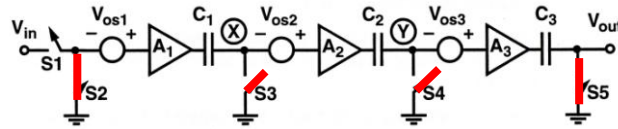
- Feedthrough from S3 → offset on X
- Switch offset, ϵ_3 induced on node X
- Since S4 remains closed, offset associated with ϵ_3 → stored on C2

$$V_X = \epsilon_3$$

$$V_{C1} = A_1 \times V_{os1} - \epsilon_3$$

$$V_{C2} = A_2 \times (V_{os2} + \epsilon_3)$$

Offset Cancellation Cascaded Output Series Cancellation



3- S4 → open

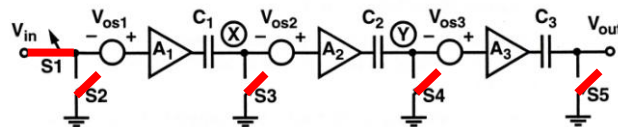
- Feedthrough from S4 → offset on Y
- Switch offset, ϵ_4 induces error on node Y
- Since S5 remains closed, offset associated with ϵ_4 → stored on C3

$$V_Y = \epsilon_4$$

$$V_{C2} = A_2 \times (V_{os2} + \epsilon_3) - \epsilon_4$$

$$V_{C3} = A_3 \times (V_{os3} + \epsilon_4)$$

Offset Cancellation Cascaded Output Series Cancellation



4- S2 → open, S5 → open, S1 → closed

- S2 open & S1 closed → since input connected to low impedance source charge injection not of major concern
- Switch offset, ϵ_5 introduced due to S5 opening

$$V_X = A_1 \times (V_{in} + V_{os1}) - V_{C1}$$

$$= A_1 \times (V_{in} + V_{os1}) - (A_1 \cdot V_{os1} - \epsilon_3)$$

$$= A_1 \cdot V_{in} + \epsilon_3$$

Offset Cancellation Cascaded Output Series Cancellation

$$\begin{aligned} V_y &= A_2 \times (V_x + V_{os2}) - V_{C2} \\ &= A_2 \times (A_1 V_{in} + \epsilon_3 + V_{os2}) - [A_2 \cdot (V_{os2} + \epsilon_3) - \epsilon_4] \\ &= A_1 \cdot A_2 \cdot V_{in} + \epsilon_4 \end{aligned}$$

$$\begin{aligned} V_{out} &= A_3 \times (V_y + V_{os3}) - V_{C3} \\ &= A_3 \cdot (A_2 \times A_1 V_{in} + \epsilon_4 + V_{os3}) - [A_3 \cdot (V_{os3} + \epsilon_4) - \epsilon_5] \\ &= A_1 \cdot A_2 \cdot A_3 \cdot V_{in} + \epsilon_5 \end{aligned}$$

Offset Cancellation Cascaded Output Series Cancellation

$$\begin{aligned} V_{out} &= A_1 \cdot A_2 \cdot A_3 \cdot (V_{in} + \epsilon_5 / A_1 \cdot A_2 \cdot A_3) \\ \text{Input-Referred Offset} &= \epsilon_5 / A_1 \cdot A_2 \cdot A_3 \end{aligned}$$

Example:

3-stage open-loop differential amplifier with series offset cancellation + output amplifier (see Ref.)

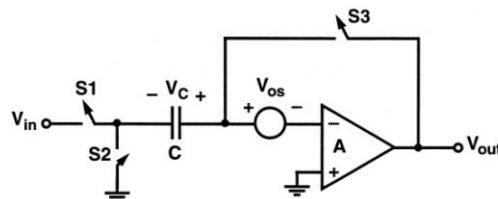
$$\begin{aligned} A_{\text{Total}}(\text{DC}) &= 2 \times 10^6 = 126\text{dB} \\ \text{Input-referred offset} &< 5\mu\text{V} \end{aligned}$$

Ref: :R. Poujois and J. Borel, "A low drift fully integrated MOSFET operational amplifier," *IEEE Journal of Solid-State Circuits*, vol. 13, pp. 499 - 503, August 1978.

Offset Cancellation Output Series Cancellation

- Advantages:
 - Almost complete cancellation
 - Closed-loop stability not required
- Disadvantages:
 - Gain per stage must be small
 - Offset storage C in the signal path → could slow down overall performance

Offset Cancellation Input Series Cancellation

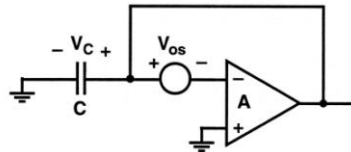


Ref: :R. Poujois and J. Borel, "A low drift fully integrated MOSFET operational amplifier," *IEEE Journal of Solid-State Circuits*, vol. 13, pp. 499 - 503, August 1978.

Offset Cancellation Input Series Cancellation

1- Store offset

S1 = 0 (off)
S2, S3 = 1 (conducting)



Note: Mandates
closed-loop
stability

$$V_C = -A(V_C - V_{os})$$

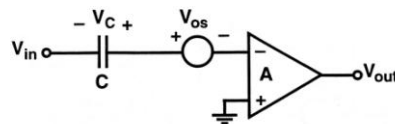
$$= \left(\frac{A}{A+1}\right)V_{os}$$

Ref: R. Poujois and J. Borel, "A low drift fully integrated MOSFET operational amplifier," *IEEE Journal of Solid-State Circuits*, vol. 13, pp. 499 - 503, August 1978.

Offset Cancellation Input Series Cancellation

2- Amplify

S2, S3 → open
S1 → closed



$$V_{out} = -A(V_{in} + V_C - V_{os}) = -A\left[V_{in} + V_{os}\left(\frac{A}{A+1} - 1\right)\right]$$

$$\therefore V_{out} = -A\left(V_{in} - \frac{V_{os}}{A+1}\right)$$

and

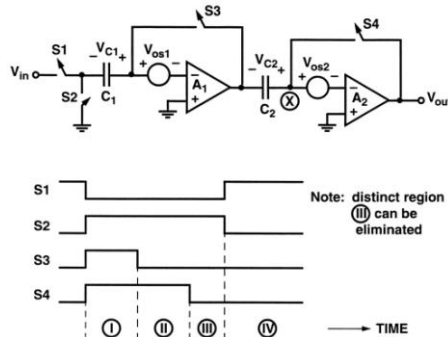
$$\text{Input-Referred Offset} = \frac{V_{os}}{A+1}$$

Example: A=4
→ Input-referred
offset = $V_{os}/5$

Offset Cancellation Cascaded Input Series Cancellation

$$V_{out} = A_1 A_2 \left[V_{in} + \frac{V_{os2}}{A_1(A_2+1)} - \frac{\epsilon_4}{A_1} \right]$$

Input referred offset: $\frac{V_{os2}}{A_1(A_2+1)} - \frac{\epsilon_4}{A_1}$



$\epsilon_4 \rightarrow$ charge injection effect associated with opening of S4

Offset Cancellation Input Series Cancellation

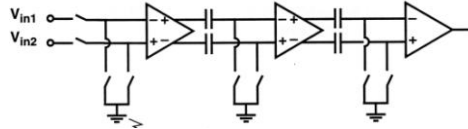
- Advantages:
 - In applications such as C-array successive approximation ADCs can use C-array to store offset
- Disadvantages:
 - Cancellation not complete
 - Requires closed loop stability
 - Offset storage C in the signal path- could slow down overall performance

CMOS Comparators

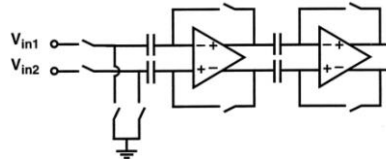
Cascade of Gain Stages

Fully differential gain stages \rightarrow 1st order cancellation of switch feedthrough offset

1- Output series offset cancellation



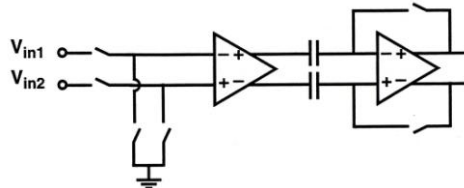
2- Input series offset cancellation



CMOS Comparators

Cascade of Gain Stages

3- Combined input & output series offset cancellation

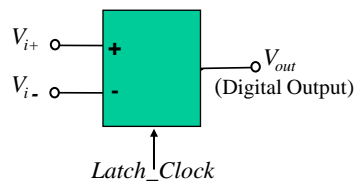


V_{os1} & V_{os2} are both stored on a single pair of coupling capacitors

Offset Cancellation

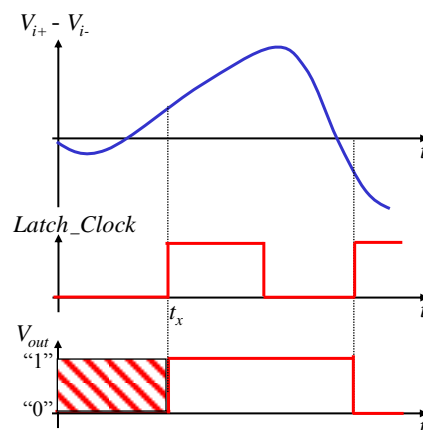
- Cancel offset by additional pair of inputs + offset storage Cs + an extra clock phase for offset storage (Lecture 18 slide 21 thru 23)

Latched Comparators



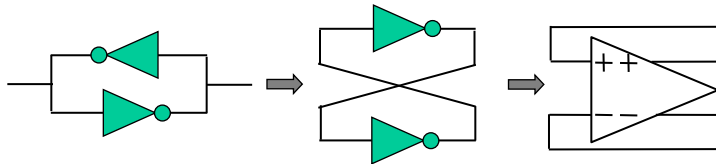
Compares two input voltages at time t_x & generates a digital output:

$$\begin{aligned} \text{If } V_{i+} - V_{i-} > 0 &\rightarrow V_{out} = \text{"1"} \\ \text{If } V_{i+} - V_{i-} < 0 &\rightarrow V_{out} = \text{"0"} \end{aligned}$$



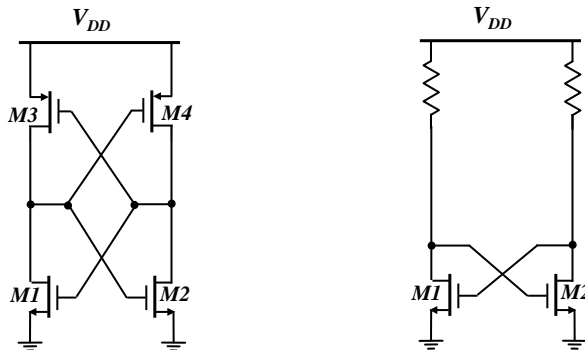
CMOS Latched Comparators

Comparator amplification need not be linear
→ can use a latch → regeneration



- Latch → Amplification + positive feedback
- Since uses positive feedback, have to reset the latch prior to each comparison

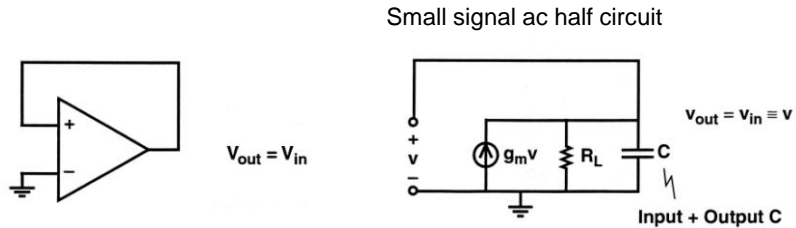
Simplest Form of CMOS Latch



CMOS Latched Comparators

Small Signal Model

Latch can be modeled as a:
 → Single-pole amp + positive feedback



CMOS Latched Comparator

Latch Delay

$$g_m V = \frac{V}{R_L} + C \frac{dV}{dt}$$

$$\frac{g_m}{C} \left(1 - \frac{1}{g_m R_L} \right) V = \frac{dV}{dt} \quad \frac{g_m}{C} \left(1 - \frac{1}{g_m R_L} \right) dt = \frac{dV}{V}$$

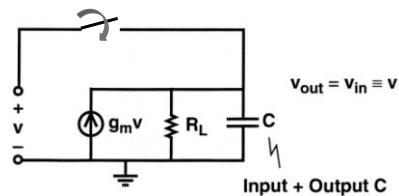
Integrating both sides: $\frac{g_m}{C} \left(1 - \frac{1}{g_m R_L} \right) \int_{t_1}^{t_2} dt = \int_{V_1}^{V_2} \frac{1}{V} dV \quad \left(\int_b^a \frac{1}{x} dx = \ln x|_b^a = \ln a - \ln b = \ln \frac{a}{b} \right)$

Latch Delay:

$$t_D = t_2 - t_1 = \frac{C}{g_m} \left(\frac{1}{1 - \frac{1}{g_m R_L}} \right) \ln \left(\frac{V_2}{V_1} \right)$$

For $g_m R_L \gg 1$

$$t_D \approx \frac{C}{g_m} \ln \left(\frac{V_2}{V_1} \right)$$



CMOS Latched Comparators

$$t_D \approx \frac{C}{g_m} \ln \left(\frac{V_2}{V_1} \right)$$

$$\frac{V_2}{V_1} \rightarrow \text{Latch Gain} = A_L$$

$$\rightarrow t_D \approx \frac{C}{g_m} \ln A_L$$

$$\tau_D(\text{3-stage amp}) = 18.2(C/g_m) \rightarrow$$

Normalized Latch Delay

$A_L = \frac{V_2}{V_1}$	$\frac{t_D}{(C/g_m)}$
10	2.3
100	4.6
1000	6.9
10K	9.2

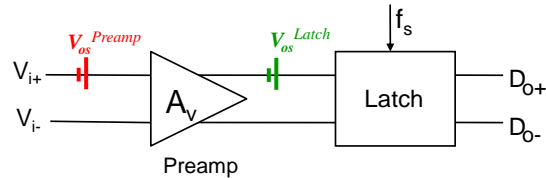
Compared to a 3-stage open-loop cascade of amps for equal overall gain of 1000

→ Latch faster by about x3

Latch-Only Comparator

- Much faster compared to cascade of open-loop amplifiers
- Since uses positive feedback, have to reset the latch prior to each comparison
- Main problem associated with latch-only comparator topology:
 - High input-referred offset voltage (as high as 100mV!)
 - Solution:
 - Use low-offset preamplifier to amplify the signal and reduce overall input-referred offset

Pre-Amplifier + Latch Overall Input-Referred Offset



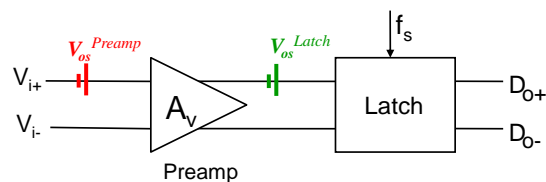
Latch offset attenuated by preamp gain when referred to preamp input.
Assuming the two offset sources are uncorrelated:

$$\sigma_{\text{Input-Referred_Offset}} = \sqrt{\sigma_{V_{os_Preamp}}^2 + \frac{1}{A_{Preamp}^2} \sigma_{V_{os_Latch}}^2}$$

$$\text{Example: } \sigma_{V_{os_Preamp}} = 4\text{mV} \ \& \ \sigma_{V_{os_Latch}} = 50\text{mV} \ \& \ A_{Preamp} = 10$$

$$\sigma_{\text{Input-Referred_Offset}} = \sqrt{4^2 + \frac{1}{10^2} 50^2} = 6.4\text{mV}$$

Pre-Amplifier + Latch Overall Input-Referred Offset



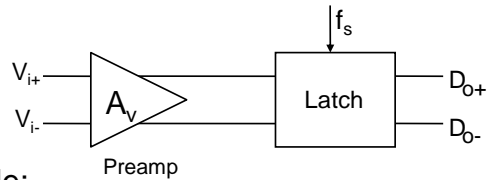
Latch offset attenuated by preamp gain when referred to preamp input.
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$$\sigma_{\text{Input-Referred_Offset}} = \sqrt{\sigma_{V_{os_Preamp}}^2 + \frac{1}{A_{Preamp}^2} \sigma_{V_{os_Latch}}^2}$$

$$\text{Example: } \sigma_{V_{os_Preamp}} = 4\text{mV} \ \& \ \sigma_{V_{os_Latch}} = 50\text{mV} \ \& \ A_{Preamp} = 10$$

$$\sigma_{\text{Input-Referred_Offset}} = \sqrt{4^2 + \frac{1}{10^2} 50^2} = 6.4\text{mV}$$

Pre-Amplifier Tradeoffs



- Example:

- Latch offset 50 to 100mV
- Preamp DC gain 10X
- Preamp input-referred latch offset 5 to 10mV
- Input-referred preamplifier offset 2 to 10mV
- Overall input-referred offset 5.5 to 14mV

→ Addition of preamp reduces the latch input-referred offset reduced by ~7 to 9X → ~allows extra 3-bit resolution for ADC!

Comparator Preamplifier Gain-Speed Tradeoffs

- Amplifier maximum Gain-Bandwidth product (f_u) or a given technology, typically a function of maximum device f_t

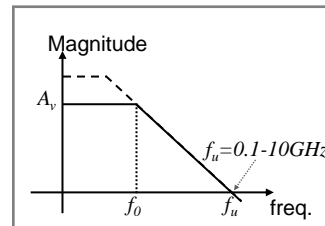
f_u = unity gain frequency, f_0 = $-3dB$ frequency & τ_0 = settling time

$$f_0 = \frac{f_u}{A_{\text{preamp}}}$$

For example assuming preamp has a gain of 10:

$$f_0 = \frac{f_u}{A_{\text{preamp}}} = \frac{1\text{GHz}}{10} = 100\text{MHz}$$

$$\tau_0 = \frac{1}{2\pi f_0} = \frac{A_{\text{preamp}}}{2\pi f_u} = 1.6\text{nsec}$$



- Tradeoff:

- To reduce the effect of latch offset → high preamp gain desirable
- Fast comparator → low preamp gain
- Choice of preamp gain: compromise speed v.s. input-referred latch offset

CMOS Preamplifier + Latch Type Comparator Delay in Response

Latch delay previously found:

$$\tau_D \approx \frac{C}{g_m} \ln\left(\frac{V_2}{V_1}\right)$$

Assuming gain of A_v for the preamplifier then $V_1 = A_v \times V_{in}$

$$\tau_D \approx \frac{C}{g_m} \ln\left(\frac{V_0}{A_v V_{in}}\right)$$

Latched Comparator Including Preamplifier Example

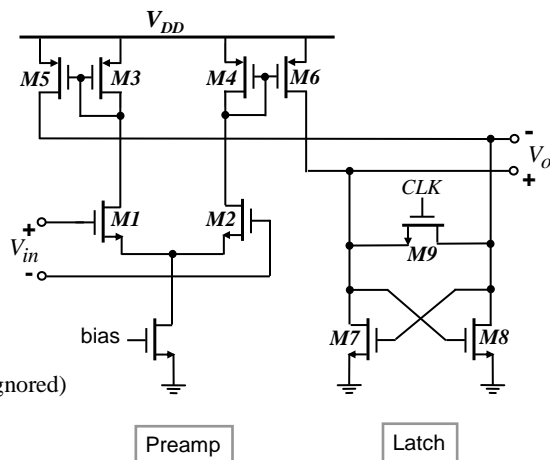
Preamplifier gain:

$$A_v = \frac{g_m^{M1}}{g_m^{M3}} = \frac{(V_{GS}^{M3} - V_{th}^{M3})}{(V_{GS}^{M1} - V_{th}^{M1})}$$

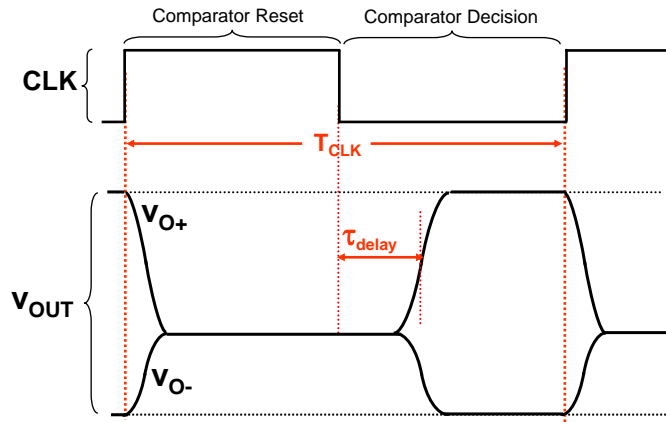
Comparator delay:

(for simplicity, preamp delay ignored)

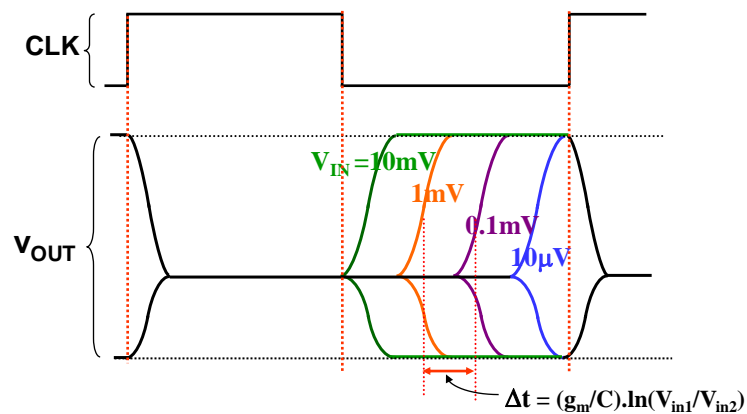
$$\tau_D \approx \frac{C}{g_m} \ln\left(\frac{V_0}{A_v V_{in}}\right)$$



Comparator Dynamic Behavior

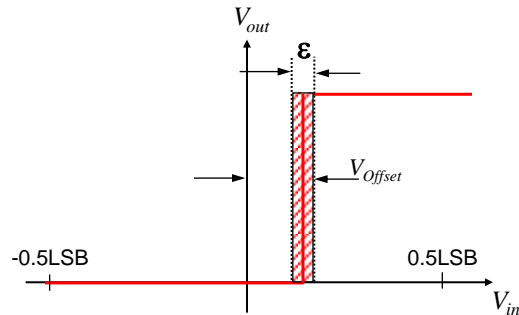


Comparator Resolution



Note: For small V_{in} the comparator may not be able to make a decision within the allotted time → output ambiguous

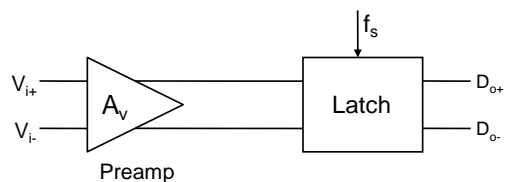
Comparator Voltage Transfer Function Non-Idealities



V_{Offset} → Comparator offset voltage

ϵ → Meta-Stable region (output ambiguous)

Latched Comparator

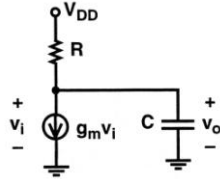


Important features:

- Overdrive recovery
- Maximum clock rate $f_s \rightarrow$ settling time, slew rate, small signal bandwidth, overdrive recovery
- Resolution \rightarrow gain, offset
- Input capacitance (and linearity of input capacitance!)
- Power dissipation
- Input common-mode range and CMR
- Kickback noise
- ...

Comparator Overdrive Recovery

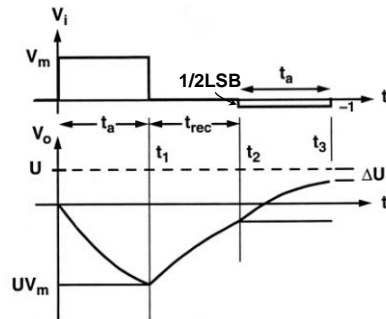
Linear model for a single-pole amplifier:



$U \rightarrow$ amplification after time t_a

During reset amplifier settles exponentially to its zero input condition with $\tau_0 = RC$

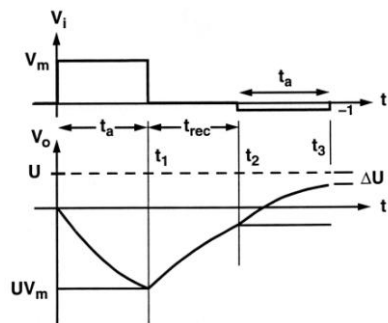
Assume $V_m \rightarrow$ maximum input normalized to $1/2\text{LSB}$ ($=1$)



Example: Worst case input/output waveforms

Previous input \rightarrow max. possible e.g. VFS
Current input \rightarrow min. input-referred signal (0.5LSB)

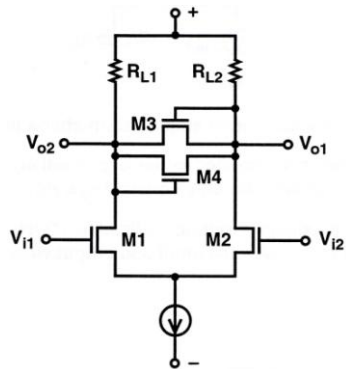
Comparators Overdrive Recovery



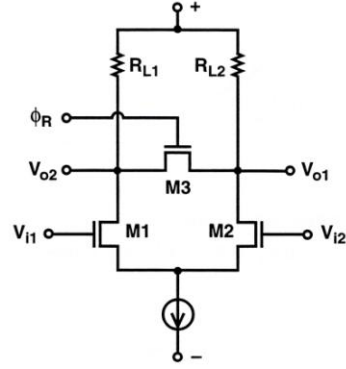
Example: Worst case input/output waveforms

- If recovery time is not long enough to allow output to discharge (recover) from previous state- then it may not be able to resolve the current low-level input \rightarrow error
- To minimize this effect:
 1. Passive clamp
 2. Active restore
 3. Low gain/stage

Comparators Overdrive Recovery Limiting Output Voltage

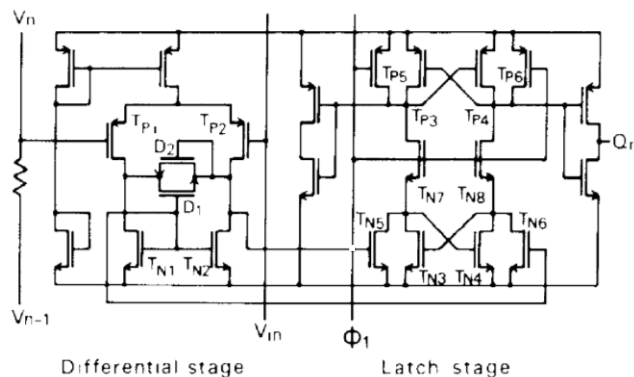


Clamp
Adds parasitic capacitance



Active Restore
After outputs are latched by following stage
→ Activate ϕ_R & equalize output nodes

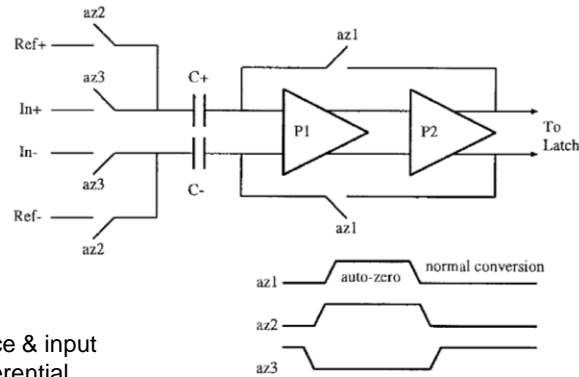
CMOS Comparator Example Flash ADC



- Flash ADC: 8bits, $\pm 1/2\text{LSB}$ INL @ $f_s=15\text{MHz}$ ($V_{\text{ref}}=3.8\text{V}$, $\text{LSB}=15\text{mV}$)
- No offset cancellation

Ref: A. Yukawa, "A CMOS 8-Bit High-Speed A/D Converter IC," JSSC June 1985, pp. 775-9

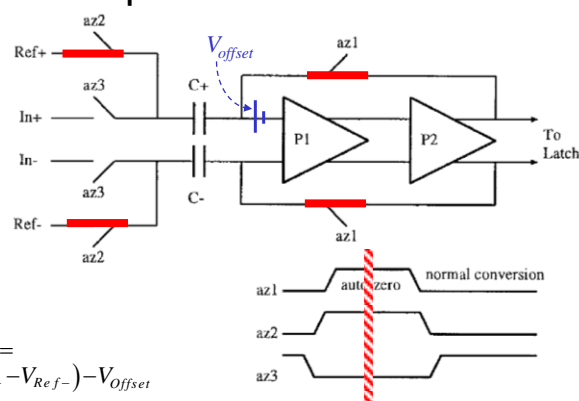
Comparator with Auto-Zero



Note:
Reference & input
both differential

Ref: I. Mehr and L. Singer, "A 500-Msample/s, 6-Bit Nyquist-Rate ADC for Disk-Drive Read-Channel Applications," JSSC July 1999, pp. 912-20.

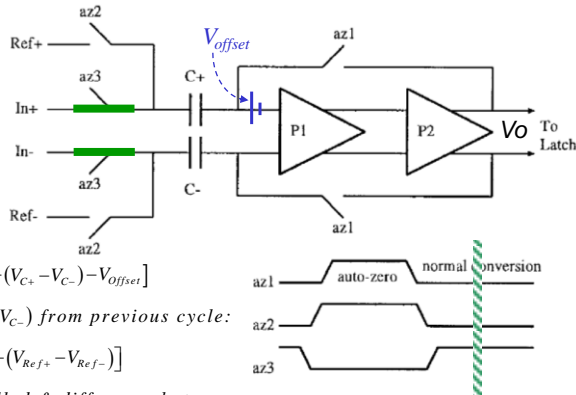
Flash ADC Comparator with Auto-Zero



$$V_{C+} - V_{C-} = (V_{Ref+} - V_{Ref-}) - V_{Offset}$$

Ref: I. Mehr and D. Dalton, "A 500-Msample/s, 6-Bit Nyquist-Rate ADC for Disk-Drive Read-Channel Applications," JSSC July 1999, pp. 912-20.

Flash ADC Comparator with Auto-Zero



$$V_o = A_{P1} * A_{P2} [(V_{In+} - V_{In-}) - (V_{C+} - V_{C-}) - V_{Offset}]$$

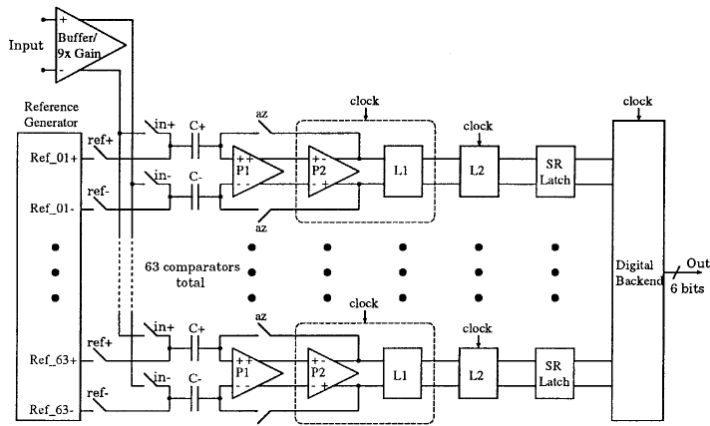
Substituting for $(V_{C+} - V_{C-})$ from previous cycle:

$$V_o = A_{P1} * A_{P2} [(V_{In+} - V_{In-}) - (V_{Ref+} - V_{Ref-})]$$

Note: Offset is cancelled & difference between input & reference established

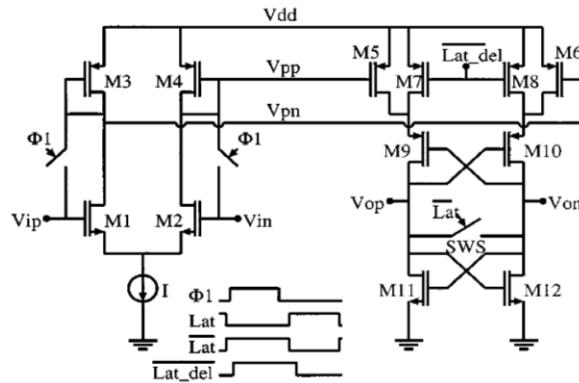
Ref: I. Mehr and D. Dalton, "A 500-Msample/s, 6-Bit Nyquist-Rate ADC for Disk-Drive Read-Channel Applications," JSSC July 1999, pp. 912-20.

Flash ADC Using Comparator with Auto-Zero



Ref: I. Mehr and D. Dalton, "A 500-Msample/s, 6-Bit Nyquist-Rate ADC for Disk-Drive Read-Channel Applications," JSSC July 1999, pp. 912-20.

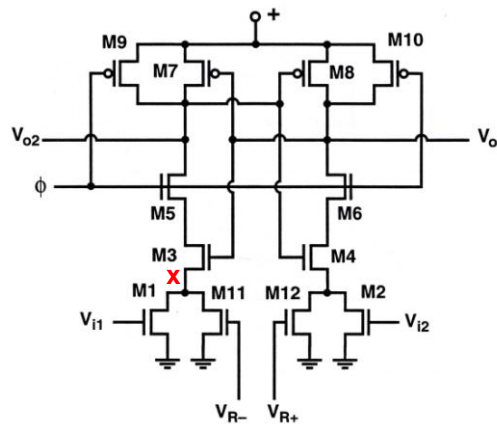
Auto-Zero Implementation



Ref: I. Mehr and L. Singer, "A 55-mW, 10-bit, 40-Msample/s Nyquist-Rate CMOS ADC," JSSC March 2000, pp. 318-25

Comparator Example

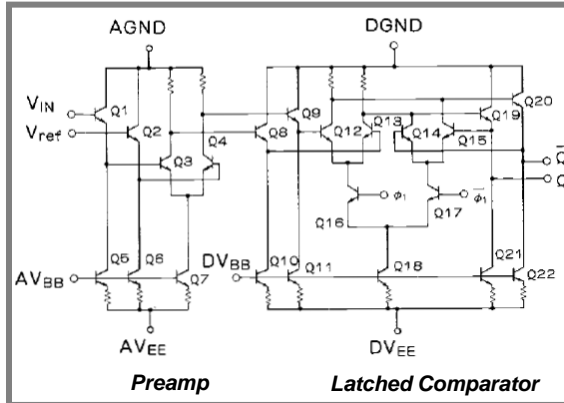
- Variation on Yukawa latch used w/o preamp
- Good for low resolution ADCs (in this case 1.5bit/stage for a pipeline we will see later are tolerant of high offset)
- Note: M1, M2, M11, M12 operate in triode mode
- M11 & M12 width chosen to set comparator threshold
- Conductance at node X is sum of G_{M1} & G_{M11}



Ref: T. B. Cho and P. R. Gray, "A 10 b, 20 Msample/s, 35 mW pipeline A/D converter," IEEE Journal of Solid-State Circuits, vol. 30, pp. 166 - 172, March 1995

Bipolar Comparator Example

- Used in 8bit 400Ms/s & 6bit 2Gb/s flash ADC
- Signal amplification during ϕ_1 high, latch operates when ϕ_1 low
- Input buffers suppress kick-back & input current
- Separate ground and supply buses for front-end preamp \rightarrow kick-back noise reduction



Ref: Y. Akazawa, et al., "A 400MSPS 8b flash AD conversion LSI," *IEEE International Solid-State Circuits Conference*, vol. XXX, pp. 98 - 99, February 1987

Ref: T. Wakimoto, et al., "Si bipolar 2GS/s 6b flash A/D conversion LSI," *IEEE International Solid-State Circuits Conference*, vol. XXXI, pp. 232 - 233, February 1988