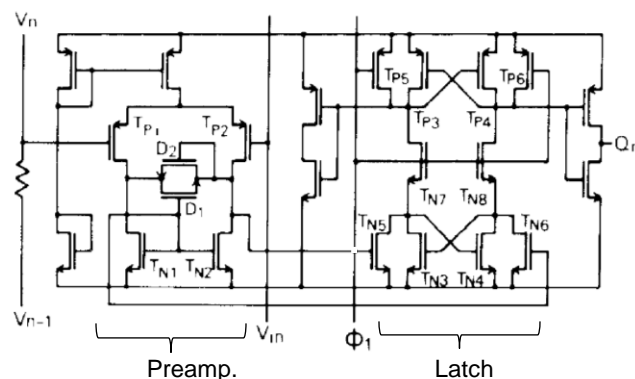


# Lecture 20

## Analog-to-Digital Converters (continued)

- Comparator design (continued)
  - Comparator architecture examples
- Techniques to reduce flash ADC complexity
  - Interpolating
  - Folding
  - Interpolating & folding
- Residue Type ADCs
  - Two-Step flash
  - Pipelined ADCs
    - Architecture basics
    - Effect of sub-ADC, sub-DAC, gain stage non-idealities on overall ADC performance

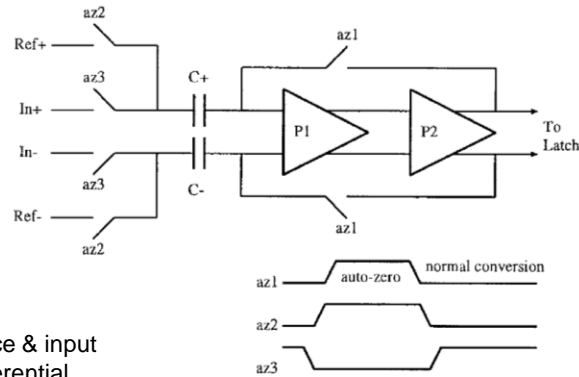
## CMOS Comparator Example Flash ADC



- Flash ADC: 8bits,  $\pm 1/2$ LSB INL @  $f_s=15$ MHz ( $V_{ref}=3.8$ V, LSB $\sim 15$ mV)
- No offset cancellation

Ref: A. Yukawa, "A CMOS 8-bit High-Speed A/D Converter IC," JSSC June 1985, pp. 775-9

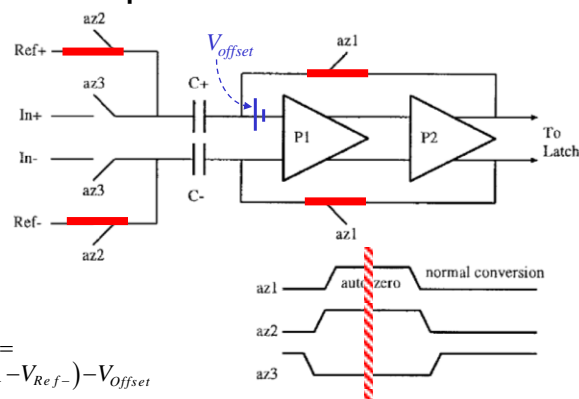
# Comparator with Auto-Zero



Note:  
Reference & input  
both differential

Ref: I. Mehr and L. Singer, "A 500-Msample/s, 6-Bit Nyquist-Rate ADC for Disk-Drive Read-Channel Applications," JSSC July 1999, pp. 912-20.

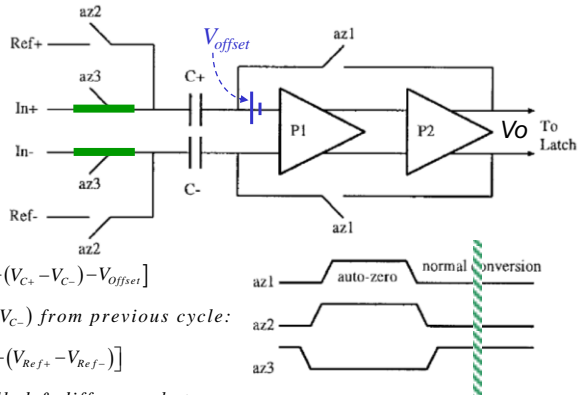
# Flash ADC Comparator with Auto-Zero



$$V_{C+} - V_{C-} = (V_{Ref+} - V_{Ref-}) - V_{Offset}$$

Ref: I. Mehr and D. Dalton, "A 500-Msample/s, 6-Bit Nyquist-Rate ADC for Disk-Drive Read-Channel Applications," JSSC July 1999, pp. 912-20.

## Flash ADC Comparator with Auto-Zero



$$V_o = A_{P1} * A_{P2} [(V_{In+} - V_{In-}) - (V_{C+} - V_{C-}) - V_{Offset}]$$

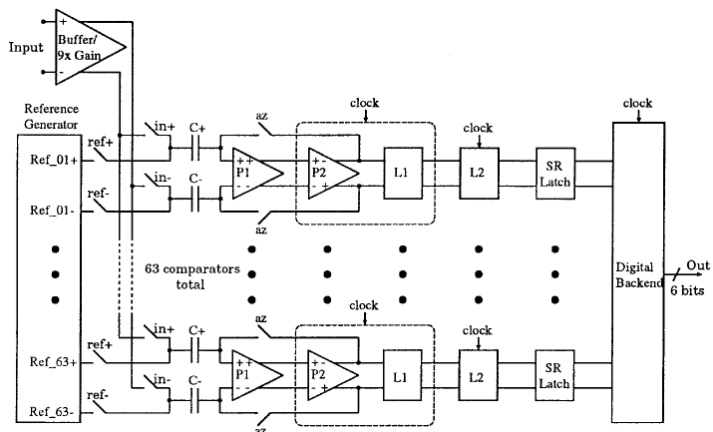
Substituting for  $(V_{C+} - V_{C-})$  from previous cycle:

$$V_o = A_{P1} * A_{P2} [(V_{In+} - V_{In-}) - (V_{Ref+} - V_{Ref-})]$$

Note: Offset is cancelled & difference between input & reference established

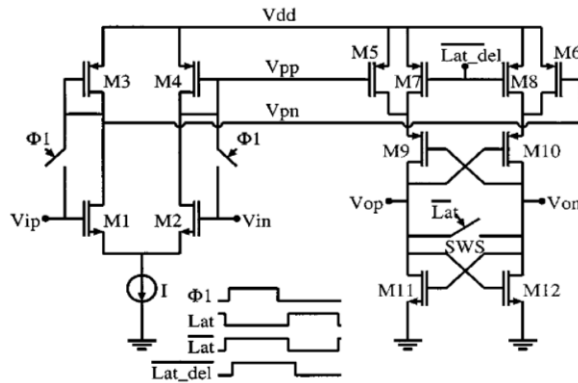
Ref: I. Mehr and D. Dalton, "A 500-Msample/s, 6-Bit Nyquist-Rate ADC for Disk-Drive Read-Channel Applications," JSSC July 1999, pp. 912-20.

## Flash ADC Using Comparator with Auto-Zero



Ref: I. Mehr and D. Dalton, "A 500-Msample/s, 6-Bit Nyquist-Rate ADC for Disk-Drive Read-Channel Applications," JSSC July 1999, pp. 912-20.

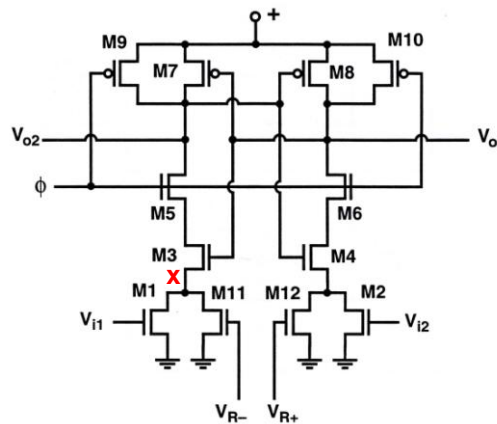
# Auto-Zero Implementation



Ref: I. Mehr and L. Singer, "A 55-mW, 10-bit, 40-Msample/s Nyquist-Rate CMOS ADC," JSSC March 2000, pp. 318-25

# Comparator Example

- Variation on Yukawa latch used w/o preamp
- Good for low resolution ADCs (in this case 1.5bit/stage for a pipeline we will see later are tolerant of high offset)
- Note: M1, M2, M11, M12 operate in triode mode
- M11 & M12 added to vary comparator threshold
- Conductance at node X is sum of  $G_{M1}$  &  $G_{M11}$



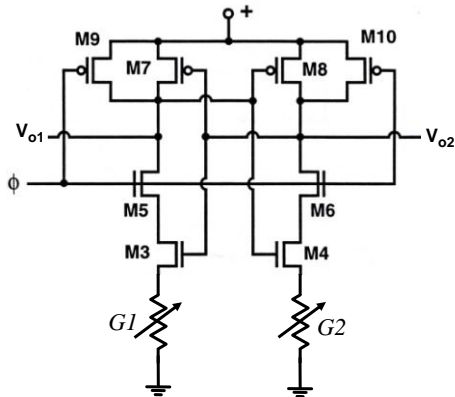
Ref: T. B. Cho and P. R. Gray, "A 10 b, 20 Msample/s, 35 mW pipeline A/D converter," IEEE Journal of Solid-State Circuits, vol. 30, pp. 166 - 172, March 1995

## Comparator Example (continued)

- M1, M2, M11, M12 operate in triode mode with all having equal L
- Conductance of input devices:
 
$$G_1 = \frac{\mu C_{ox}}{L} \times [W_1(V_{I1} - V_{th}) + W_{I1}(V_{R+} - V_{th})]$$

$$G_2 = \frac{\mu C_{ox}}{L} \times [W_2(V_{I2} - V_{th}) + W_{I2}(V_{R+} - V_{th})]$$

$$\rightarrow \Delta G = \frac{\mu C_{ox}}{L} \times \left[ (V_{I1} - V_{I2}) - \frac{W_{I1}}{W_1} (V_{R+} - V_{R-}) \right]$$



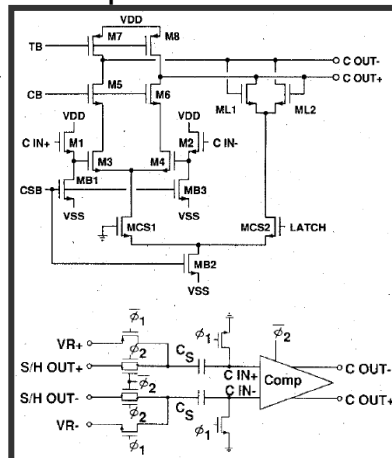
- To 1st order, for  $W1 = W2$  &  $W_{I1} = W_{I2}$ 

$$V_{th}^{latch} = W_{I1}/W_1 \times V_R$$
 where  $V_R = V_{R+} - V_{R-}$ .
  - $\rightarrow V_R$  fixed,  $W_{I1}, I_2$  varied from comparator to comparator  $\rightarrow$  Eliminates need for resistive divider

Ref: T. B. Cho and P. R. Gray, "A 10 b, 20 Msample/s, 35 mW pipeline A/D converter," *IEEE Journal of Solid-State Circuits*, vol. 30, pp. 166 - 172, March 1995

## Comparator Example

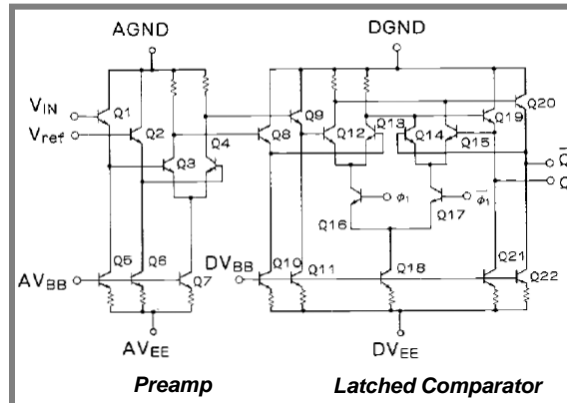
- Used in a pipelined ADC with digital correction
  - $\rightarrow$  No offset cancellation required
- Differential reference & input
- M7, M8 operate in triode region
- Preamp gain  $\sim 10$
- Input buffers suppress kick-back
- $\phi_1$  high  $\rightarrow C_s$  charged to  $VR$  &  $\phi_{2B}$  is also high  $\rightarrow$  current diverted to latch  $\rightarrow$  comparator output in hold mode
- $\phi_2$  high  $\rightarrow C_s$  connected to S/Hout & comparator input ( $VR$ -S/Hout), current sent to preamp  $\rightarrow$  comparator in amplify mode



Ref: S. Lewis, et al., "A Pipelined 5-Msample/s 9-bit Analog-to-Digital Converter" *IEEE JSSC*, NO. 6, Dec. 1987

## Bipolar Comparator Example

- Used in 8bit 400Ms/s & 6bit 2Gb/s flash ADC
- Signal amplification during  $\phi_1$  high, latch operates when  $\phi_1$  low
- Input buffers suppress kick-back & input current
- Separate ground and supply buses for front-end preamp  $\rightarrow$  kick-back noise reduction



Ref: Y. Akazawa, et al., "A 400MSPS 8b flash AD conversion LSI," *IEEE International Solid-State Circuits Conference*, vol. XXX, pp. 98 - 99, February 1987

Ref: T. Wakimoto, et al., "Si bipolar 2GS/s 6b flash A/D conversion LSI," *IEEE International Solid-State Circuits Conference*, vol. XXXI, pp. 232 - 233, February 1988

## Reducing Flash ADC Complexity

E.g. 10-bit "straight" flash

- Input range: 0 ... 1V
- LSB =  $\Delta$ :  $\sim 1\text{mV}$
- Comparators: 1023 with offset  $< 1/2$  LSB
- Assuming  $C_{in}$  for each comparator is 0.1pF & power 3mW
  - Total input capacitance:  $1023 * 100\text{fF} = 102\text{pF}$
  - Power:  $1023 * 3\text{mW} = 3\text{W}$
- $\rightarrow$  High power dissipation & large area & high input cap.

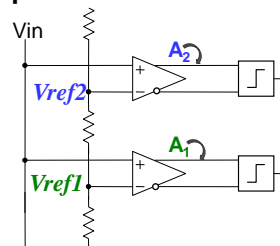
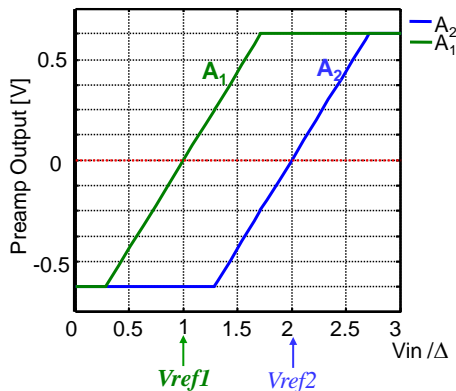
Techniques to reduce complexity & power dissipation :

- Interpolation
- Folding
- Folding & Interpolation
- Two-step, pipelining

# Interpolation

- Idea
  - Reduce number of preamps & instead interpolate between preamp outputs
- Reduced number of preamps
  - Reduced input capacitance
  - Reduced area, power dissipation
- Same number of latches ( $2^B-1$ )
- Important “side-benefit”
  - Decreased sensitivity to preamp offset  
→ Improved DNL

# Flash ADC Preamp Output

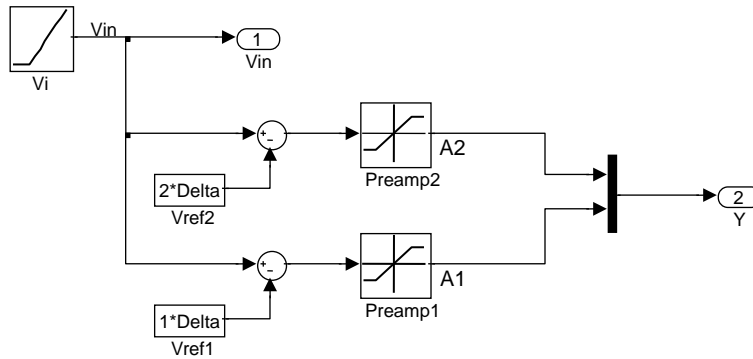


Zero crossings (to be detected by latches) at  $V_{in} =$

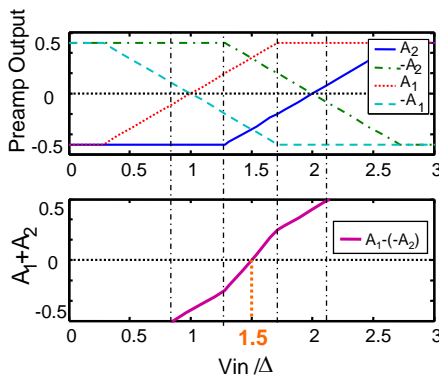
$$V_{ref1} = 1\Delta$$

$$V_{ref2} = 2\Delta$$

# Simulink Model



# Differential Preamp Output



Differential output crossings

@  $V_{in} =$   
 $V_{ref1} = 1 \Delta$   
 $V_{ref2} = 2 \Delta$

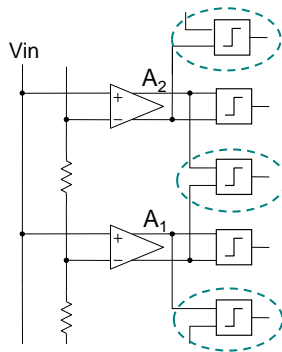
Note: Additional crossing of

$A_1 \& -A_2$  ( $A_2 \& -A_1$ )  
 $\rightarrow A_1 - (-A_2) = A_1 + A_2$   
 $\rightarrow$  cross zero at:

$V_{ref12} = 0.5 * (1+2) \Delta = 1.5 \Delta$



# Interpolation in Flash ADC



Half as many reference voltages and preamps  
Interpolation factor: x2

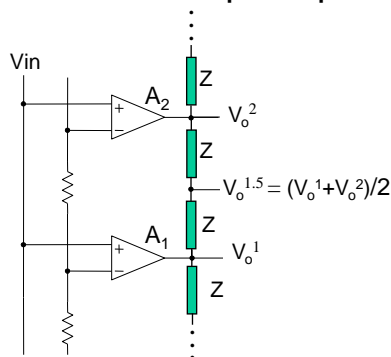
Example: For 10bit straight Flash ADC need  $2^B=1024$  preamps compared  $2^{B-1}=512$  for x2 interpolation

Possible to accomplish higher interpolation factor

→ Interpolation at the output of preamps

Compare  $A_2$  &  $-A_1$   
→ Comparator output is sign of  $A_1+A_2$

## Interpolation in Flash ADC Preamp Output Interpolation



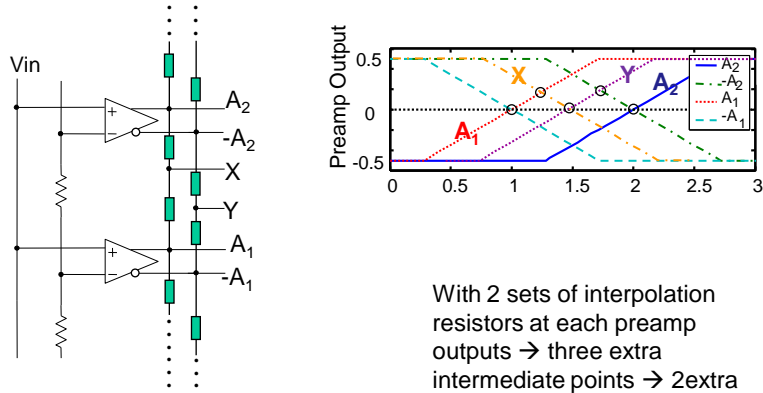
Interpolate between two consecutive output via impedance  $Z$

Choices of  $Z$ :

1. Resistors (Kimura)
2. Capacitors (Kusumoto)
3. Current mode (Roovers)

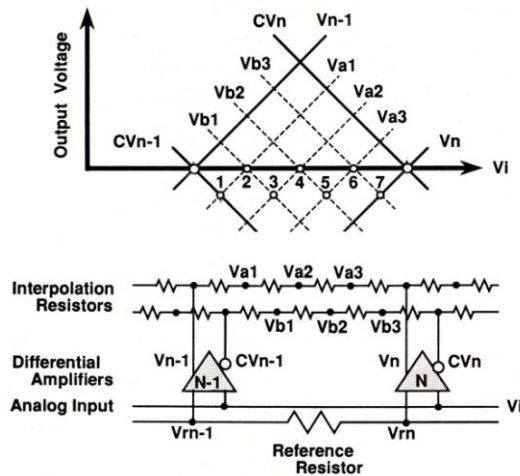
Ref: H. Kimura et al, "A 10-b 300-MHz Interpolated-Parallel A/D Converter," JSSC, pp. 438-446, April 1993  
K. Kusumoto et al, "A 10-b 20-MHz 30-mW pipelined interpolating CMOS ADC," JSSC, pp.1200 -1206, December 1993.  
R. Roovers et al, "A 175 Ms/s, 6 b, 160 mW, 3.3 V CMOS A/D converter," JSSC, pp. 938 - 944, July 1996.

## Interpolation in Flash ADC Preamp Output Interpolation



With 2 sets of interpolation resistors at each preamp outputs → three extra intermediate points → 2 extra bits

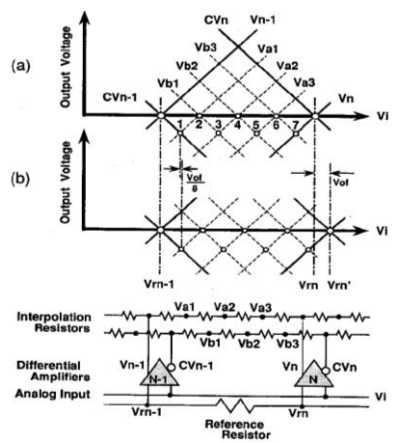
## Higher Order Resistive Interpolation



- Resistors produce additional levels
- With 4 resistors per side, the “interpolation factor”  $M=8$  → extra 3bits
- ( $M \rightarrow$  ratio of latches/preamps)

Ref: H. Kimura et al, “A 10-b 300-MHz Interpolated-Parallel A/D Converter,” JSSC April 1993, pp. 438-446

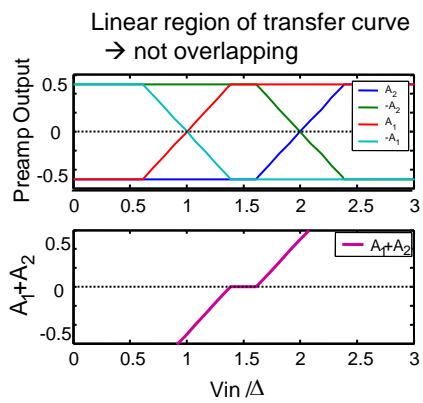
# Preamp Output Interpolation DNL Improvement



- Preamp offset distributed over M resistively interpolated voltages:  
→ Impact on DNL divided by M
- Latch offset divided by gain of preamp  
→ Use "large" preamp gain  
→ Next: Investigate how large preamp gain can be

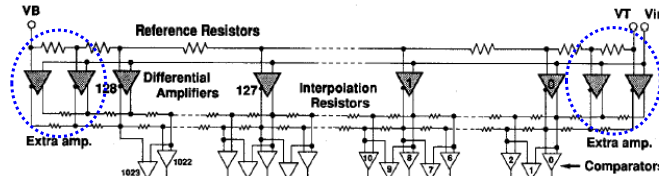
Ref: H. Kimura et al, "A 10-b 300-MHz Interpolated-Parallel A/D Converter," JSSC April 1993, pp. 438-446

# Preamp Input Range



- If linear region of preamp transfer curve do not overlap  
→ Dead-zone in the interpolated transfer curve!  
Results in error
- Linear consecutive preamp input ranges must overlap  
i.e. input range w/o output saturation  $> \Delta$
- Sets upper bound on preamp gain: **Preamp<sub>gain</sub>  $< V_{DD} / \Delta$**

# Interpolated-Parallel ADC



- 10-bit overall resolution:
- 7-bit flash (127 preamps and 128 resistors for reference V) & x8 interpolation
- Use of Gray Encoder minimizes effect of sparkle code & meta-stability

Ref: H. Kimura et al, "A 10-b 300-MHz Interpolated-Parallel A/D Converter," JSSC April 1993, pp. 438-446

# Measured Performance

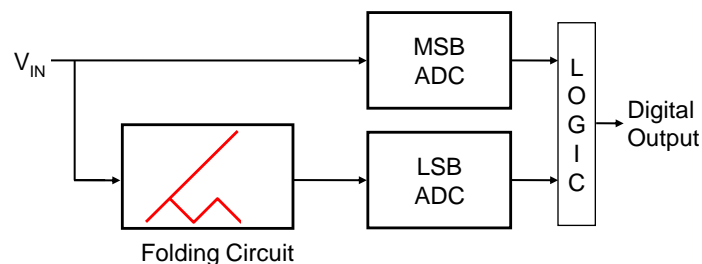
<b>Resolution</b>	<b>10 b</b>	<b>(7+3)</b>
<b>Maximum conversion frequency</b>	<b>300 MHz</b>	
<b>Integral non-linearity</b>	<b>±1.0 LSB</b>	
<b>Differential non-linearity</b>	<b>±0.4 LSB</b>	
<b>SNR/THD</b>	<b>10MHz input</b>	<b>56/-59 dB</b>
	<b>50MHz input</b>	<b>48/-47 dB</b>
<b>Input capacitance</b>	<b>8 pF</b>	<b>Low input capacitance</b>
<b>Input range</b>	<b>2 V</b>	<b>1LSB=2mV</b>
<b>Power supply</b>	<b>-5.2V</b>	
<b>Power dissipation</b>	<b>4.0W</b>	
<b>Chip size</b>	<b>9.0 × 4.2 mm<sup>2</sup></b>	
<b>Element count</b>	<b>36,000</b>	
<b>Technology</b>	<b>1.0 μm bipolar:ft=25GHz</b>	

Ref: H. Kimura et al, "A 10-b 300-MHz Interpolated-Parallel A/D Converter," JSSC April 1993, pp. 438-446

# Interpolation Summary

- Consecutive preamp transfer curve linear region need to have overlap → Limits gain of preamp to  $\sim V_{DD}/\Delta$
- The added impedance at the output of the preamp typically reduces the bandwidth and affects the maximum achievable frequencies
- DNL due to preamp offset reduced by interpolation factor M
- Interpolation reduces # of preamps and thus reduces input C- however, the # of required latches the same as “straight” Flash → Use folding to reduce the # of latches

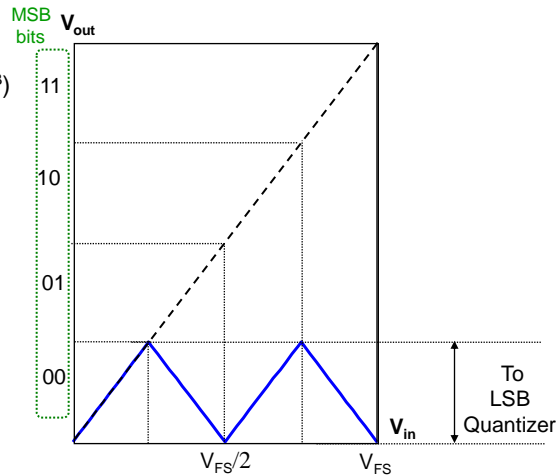
# Folding Converter



- Two ADCs operating in parallel
  - MSB ADC
  - Folder + LSB ADC
- Significantly fewer comparators compared to flash
- Medium fast
- Typically, nonidealities in folder limit resolution

## Example: Folding Factor of 4

- Folding factor:  
→ number of folds ( $2^{\text{MSB bits}}$ )
- Folder maps input to smaller range
- MSB ADC determines which fold input is in
- LSB ADC determines position within fold
- Logic circuit combines LSB and MSB results

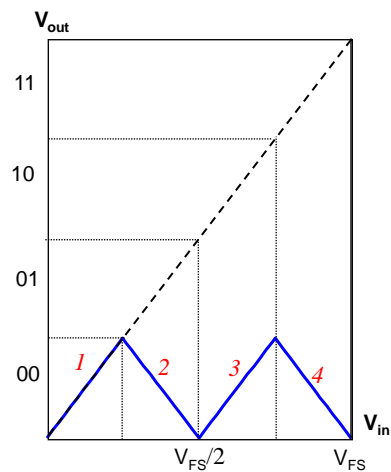


## Example: Folding Factor of 4

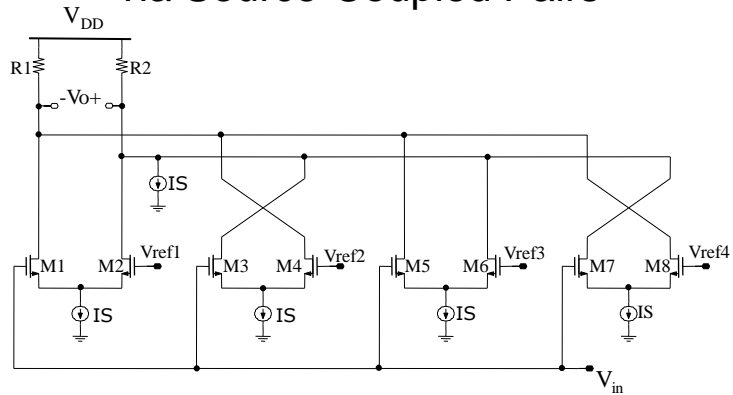
- How are folds generated?

$$\begin{aligned} \text{Fold 1} &\rightarrow V_{out} = + V_{in} \\ \text{Fold 2} &\rightarrow V_{out} = - V_{in} + V_{FS}/2 \\ \text{Fold 3} &\rightarrow V_{out} = + V_{in} - V_{FS}/2 \\ \text{Fold 4} &\rightarrow V_{out} = - V_{in} + V_{FS} \end{aligned}$$

- Note: Sign change every other fold + reference shift



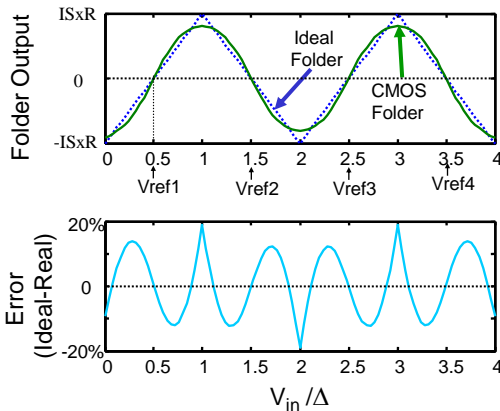
## Generating Folds via Source-Coupled Pairs



$V_{ref1} < V_{ref2} < V_{ref3} < V_{ref4}$

As  $V_{in}$  changes, only one of M1, M3, M5, M7 is on depending on the input level

## CMOS Folder Output

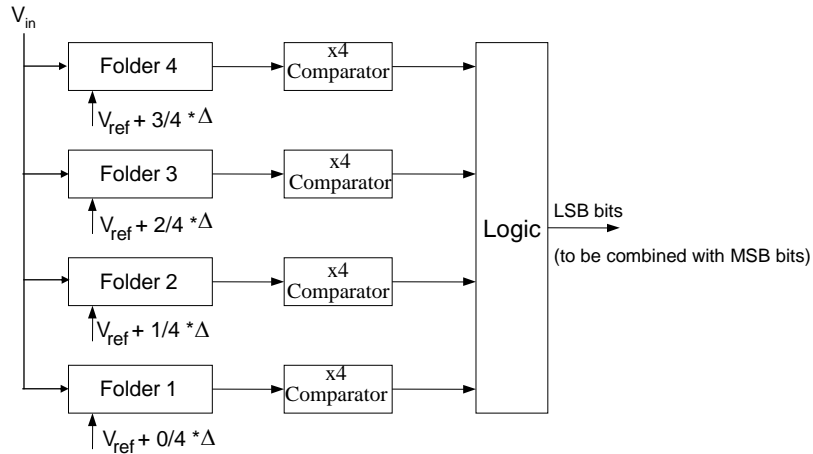


CMOS folder transfer curve max. min. portions:

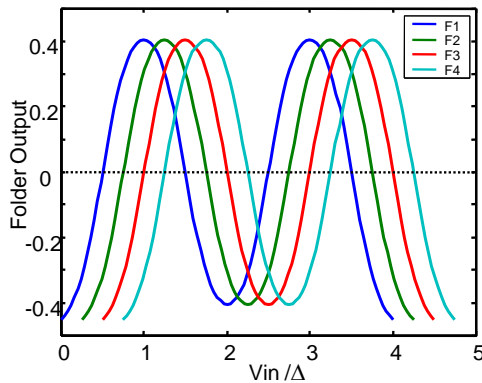
- Rounded
- Accurate only at zero-crossings

In fact, most folding ADCs do not use the folds, but only the zero-crossings!

## Parallel Folders Using Only Zero-Crossings



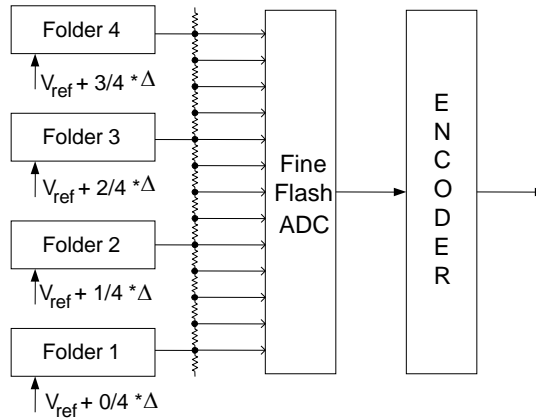
## Parallel Folder Outputs



- 4 folders with 4 folds each
- 16 zero crossings
- → 4 LSB bits
- Higher resolution
  - More folders → Large complexity
  - Better solution: Combine with interpolation

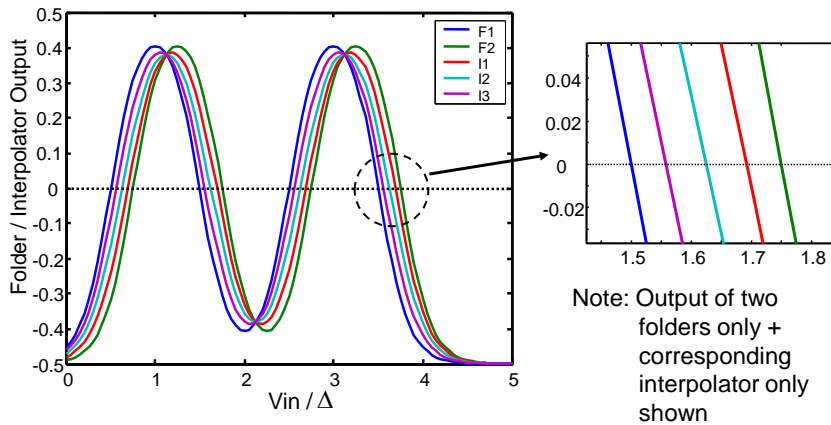


# Folding & Interpolation



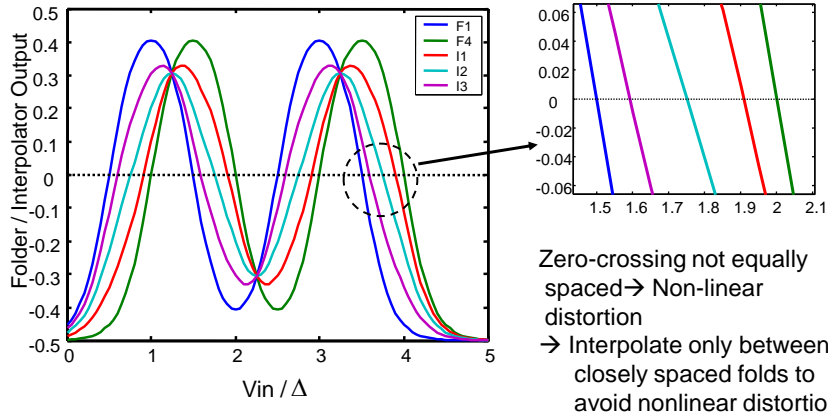
# Folder / Interpolator Output

Example: 4 Folders + 4 Resistive Interpolator per Stage

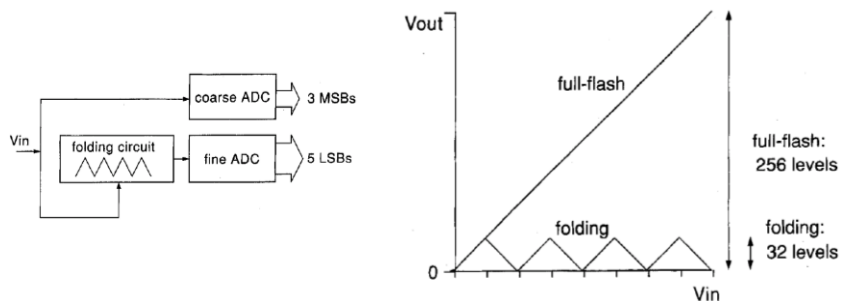


## Folder / Interpolator Output

Example: 2 Folders + 8 Resistive Interpolator per Stage

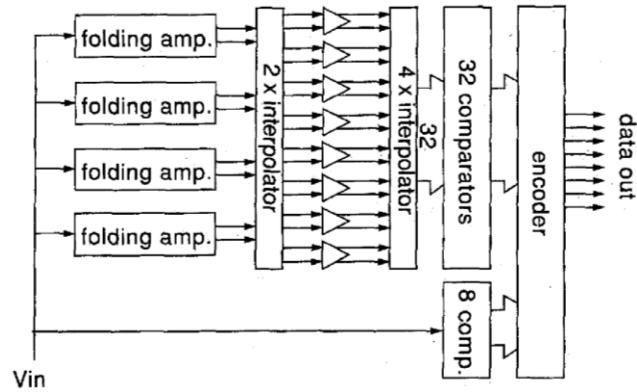


## A 70-MS/s 110-mW 8-b CMOS Folding and Interpolating A/D Converter



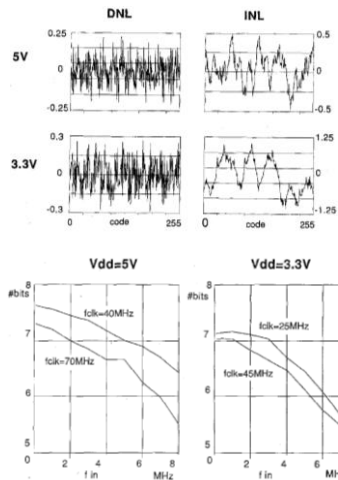
Ref: B. Nauta and G. Venes, JSSC Dec 1985, pp. 1302-8

## A 70-MS/s 110-mW 8-b CMOS Folding and Interpolating A/D Converter



Note:  
Total of **40** (MSB=8, LSB=32) comparators compared to  $2^8-1=$  **255** for straight flash

## A 70-MS/s 110-mW 8-b CMOS Folding and Interpolating A/D Converter



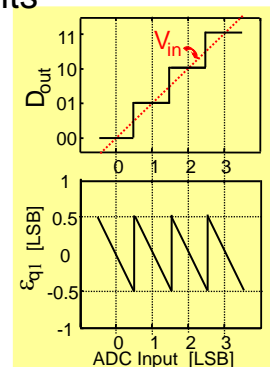
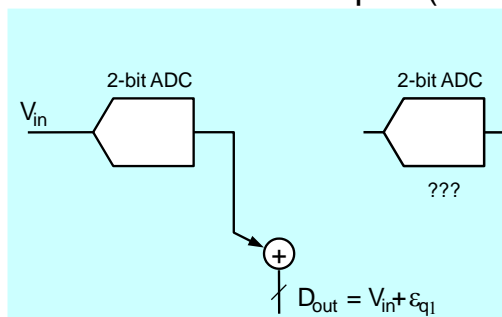
parameter	
resolution	8bit
input capacitance	4.8 pF <span style="color:red">←</span>
reference ladder resistance	720 Ω
active area	0.7 mm <sup>2</sup>
technology	0.8μm, 1 poly, 2 metal, CMOS
supply voltage	$V_{dd}=5V$ $V_{dd}=3.3V$
analog input	2V <sub>pp</sub> 1.4V <sub>pp</sub>
Integral nonlinearity	$\pm 0.5LSB$ $\pm 1.0LSB$ <span style="color:red">←</span>
Differential nonlinearity	$\pm 0.2LSB$ $\pm 0.3LSB$ <span style="color:red">←</span>
max. clock frequency	70MHz    45MHz
power dissipation	110mW    45mW

Ref: B. Nauta and G. Venes, JSSC Dec 1985, pp. 1302-8

# ADC Architectures

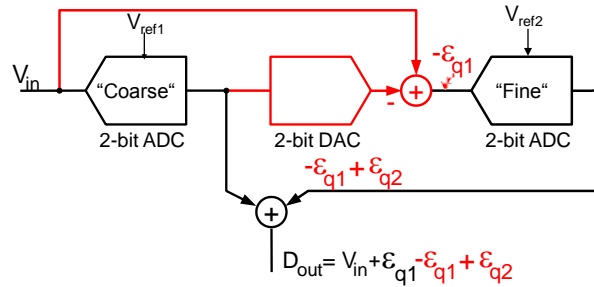
- Slope type converters
- Successive approximation
- Flash
- Interpolating & Folding
- ➔ • Residue type ADCs
  - Two-step Flash
  - Pipelined ADCs
  - ...
- Time-interleaved / parallel converter
- Oversampled ADCs

## Two-Step Example: (2+2)Bits



- Using only one ADC: output contains large quantization error
- "Missing voltage" or "residue" ( $-\epsilon_{q1}$ )
- Idea: Use second ADC to quantize and add  $-\epsilon_{q1}$

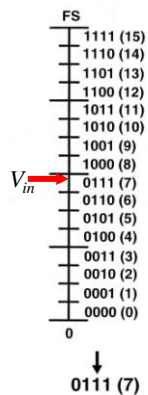
## Two Stage Example



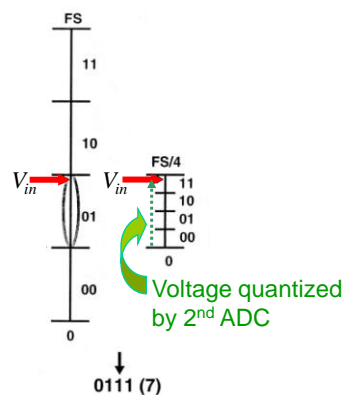
- Use DAC to compute missing voltage
- Add quantized representation of missing voltage
- Why does this help? How about  $\epsilon_{q2}$ ?
- Since maximum voltage at input of the 2<sup>nd</sup> ADC is  $V_{ref1}/4$  then for 2<sup>nd</sup> ADC  $V_{ref2} = V_{ref1}/4$  and thus  $\epsilon_{q2} = \epsilon_{q1}/4 = V_{ref1}/16 \rightarrow$  4bit overall resolution

## Two Step (2+2) Flash ADC

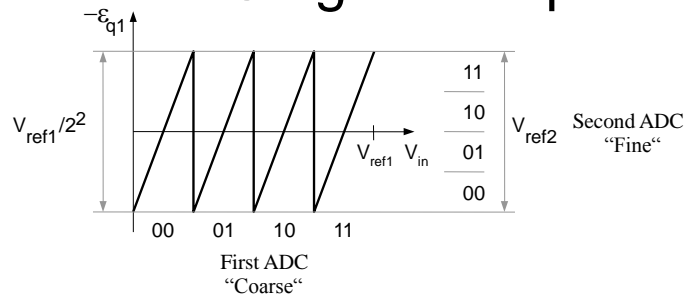
4-bit Straight Flash ADC



Ideal 2-step Flash ADC



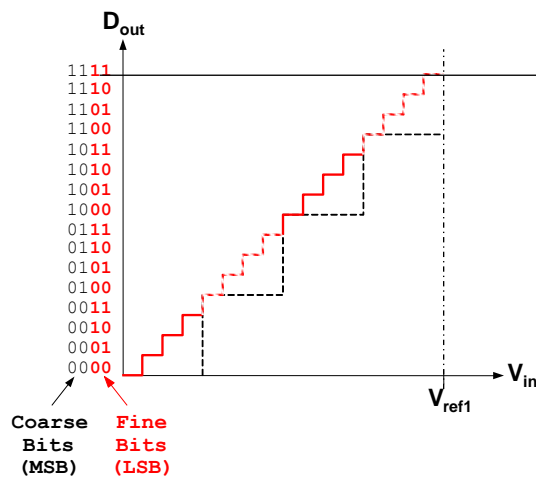
# Two Stage Example



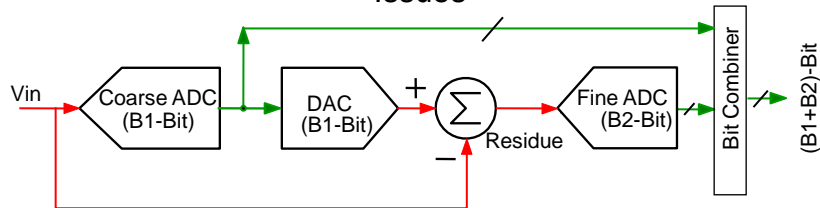
- Fine ADC is re-used  $2^2$  times
- Fine ADC's full scale range needs to span only 1 LSB of coarse quantizer

$$\epsilon_{q2} = \frac{V_{ref2}}{2^2} = \frac{V_{ref1}}{2^2 \cdot 2^2}$$

# Two-Stage (2+2) ADC Transfer Function

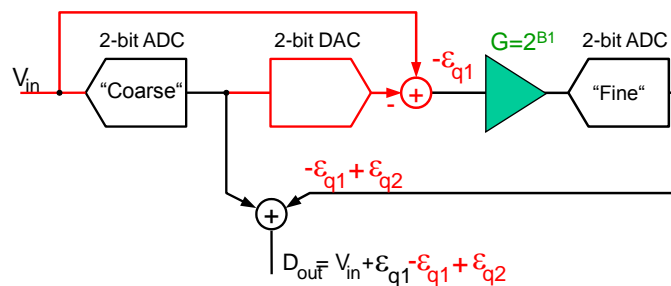


## Residue or Multi-Step Type ADC Issues



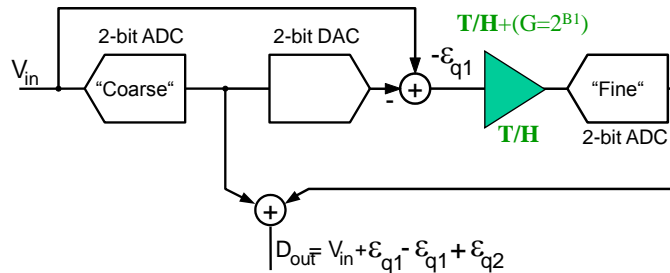
- Operation:
  - Coarse ADC determines MSBs
  - DAC converts the coarse ADC output to analog- Residue is found by subtracting ( $V_{in} - V_{DAC}$ )
  - Fine ADC converts the residue and determines the LSBs
  - Bits are combined in digital domain
- Issue:
  1. Fine ADC has to have precision in the order of overall ADC 1/2LSB
  2. Speed penalty  $\rightarrow$  Need at least 1 clock cycle per extra series stage to resolve one sample

## Solution to Issue (1) Reducing Precision Required for Fine ADC



- Accuracy needed for fine ADC relaxed by introducing inter-stage gain
  - Example: By adding gain of  $x(G=2^{B1}=4)$  prior to fine ADC in (2+2)bit case, precision required for fine ADC is reduced to 2-bit only!
  - Additional advantage- coarse and fine ADC can be identical stages

## Solution to Issue (2) Increasing ADC Throughput



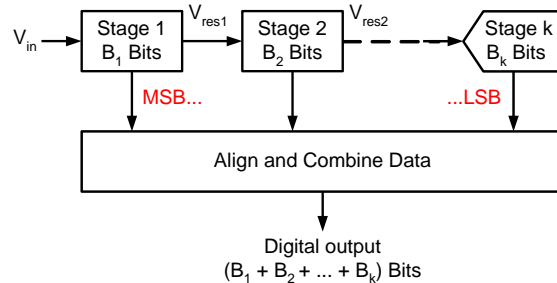
- Conversion time significantly decreased by employing T/H between stages
  - All stages busy at all times → operation concurrent
  - During one clock cycle coarse & fine ADCs operate concurrently:
    - First stage samples/converts/generates residue of input signal sample #  $n$
    - While 2<sup>nd</sup> stage samples/converts residue associated with sample #  $n-1$

## Residue Type ADCs

- Two-Step flash
- ➔ • Pipelined ADCs
  - Basic operation
  - Effect of sub-ADC, sub-DAC, gain stage non-idealities on overall ADC performance
    - Error correction by adding redundancy
    - Digital calibration
    - Correction for inter-stage gain nonlinearity
  - Implementation
    - Practical circuits
    - Stage scaling
    - Combining the bits
    - Stage implementation
      - Circuits
      - Noise budgeting
    - How many bits per stage?

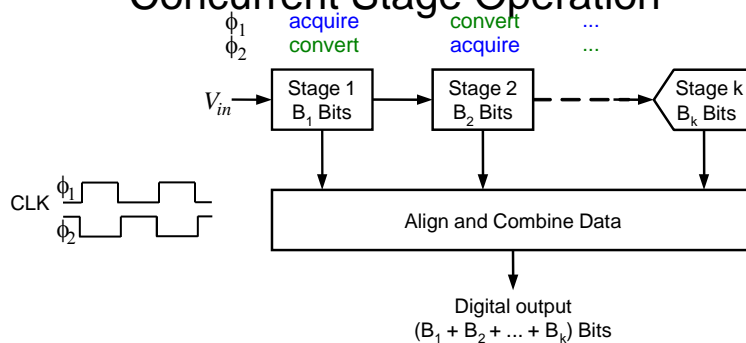


## Pipeline ADC Block Diagram



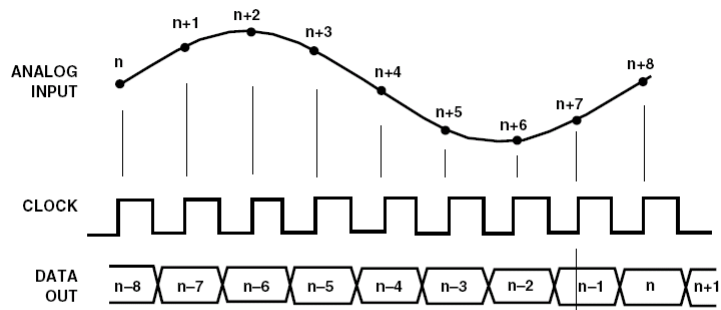
- Idea: Cascade several low resolution stages to obtain high overall resolution (e.g. 10bit ADC can be built with series of 10 ADCs each 1-bit only!)
- Each stage performs coarse A/D conversion and computes its quantization error, or "residue"
- All stages operate concurrently

## Pipeline ADC Concurrent Stage Operation



- Stages operate on the input signal like a shift register
- New output data **every** clock cycle, but each stage introduces at least  $\frac{1}{2}$  clock cycle latency

## Pipeline ADC Latency



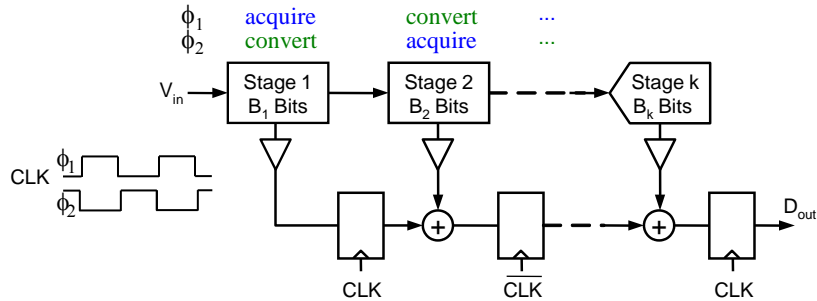
Note: One conversion per clock cycle & 8 clock cycle latency

[Analog Devices, AD 9226 Data Sheet]

## Pipeline ADC Characteristics

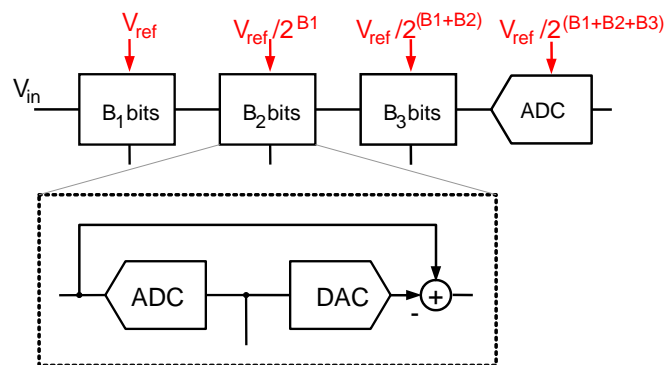
- Number of components (stages) grows linearly with resolution
- Pipelining
  - Trading latency for overall component count
  - Latency may be an issue in e.g. control systems
  - Throughput limited by speed of one stage → Fast
- Versatile: 8...16bits, 1...400MS/s
- One important feature of pipeline ADC: many analog circuit non-idealities can be corrected digitally

## Pipeline ADC Digital Data Alignment



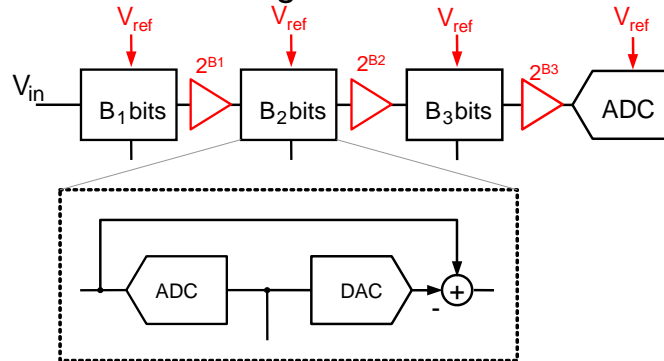
- Digital shift register aligns sub-conversion results in time

## Cascading More Stages



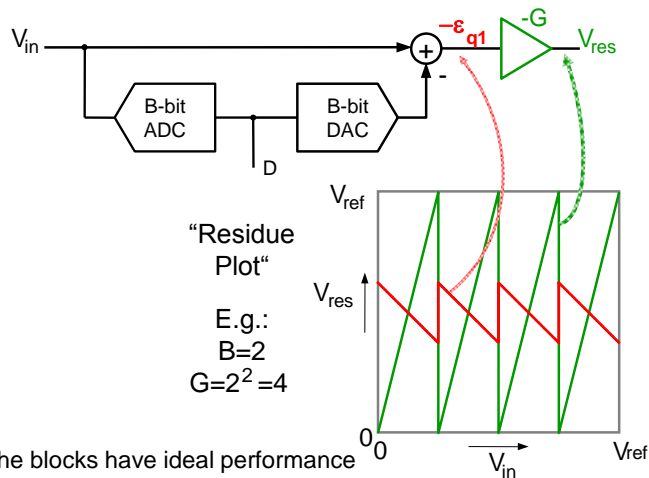
- LSB of last stage becomes very small
- All stages need to have full precision
- Impractical to generate several  $V_{ref}$

## Pipeline ADC Inter-Stage Gain Elements



- Practical pipelines by adding inter-stage gain  $\rightarrow$  use single  $V_{ref}$
- Precision requirements decrease down the pipe
  - Advantageous for noise, matching (later), power dissipation

## Complete Pipeline Stage

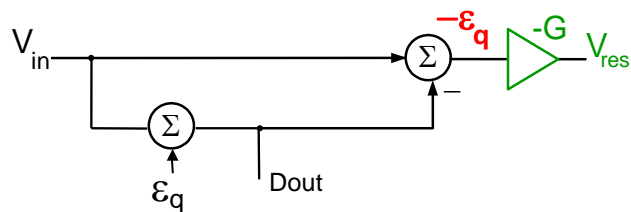


Note: None of the blocks have ideal performance  
Question: What is the effect of the non-idealities?

# Pipeline ADC Errors

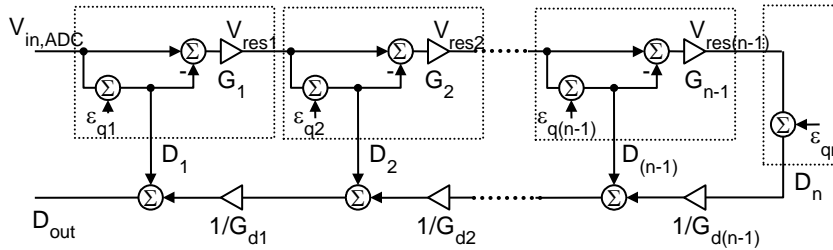
- Non-idealities associated with sub-ADCs, sub-DACs and gain stages → error in overall pipeline ADC performance
- Need to find means to tolerate/correct errors
- Important sources of error
  - Sub-ADC errors- comparator offset
  - Gain stage offset
  - Gain stage gain error
  - Sub-DAC error

## Pipeline ADC Single Stage Model



$$V_{res} = G \times \epsilon_q$$

## Pipeline ADC Multi-Stage Model



$$D_{out} = V_{in,ADC} + \epsilon_{q1} \left( 1 - \frac{G_1}{G_{d1}} \right) + \frac{\epsilon_{q2}}{G_{d1}} \left( 1 - \frac{G_2}{G_{d2}} \right) + \dots + \frac{\epsilon_{q(n-1)}}{\prod_{j=1}^{n-2} G_{dj}} \left( 1 - \frac{G_{(n-1)}}{G_{d(n-1)}} \right) + \frac{\epsilon_{qn}}{\prod_{j=1}^{n-1} G_{dj}}$$

## Pipeline ADC Model

- If the "Analog" and "Digital" gain/loss is precisely matched:

$$D_{out} = V_{in,ADC} + \frac{\epsilon_{qn}}{\prod_{j=1}^{n-1} G_j} \quad \text{where } \epsilon_{qn} = \frac{V_{ref}}{2^{B_n}} \quad \& B_n = \# \text{ of bits in final stage}$$

$$D.R. = 20 \log \frac{\text{rms FS Signal}}{\text{rms Quant. Noise}} = 20 \log \frac{\frac{V_{ref}}{2\sqrt{2}}}{\frac{V_{ref}}{\sqrt{12} \times 2^{B_n} \prod_{j=1}^{n-1} G_j}} = 20 \log \left( \sqrt{\frac{3}{2}} \times 2^{B_n} \times \prod_{j=1}^{n-1} G_j \right)$$

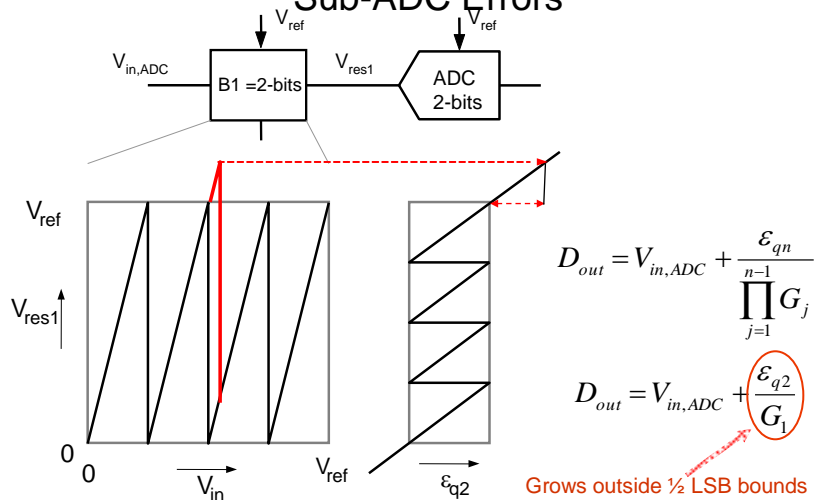
$$B_{ADC} \approx \log_2 \left( 2^{B_n} \times \prod_{j=1}^{n-1} G_j \right)$$

$$B_{ADC} \approx B_n + \log_2 \prod_{j=1}^{n-1} G_j$$

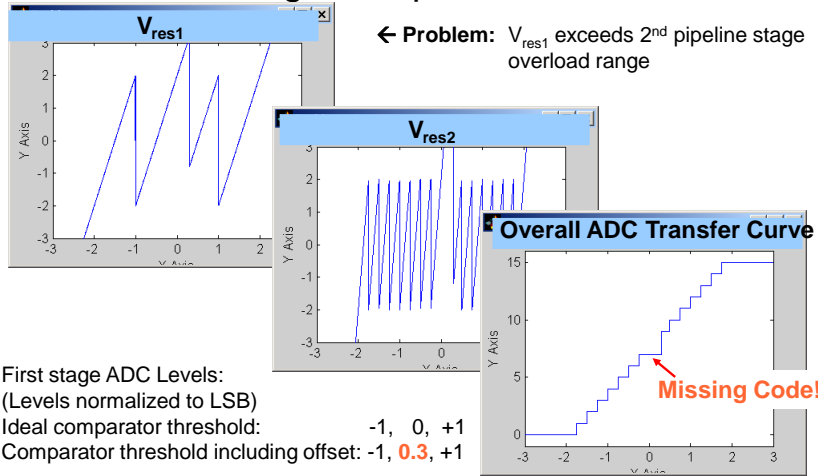
## Pipeline ADC Observations

- The aggregate **ADC resolution is independent of sub-ADC resolution!**
- *Effective* stage resolution  $B_j = \log_2(G_j)$
- **Overall conversion error does not (directly) depend on sub-ADC errors!**
- Only error term in  $D_{out}$  contains quantization error associated with the last stage
- So why do we care about sub-ADC errors?
  - Go back to two stage example

## Pipeline ADC Sub-ADC Errors



## Pipeline ADC 1<sup>st</sup>-Stage Comparator Offset

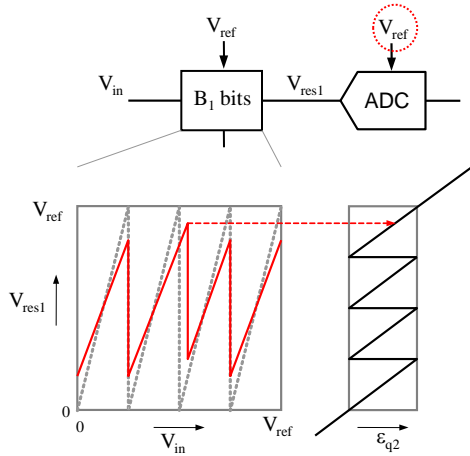


## Pipeline ADC Three Ways to Deal with Sub-ADC Errors

- All involve “sub-ADC redundancy”
- Redundancy in stage that produces errors
  - Choose gain for residue to be processed by the 2<sup>nd</sup> stage  $< 2^{B_1}$
  - Higher resolution sub-ADC & sub-DAC
- Redundancy in succeeding stage(s)



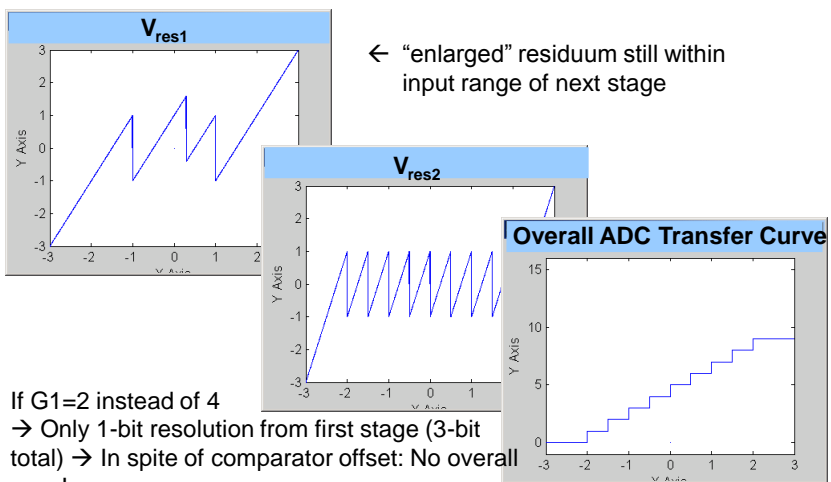
## (1) Inter-Stage Gain Following 1<sup>st</sup> Stage $< 2^{B_1}$



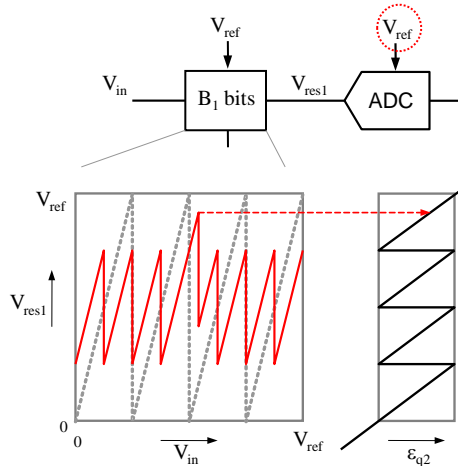
- Choose  $G_1$  less than  $2^{B_1}$
- Effective stage resolution could become non-integer  
 $B_{1\text{eff}} = \log_2 G_1$
- E.g. if  $G_1 = 3.8 \rightarrow B_{1\text{eff}} = 1.8\text{bit}$

-Ref: A. Karanicolas et. al.,  
JSSC 12/1993

## Correction Through Redundancy



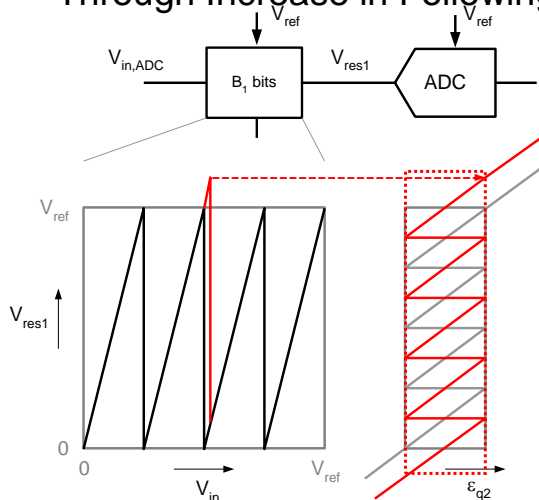
## (2) Higher Resolution Sub-ADC



- Keep  $G_1$  precise power of two (e.g.  $G_1=4$ )
- Add extra decision levels in sub-ADC (e.g. add 1 extra bit to 1<sup>st</sup> stage)
- E.g.  $B_1=B_{1\text{eff}}+1$

-Ref: Singer et. al., VSLI 1996

## (3) Over-Range Accommodation Through Increase in Following Stage Resolution



- No redundancy in stage with errors
- Add extra decision levels in succeeding stage

❖ Ref: Opris et. al., JSSC 12/1998