EE247
Administrative

– Homework #1 will be posted on EE247 site and is due Sept. 9th

– Office hours held @ 201 Cory Hall:
  - Tues. and Thurs.: 4 to 5pm

EE247
Lecture 3

- Active Filters
  - Active biquads-
  - How to build higher order filters?
    - Integrator-based filters
      - Signal flowgraph concept
      - First order integrator-based filter
      - Second order integrator-based filter & biquads
  - High order & high Q filters
    - Cascaded biquads & first order filters
      - Cascaded biquad sensitivity to component mismatch
    - Ladder type filters

Filters

2^{nd} Order Transfer Functions (Biquads)

- Biquadratic (2^{nd} order) transfer function:

$$H(s)=\frac{1}{1+\frac{s}{\omega_P Q_P}+\frac{s^2}{\omega_P^2}}$$

$$|H(j\omega)|=\frac{1}{\sqrt{\left(1-\frac{\omega^2}{\omega_P^2}\right)^2+\left(\frac{\omega}{\omega_P Q_P}\right)^2}} \rightarrow \begin{cases} |H(j\omega)|_{\omega=0}=1 \\ |H(j\omega)|_{\omega\to\infty}=0 \\ |H(j\omega)|_{\omega=\omega_P}=Q_P \end{cases}$$

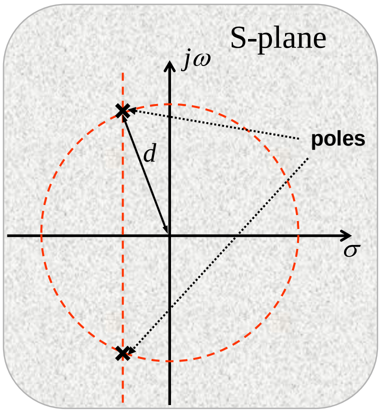
$$\text{Biquad poles @: } s=-\frac{\omega_P}{2Q_P}\left(1\pm\sqrt{1-4Q_P^2}\right)$$

Note: for $Q_P \le \frac{1}{2}$ *poles are real, complex otherwise*

Biquad Complex Poles

$Q_P > \frac{1}{2} \rightarrow$ Complex conjugate poles:

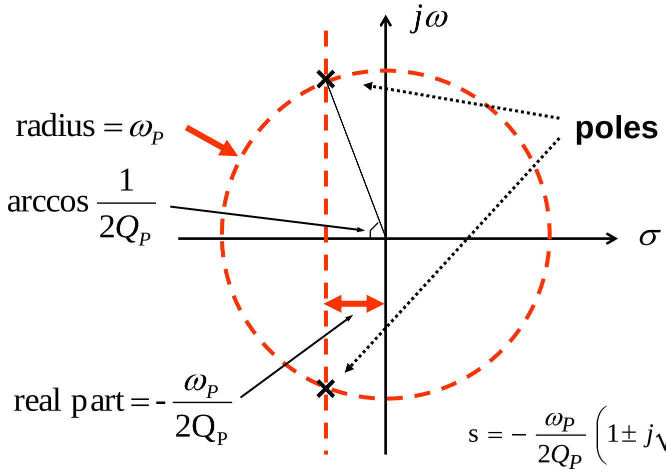
$$s=-\frac{\omega_P}{2Q_P}\left(1\pm j\sqrt{4Q_P^2-1}\right)$$



Distance from origin in s-plane:

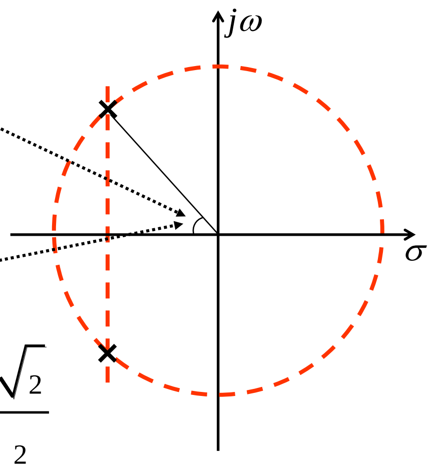
$$d^2=\left(\frac{\omega_P}{2Q_P}\right)^2\left(1+4Q_P^2-1\right)$$
$$=\omega_P^2$$

s-Plane



$$s = -\frac{\omega_P}{2Q_P}\left(1 \pm j\sqrt{4Q_P^2 - 1}\right)$$

Example
2nd Order Butterworth



$\arccos \frac{1}{2Q_P}$

Since 2nd order Butterworth:

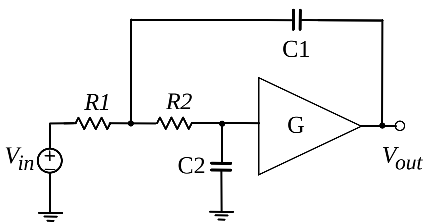
$\angle = 45$ deg *ree*

$$\rightarrow Cos\,\frac{\pi}{4} = \frac{\sqrt{2}}{2} \rightarrow Q_p = \frac{\sqrt{2}}{2}$$

Implementation of Biquads

- Passive RC: only *real poles* → can't implement *complex conjugate poles*
- Terminated LC
  - Low power, since it is passive
  - Only fundamental noise sources → load and source resistance
  - As previously analyzed, not feasible in the monolithic form for $f < 350MHz$
- Active Biquads
  - Many topologies can be found in filter textbooks!
  - Widely used topologies:
    - Single-opamp biquad: *Sallen-Key*
    - Multi-opamp biquad: *Tow-Thomas*
    - Integrator based biquads

Active Biquad
Sallen-Key Low-Pass Filter



$$H(s) = \frac{G}{1 + \frac{s}{\omega_P Q_P} + \frac{s^2}{\omega_P^2}}$$

$$\omega_P = \frac{1}{\sqrt{R_1 C_1 R_2 C_2}}$$

$$Q_P = \frac{\omega_P}{\frac{1}{R_1 C_1} + \frac{1}{R_2 C_1} + \frac{1-G}{R_2 C_2}}$$

- Single gain element
- Can be implemented both in discrete & monolithic form
- "Parasitic sensitive"
- Versions for LPF, HPF, BP, …

  → Advantage: Only one opamp used

  → Disadvantage: Sensitive to parasitic – all pole no finite zeros

Addition of Imaginary Axis Zeros

- Sharpen transition band
- Can “notch out” interference
  - Band-reject filter
- High-pass filter (HPF)

$$H(s) = K\frac{1+\left(\frac{s}{\omega_Z}\right)^2}{1+\frac{s}{\omega_P Q_P}+\left(\frac{s}{\omega_P}\right)^2}$$

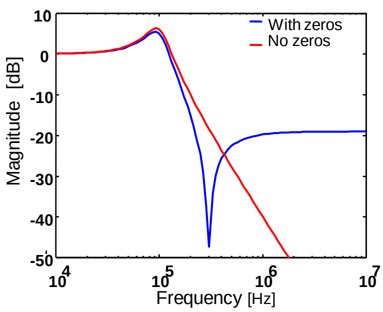
$$\left|H(j\omega)\right|_{\omega\to\infty} = K\left(\frac{\omega_P}{\omega_Z}\right)^2$$

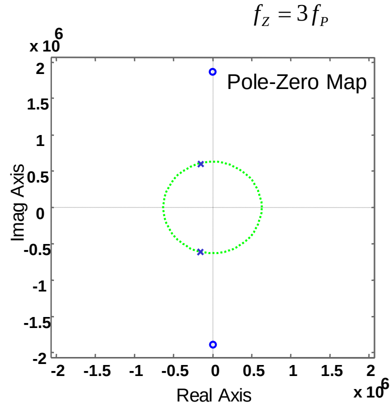
Note: Always represent transfer functions as a product of a gain term, poles, and zeros (pairs if complex). Then all coefficients have a physical meaning, and readily identifiable units.

Imaginary Zeros

- Zeros substantially sharpen transition band
- At the expense of reduced stop-band attenuation at high frequencies

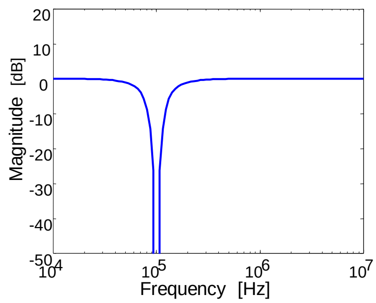
$$f_P = 100kHz$$
$$Q_P = 2$$
$$f_Z = 3f_P$$

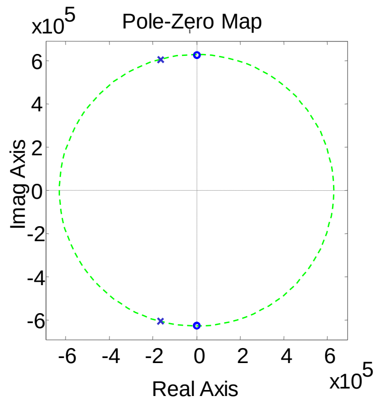




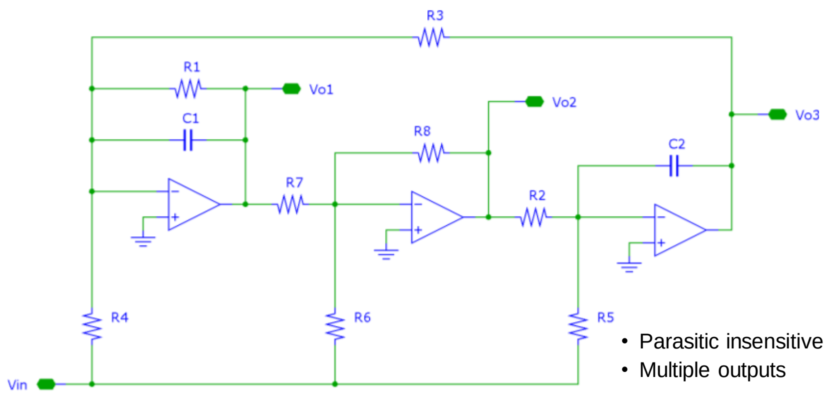
Moving the Zeros

$f_P = 100kHz$
$Q_P = 2$
$\underline{f_Z = f_P}$





Tow-Thomas Active Biquad



- Parasitic insensitive
- Multiple outputs

Ref: P. E. Fleischer and J. Tow, “Design Formulas for biquad active filters using three operational amplifiers,” Proc. IEEE, vol. 61, pp. 662-3, May 1973.

Frequency Response

$$\frac{V_{o1}}{V_{in}} = -k_2 \frac{(b_2 a_1 - b_1)s + (b_2 a_0 - b_0)}{s^2 + a_1 s + a_0}$$

$$\frac{V_{o2}}{V_{in}} = \frac{b_2 s^2 + b_1 s + b_0}{s^2 + a_1 s + a_0}$$

$$\frac{V_{o3}}{V_{in}} = -\frac{1}{k_1 \sqrt{a_0}} \frac{(b_0 - b_2 a_0)s + (a_1 b_0 - a_0 b_1)}{s^2 + a_1 s + a_0}$$

- V_{o2} implements a general biquad section with arbitrary poles and zeros
- V_{o1} and V_{o3} realize the same poles but are limited to at most one finite zero
- Possible to use combination of 3 outputs

Component Values

$$b_0 = \frac{R_8}{R_3 R_5 R_7 C_1 C_2}$$

$$b_1 = \frac{1}{R_1 C_1}\left(\frac{R_8}{R_6} - \frac{R_1 R_8}{R_4 R_7}\right)$$

$$b_2 = \frac{R_8}{R_6}$$

$$a_0 = \frac{R_8}{R_2 R_3 R_7 C_1 C_2}$$

$$a_1 = \frac{1}{R_1 C_1}$$

$$k_1 = \sqrt{\frac{R_2 R_8 C_2}{R_3 R_7 C_1}}$$

$$k_2 = \frac{R_7}{R_8}$$

given a_i, b_i, k_i, C_1, C_2 and R_8

$$R_1 = \frac{1}{a_1 C_1}$$

$$R_2 = \frac{k_1}{\sqrt{a_0} C_2}$$

$$R_3 = \frac{1}{k_1 k_2} \frac{1}{\sqrt{a_0} C_1}$$

$$R_4 = \frac{1}{k_2} \frac{1}{a_1 b_2 - b_1} \frac{1}{C_1}$$

$$R_5 = \frac{k_1 \sqrt{a_0}}{b_0 C_2}$$

$$R_6 = \frac{R_8}{b_2}$$

$$R_7 = k_2 R_8$$

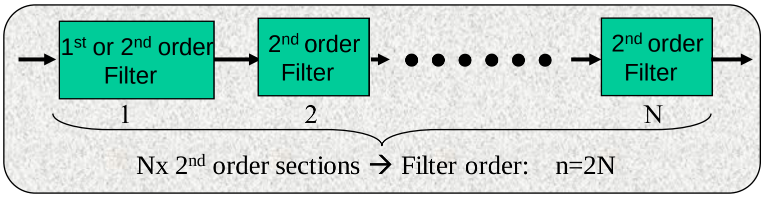
it follows that

$$\omega_P = \sqrt{\frac{R_8}{R_2 R_3 R_7 C_1 C_2}}$$

$$Q_P = \omega_P R_1 C_1$$

Higher-Order Filters in the Integrated Form

- One way of building higher-order filters (n>2) is via cascade of 2nd order biquads & 1st order , e.g. Sallen-Key,or Tow-Thomas, & RC



Cascade of 1st and 2nd order filters:

☺ Easy to implement

☹ Highly sensitive to component mismatch -good for low Q filters only

→ For high Q applications good alternative: Integrator-based ladder filters

Integrator Based Filters

- Main building block for this category of filters
  → Integrator
- By using signal flowgraph techniques
  → Conventional RLC filter topologies can be converted to integrator based type filters
- How to design integrator based filters?
  - Introduction to signal flowgraph techniques
  - 1st order integrator based filter
  - 2nd order integrator based filter
  - High order and high Q filters

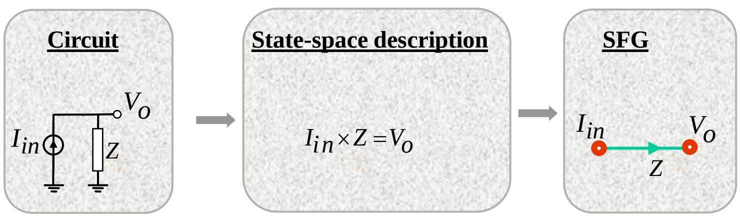
What is a Signal Flowgraph (SFG)?

- SFG → Topological network representation consisting of nodes & branches- used to convert one form of network to a more suitable form (e.g. passive RLC filters to integrator based filters)
- Any network described by a set of linear differential equations can be expressed in SFG form
- For a given network, many different SFGs exists
- Choice of a particular SFG is based on practical considerations such as type of available components

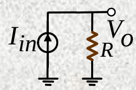
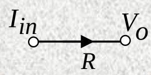
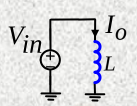
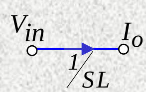
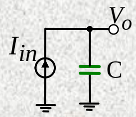
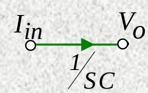
*Ref: W.Heinlein & W. Holmes, "Active Filters for Integrated Circuits", Prentice Hall, Chap. 8, 1974.

What is a Signal Flowgraph (SFG)?

- Signal flowgraph technique consist of **nodes & branches**:
  - **Nodes** represent variables (*V & I* in our case)
  - **Branches** represent transfer functions (we will call the transfer function *branch multiplication factor* or *BMF*)
- To convert a network to its SFG form, *KCL* & *KVL* is used to derive state space description
- Simple example:

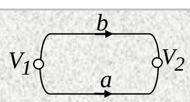


Signal Flowgraph (SFG) Examples

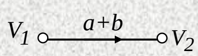
Circuit	State-space description	SFG
	$I_{in} \times R = V_o$	
	$V_{in} \times \frac{1}{SL} = I_o$	
	$I_{in} \times \frac{1}{SC} = V_o$	

Useful Signal Flowgraph (SFG) Rules

- Two parallel branches can be replaced by a single branch with overall *BMF* equal to **sum** of two *BMF*s

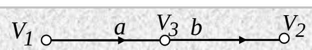


$a.V_1+b.V_1=V_2$

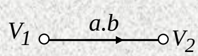


$(a+b).V_1=V_2$

- A node with only one incoming branch & one outgoing branch can be eliminated & replaced by a single branch with *BMF* equal to the **product** of the two *BMF*s



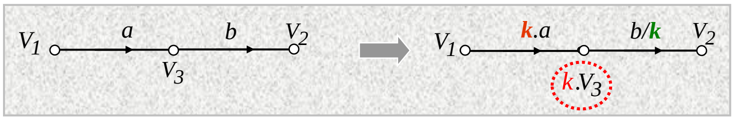
$a.V_1=V3$ (1)
$b.V_3=V_2$ (2)
Substituting for V3 from (1) in (2)



$(a.b).V_1=V_2$

Useful Signal Flowgraph (SFG) Rules

- An intermediate node can be multiplied by a factor (k). *BMF*s for incoming branches have to be multiplied by k and outgoing branches divided by k



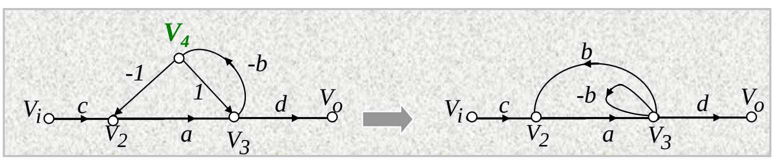
$a.V_1=V_3$ *(1)*
$b.V_3=V_2$ *(2)*

Multiply both sides of *(1)* by k
$(a.k) \, . \, V_1= k.V_3$ *(1)*

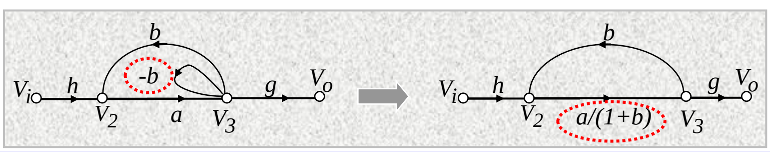
Divide & multiply left side of *(2)* by k
$(b/k) \, . \, k.V_3 = V_2$ *(2)*

Useful Signal Flowgraph (SFG) Rules

- Simplifications can often be achieved by shifting or eliminating nodes
- Example: eliminating node V_4

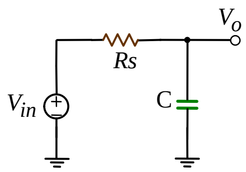


- A self-loop branch with *BMF* y can be eliminated by multiplying the *BMF* of incoming branches by $1/(1-y)$



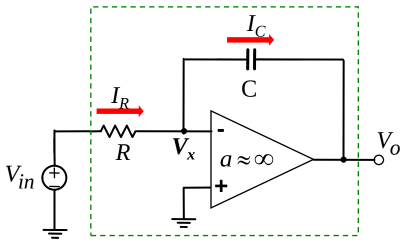
Integrator Based Filters
1st Order LPF

- Conversion of simple lowpass RC filter to integrator-based type by using signal flowgraph techniques



$$\frac{V_o}{V_{in}} = \frac{1}{1+sRC}$$

What is an Integrator?
Example: Single-Ended Opamp-RC Integrator

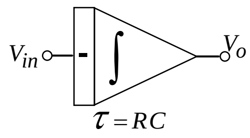


- *Node x: since opamp has high gain* $V_x = -V_o/a \rightarrow 0$
- *Node x is at "virtual ground"*
  - → *No voltage swing at* V_x *combined with high opamp input impedance*
  - → *No input opamp current*

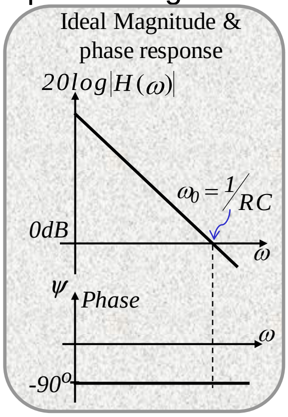
$$\frac{V_{in}}{R} = -V_o sC \quad , \quad V_o = -V_{in} \times \frac{1}{sRC} \quad , \quad V_o = -\frac{1}{RC}\int V_{in}\,dt$$

What is an Integrator?

Example: Single-Ended Opamp-RC Integrator



$$V_o / V_{in} = -\frac{1}{sRC}$$

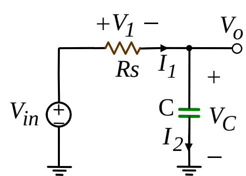


Note: Practical integrator in CMOS technology has input & output both in the form of voltage and not current → Consideration for SFG derivation

1st Order LPF

Convert RC Prototype to Integrator Based Version

1. Start from circuit prototype-
   Name voltages & currents for all components



2. Use KCL & KVL to derive state space description in such a way to have BMFs in the integrator form:
   → Capacitor voltage expressed as function of its current $V_{Cap.}=f(I_{Cap.})$
   → Inductor current as a function of its voltage $I_{Ind.}=f(V_{Ind.})$

3. Use state space description to draw signal flowgraph (SFG) (see next page)

Integrator Based Filters

First Order LPF

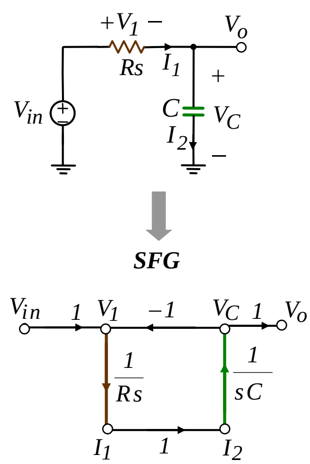
$V_1 = V_{in} - V_C$

$V_C = I_2 \times \frac{1}{sC}$ ← *Integrator form*

$V_o = V_C$

$I_1 = V_1 \times \frac{1}{Rs}$

$I_2 = I_1$



- All voltages & currents → nodes of SFG
- Voltage nodes on top, corresponding current nodes below each voltage node

Normalize

- Since integrators are the main building blocks→ require in & out signals in the form of voltage (not current)

  → Convert all currents to voltages by multiplying current nodes by a scaling resistance R^*

  → Corresponding *BMF*s should then be scaled accordingly

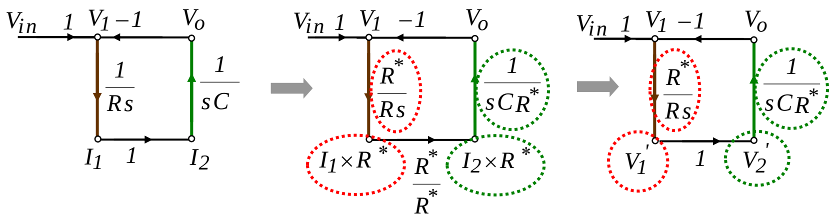
$V_1 = V_{in} - V_o$, $I_1 = \frac{V_1}{Rs}$, $V_o = \frac{I_2}{sC}$, $I_2 = I_1$

⇒ $V_1 = V_{in} - V_o$, $I_1 R^* = \frac{R^*}{Rs} V_1$, $V_o = \frac{I_2 R^*}{sC\,R^*}$, $I_2 R^* = I_1 R^*$

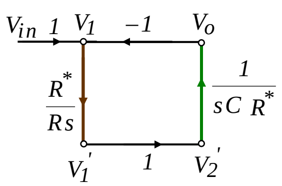
⇒ $I_x R^* = V_x'$ ⇒

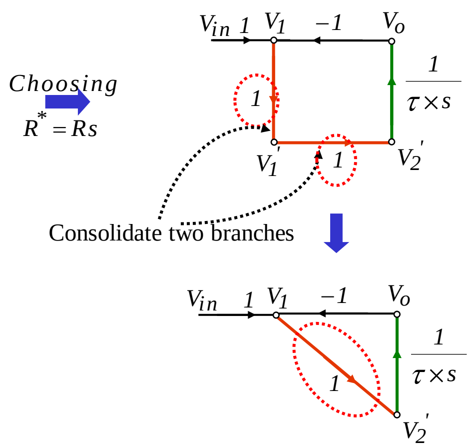
$V_1 = V_{in} - V_o$, $V_1' = \frac{R^*}{Rs} V_1$, $V_o = \frac{V_2'}{sC\,R^*}$, $V_2' = V_1'$

1^{st} Order Lowpass Filter SGF
Normalize



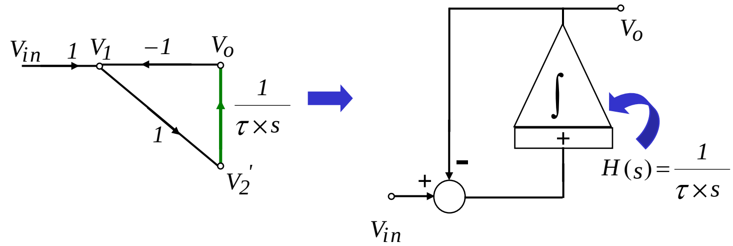
1^{st} Order Lowpass Filter SGF
Synthesis





$$R^* = Rs \quad , \; \tau = R^* \times C$$

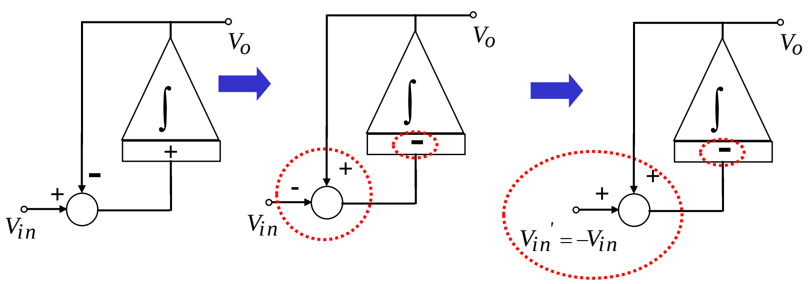
First Order Integrator Based Filter



1st Order Filter
Built with Opamp-RC Integrator

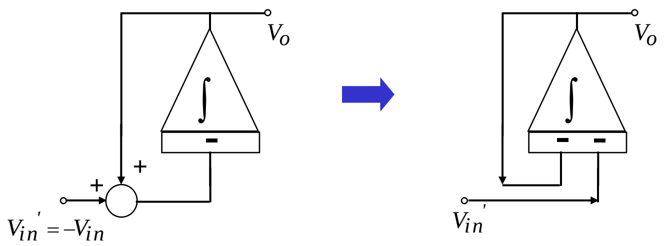
- Single-ended Opamp-RC integrator has a sign inversion from input to output

  → Convert SFG accordingly by modifying BMF

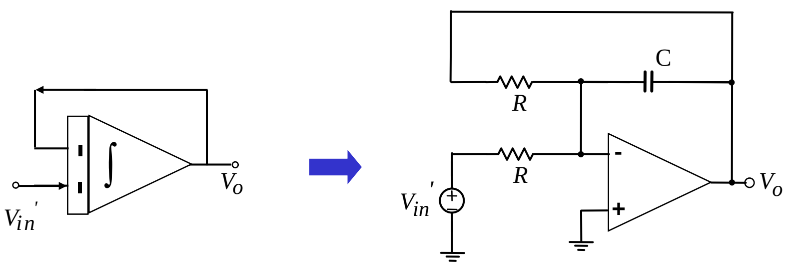


1st Order Filter
Built with Opamp-RC Integrator

- To avoid requiring an additional opamp to perform summation at the input node:



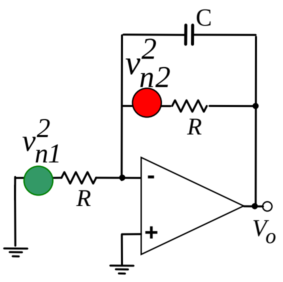
1st Order Filter
Built with Opamp-RC Integrator (continued)



$$\frac{V_o}{V_{in}'} = -\frac{1}{1+sRC}$$

Opamp-RC 1st Order Filter Noise

Identify noise sources (here it is resistors & opamp)
Find transfer function from each noise source
to the output (opamp noise next page)



$$\overline{v_o^2} = \sum_{m=1}^{k} \int_0^{\infty} |H_m(f)|^2 S_m(f)\, df$$

$$S_i(f) \rightarrow Noise\ spectral\ density\ of\ i^{th}\ noise\ source$$

$$|H_1(f)|^2 = |H_2(f)|^2 = \frac{1}{1+(2\pi fRC)^2}$$

$$v_{n1}^2 = v_{n2}^2 = 4KTR\Delta f$$

$$\sqrt{\overline{v_o^2}} = \sqrt{2\frac{kT}{C}}$$

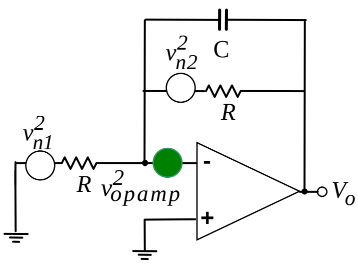
$\alpha = 2$

α

Typically, α increases as filter order increases

Opamp-RC Filter Noise Opamp Contribution

- So far only the fundamental noise sources are considered
- In reality, noise associated with the opamp increases the overall noise
- For a well-designed filter opamp is designed such that noise contribution of opamp is negligible compared to other noise sources
- The bandwidth of the opamp affects the opamp noise contribution to the total noise



Integrator Based Filter
2nd Order RLC Filter

- State space description:

$$V_R = V_L = V_C = V_o$$

$$V_C = \frac{I_C}{sC}$$

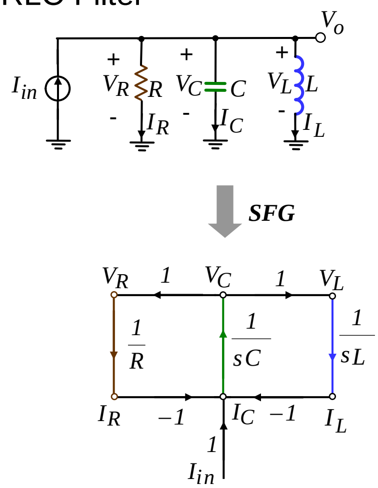
$$I_R = \frac{V_R}{R}$$

$$I_L = \frac{V_L}{sL}$$

$$I_C = I_{in} - I_R - I_L$$

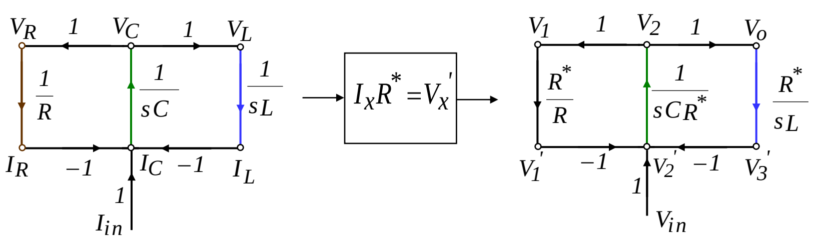
Integrator form

- Draw signal flowgraph (SFG)

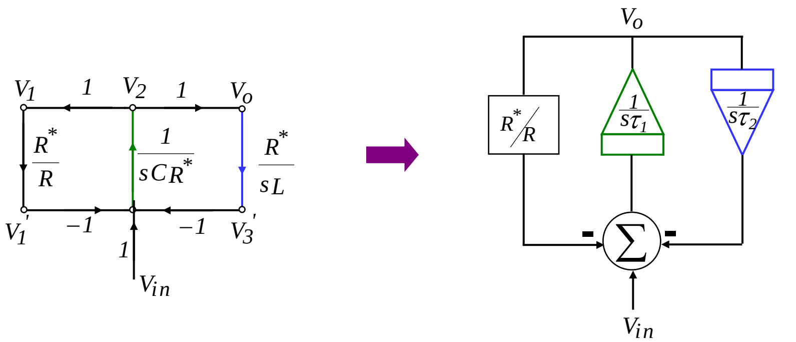


2nd Order RLC Filter SGF
Normalize

- Convert currents to voltages by multiplying all current nodes by the scaling resistance R^*

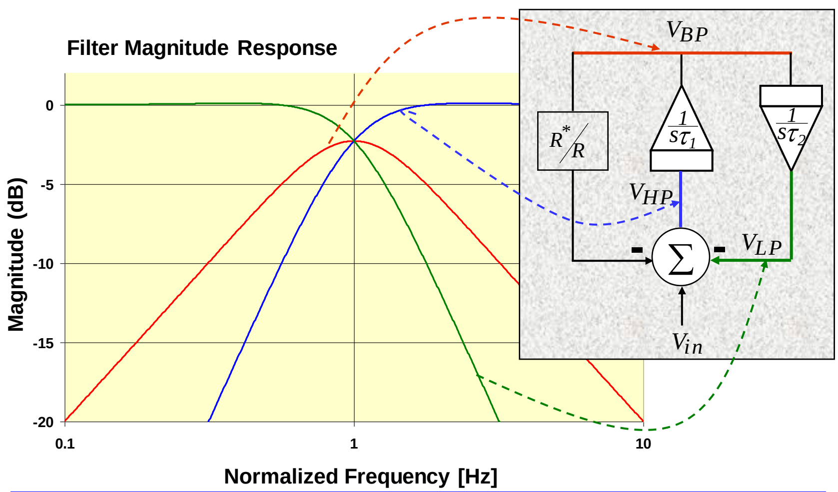


2nd Order RLC Filter SGF Synthesis



$$\tau_1 = R^* \times C \qquad \tau_2 = L \big/ R^*$$

Second Order Integrator Based Filter



Second Order Integrator Based Filter

$$\frac{V_{BP}}{V_{in}} = \frac{\tau_2 s}{\tau_1\tau_2 s^2 + \beta\tau_2 s + 1}$$

$$\frac{V_{LP}}{V_{in}} = \frac{1}{\tau_1\tau_2 s^2 + \beta\tau_2 s + 1}$$

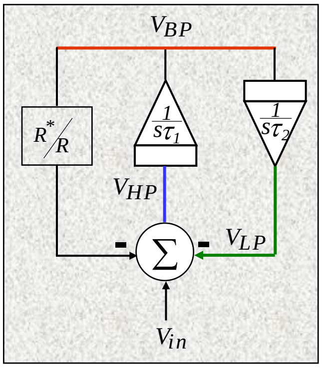
$$\frac{V_{HP}}{V_{in}} = \frac{\tau_1\tau_2 s^2}{\tau_1\tau_2 s^2 + \beta\tau_2 s + 1}$$

$$\tau_1 = R^* \times C \qquad \tau_2 = L \big/ R^*$$

$$\beta = R^* \big/ R$$

$$\omega_0 = 1 \big/ \sqrt{\tau_1\tau_2} = 1 \big/ \sqrt{LC}$$

$$Q = 1/\beta \times \sqrt{\tau_1 \big/ \tau_2}$$



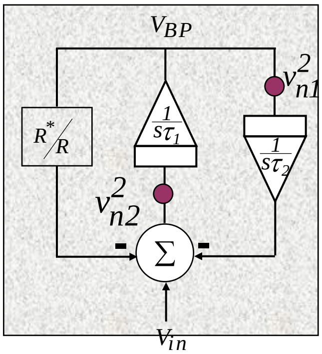
From matching point of view desirable:

$$\tau_1 = \tau_2 \rightarrow Q = R \big/ R^*$$

Second Order Bandpass Filter Noise

$$\overline{v_o^2} = \sum_{m=1}^{k} \int_0^\infty |H_m(f)|^2 S_m(f)\, df$$

- Find transfer function of each noise source to the output
- Integrate contribution of all noise sources
- Here it is assumed that opamps are noise free (not usually the case!)



$$v_{n1}^2 = v_{n2}^2 = 4KTRdf$$

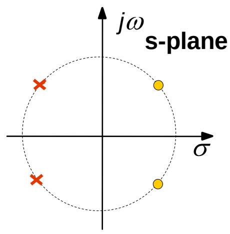
$$\sqrt{\overline{v_o^2}} = \sqrt{\underbrace{2Q}_{\alpha}\frac{kT}{C}}$$

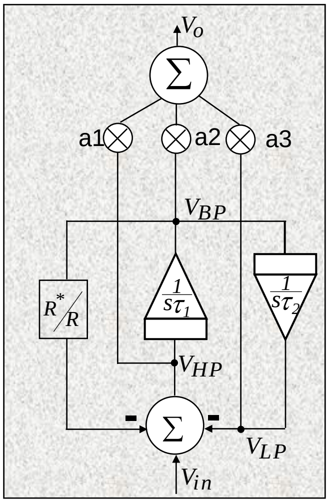
Typically, α increases as filter order increases
Note the noise power is directly proportion to *Q*

Second Order Integrator Based Filter
Biquad

- By combining outputs can generate general biquad function:

$$\frac{V_0}{V_{in}} = \frac{a_1 \tau_1 \tau_2 s^2 + a_2 \tau_2 s + a_3}{\tau_1 \tau_2 s^2 + \beta \tau_2 s + 1}$$





Summary
Integrator Based Monolithic Filters

- Signal flowgraph techniques utilized to convert RLC networks to integrator based active filters
- Each reactive element (L& C) replaced by an integrator
- Fundamental noise limitation determined by integrating capacitor value:
  - For lowpass filter: $\sqrt{\overline{v_o^2}} = \sqrt{\alpha \frac{kT}{C}}$
  - Bandpass filter: $\sqrt{\overline{v_o^2}} = \sqrt{\alpha Q \frac{kT}{C}}$

  where α is a function of filter order and topology

Higher Order Filters

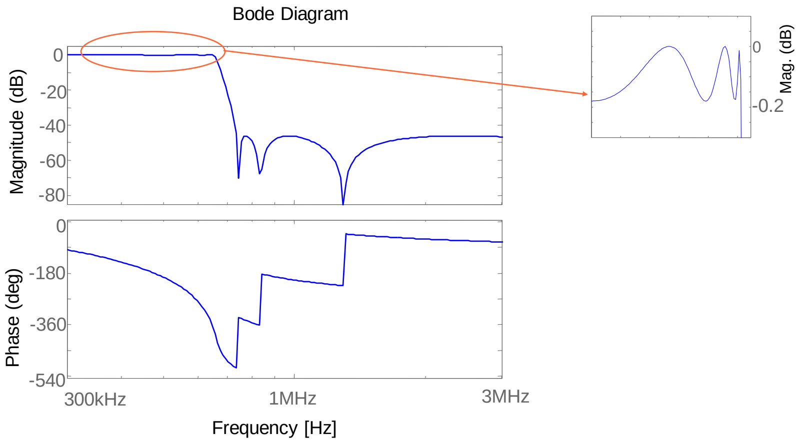
- How do we build higher order filters?
  - Cascade of biquads and 1st order sections
    - Each complex conjugate pole built with a biquad and real pole with 1st order section
    - Easy to implement
    - In the case of high order high Q filters → highly sensitive to component mismatch
  - Direct conversion of high order ladder type RLC filters
    - SFG techniques used to perform exact conversion of ladder type filters to integrator based filters
    - More complicated conversion process
    - Much less sensitive to component mismatch compared to cascade of biquads

Higher Order Filters
Cascade of Biquads

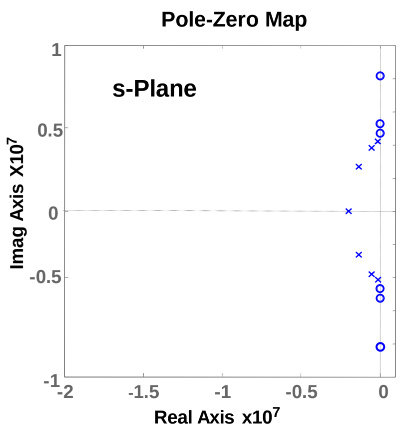
Example: LPF filter for CDMA cell phone baseband receiver

- LPF with
  - fpass = 650 kHz   Rpass = 0.2 dB
  - fstop = 750 kHz   Rstop = 45 dB
  - Assumption: Can compensate for phase distortion in the digital domain
- Matlab used to find minimum order required → 7th order Elliptic Filter
- Implementation with cascaded Biquads
  Goal: Maximize dynamic range
  - Pair poles and zeros
  - In the cascade chain place lowest Q poles first and progress to higher Q poles moving towards the output node

Overall Filter Frequency Response



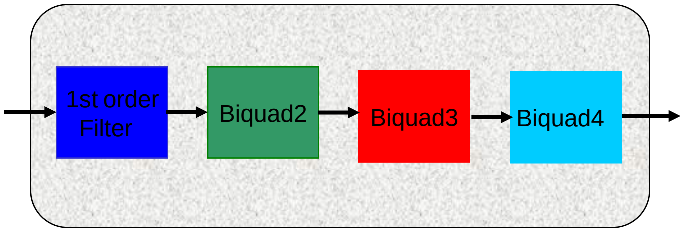
Pole-Zero Map (pzmap in Matlab)



Q_{pole}	f_{pole} [kHz]
16.7902	659.496
3.6590	611.744
1.1026	473.643
	319.568

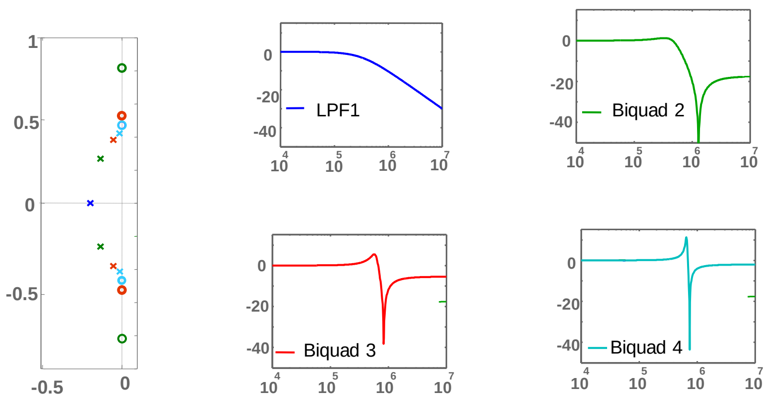
f_{zero} [kHz]
1297.5
836.6
744.0

CDMA Filter
Built with Cascade of 1st and 2nd Order Sections

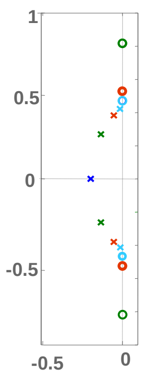


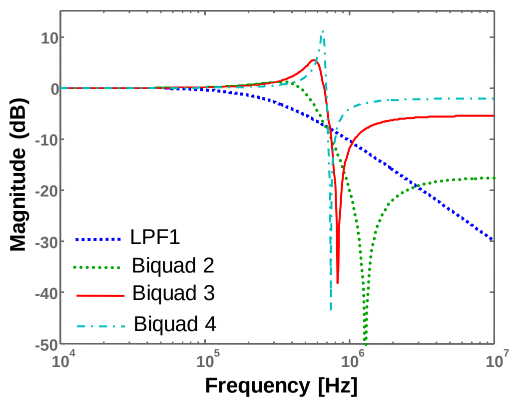
- 1st order filter implements the single real pole
- Each biquad implements a pair of complex conjugate poles and a pair of imaginary axis zeros

Biquad Response

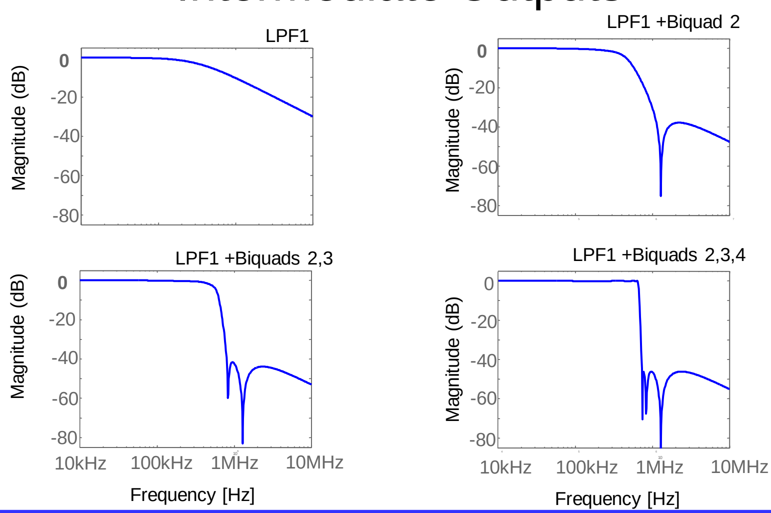


Individual Stage Magnitude Response





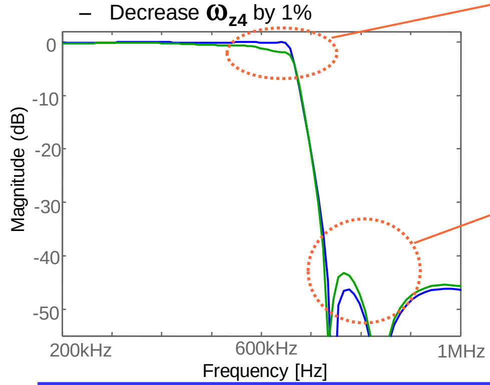
Intermediate Outputs

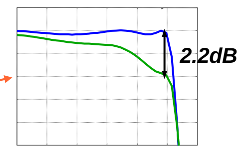


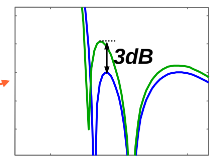
Sensitivity to Relative Component Mismatch

Component variation in Biquad 4 relative to the rest (highest Q poles):

- Increase ω_{p4} by 1%
- Decrease ω_{z4} by 1%







High Q poles → High sensitivity in Biquad realizations

High Q & High Order Filters

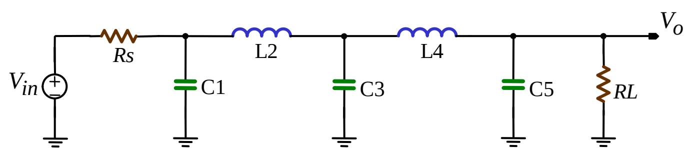
- Cascade of biquads
  - Highly sensitive to component mismatch → not suitable for implementation of high Q & high order filters
  - Cascade of biquads only used in cases where required Q for all biquads <4 (e.g. filters for disk drives)

- Ladder type filters more appropriate for high Q & high order filters (next topic)
  - Will show later →Less sensitive to component mismatch

Ladder Type Filters

- Active ladder type filters
  - For simplicity, will start with all pole ladder type filters
    - Convert to integrator based form- example shown
  - Then will attend to high order ladder type filters incorporating zeros
    - Implement the same 7th order elliptic filter in the form of ladder RLC with zeros
      - Find level of sensitivity to component mismatch
      - Compare with cascade of biquads
    - Convert to integrator based form utilizing SFG techniques
  - Effect of integrator non-Idealities on filter frequency characteristics

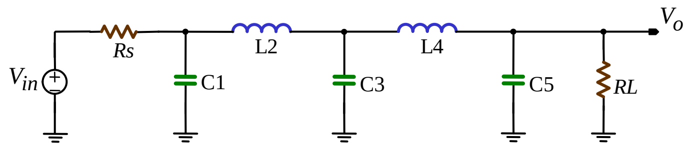
RLC Ladder Filters
Example: 5th Order Lowpass Filter



- Made of resistors, inductors, and capacitors
- Doubly terminated or singly terminated (with or w/o R_L)

Doubly terminated LC ladder filters → Lowest sensitivity to component mismatch

LC Ladder Filters



- First step in the design process is to find values for Ls and Cs based on specifications:
  - Filter graphs & tables found in:
    - A. Zverev, *Handbook of filter synthesis*, Wiley, 1967.
    - A. B. Williams and F. J. Taylor, *Electronic filter design*, 3rd edition, McGraw-Hill, 1995.
  - CAD tools
    - Matlab
    - Spice

LC Ladder Filter Design Example

Design a LPF with maximally flat passband:

f_{-3dB} = 10MHz, f_{stop} = 20MHz

R_s >27dB @ f_{stop}

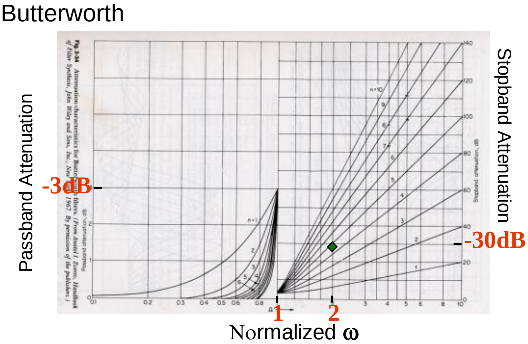
- Maximally flat passband → Butterworth
  - Find minimum filter order :
  - Here standard graphs from filter books are used

f_{stop} / f_{-3dB} = 2

R_s >27dB

Minimum Filter Order

⇨5th order Butterworth



From: Williams and Taylor, p. 2-37

LC Ladder Filter Design Example

Find values for L & C from Table: →

Note L &C values normalized to

$\omega_{-3dB} = 1$

Denormalization:

Multiply all L_{Norm}, C_{Norm} by:

$L_r = R/\omega_{-3dB}$

$C_r = 1/(R \times \omega_{-3dB})$

R is the value of the source and termination resistor (choose both 1Ω for now)

Then: $L = L_r \times L_{Norm}$

$C = C_r \times C_{Norm}$

NORMALIZED FILTER DESIGN TABLES 11.3

TABLE 11-2 Butterworth *LC* Element Values (*Continued*)

n	R_s	C_1	L_2	C_3	L_4	C_5	L_6	C_7
5	1.0000	0.6180	1.6180	2.0000	1.6180	0.6180		
	0.9000	0.4416	1.0265	1.9095	1.7562	1.3887		
	0.8000	0.4698	0.8660	2.0605	1.5443	1.7380		
	0.7000	0.5173	0.7313	2.2849	1.3326	2.1083		
	0.6000	0.5860	0.6094	2.5998	1.1255	2.5524		
	0.5000	0.6857	0.4955	3.0510	0.9237	3.1331		
	0.4000	0.8378	0.3877	3.7357	0.7274	3.9648		
	0.3000	1.0937	0.2848	4.8835	0.5367	5.3073		
	0.2000	1.6077	0.1861	7.1849	0.3518	7.9345		
	0.1000	3.1522	0.0912	14.0945	0.1727	15.7103		
	Inf.	1.5451	1.6944	1.3820	0.8944	0.3090		
6	1.0000	0.5176	1.4142	1.9319	1.9319	1.4142	0.5176	
	1.1111	0.2890	1.0403	1.3217	2.0539	1.7443	1.3347	
	1.2500	0.2445	1.1163	1.1257	2.2389	1.5498	1.6881	
	1.4286	0.2072	1.2363	0.9567	2.4991	1.3464	2.0618	
	1.6667	0.1732	1.4071	0.8011	2.8580	1.1431	2.5092	
	2.0000	0.1412	1.6531	0.6542	3.3687	0.9423	3.0938	
	2.5000	0.1108	2.0275	0.5139	4.1408	0.7450	3.9305	
	3.3333	0.0816	2.6559	0.3788	5.4325	0.5517	5.2804	
	5.0000	0.0535	3.9170	0.2484	8.0201	0.3628	7.9216	
	10.0000	0.0263	7.7053	0.1222	15.7855	0.1788	15.7375	
	Inf.	1.5529	1.7593	1.5529	1.2016	0.7579	0.2588	
7	1.0000	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450
	0.9000	0.2985	0.7111	1.4043	1.4891	2.1249	1.7268	1.2961
	0.8000	0.3215	0.6057	1.5174	1.2777	2.3338	1.5461	1.6520
	0.7000	0.3571	0.5154	1.6883	1.0910	2.6177	1.3498	2.0277
	0.6000	0.4075	0.4322	1.9284	0.9170	3.0050	1.1503	2.4771
	0.5000	0.4799	0.3536	2.2726	0.7512	3.5532	0.9513	3.0640
	0.4000	0.5899	0.2782	2.7950	0.5917	4.3799	0.7542	3.9037
	0.3000	0.7745	0.2055	3.6706	0.4373	5.7612	0.5600	5.2583
	0.2000	1.1448	0.1350	5.4267	0.2874	8.5263	0.3692	7.9079
	0.1000	2.2571	0.0665	10.7004	0.1417	16.8222	0.1823	15.7480
	Inf.	1.5576	1.7988	1.6588	1.3972	1.0550	0.6560	0.2225
n	$1/R_s$	L_1	C_2	L_3	C_4	L_5	C_6	L_7

From: Williams and Taylor, p. 11.3

LC Ladder Filter Design Example

Find values for L & C from Table: →

Normalized values:

$C1_{Norm} = C5_{Norm} = 0.618$

$C3_{Norm} = 2.0$

$L2_{Norm} = L4_{Norm} = 1.618$

Denormalization:

Since $\omega_{-3dB} = 2\pi \times 10MHz$

$L_r = R/\omega_{-3dB} = 15.9$ nH

$C_r = 1/(R \times \omega_{-3dB}) = 15.9$ nF

R =1

⇨C1=C5=9.836nF, C3=31.83nF

⇨L2=L4=25.75nH

NORMALIZED FILTER DESIGN TABLES 11.3

TABLE 11-2 Butterworth *LC* Element Values (*Continued*)

n	R_s	C_1	L_2	C_3	L_4	C_5	L_6	C_7
5	1.0000	0.6180	1.6180	2.0000	1.6180	0.6180		
	0.9000	0.4416	1.0265	1.9095	1.7562	1.3887		
	0.8000	0.4698	0.8660	2.0605	1.5443	1.7380		
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	0.6000	0.5860	0.6094	2.5998	1.1255	2.5524		
	0.5000	0.6857	0.4955	3.0510	0.9237	3.1331		
	0.4000	0.8378	0.3877	3.7357	0.7274	3.9648		
	0.3000	1.0937	0.2848	4.8835	0.5367	5.3073		
	0.2000	1.6077	0.1861	7.1849	0.3518	7.9345		
	0.1000	3.1522	0.0912	14.0945	0.1727	15.7103		
	Inf.	1.5451	1.6944	1.3820	0.8944	0.3090		
6	1.0000	0.5176	1.4142	1.9319	1.9319	1.4142	0.5176	
	1.1111	0.2890	1.0403	1.3217	2.0539	1.7443	1.3347	
	1.2500	0.2445	1.1163	1.1257	2.2389	1.5498	1.6881	
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	1.6667	0.1732	1.4071	0.8011	2.8580	1.1431	2.5092	
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	2.5000	0.1108	2.0275	0.5139	4.1408	0.7450	3.9305	
	3.3333	0.0816	2.6559	0.3788	5.4325	0.5517	5.2804	
	5.0000	0.0535	3.9170	0.2484	8.0201	0.3628	7.9216	
	10.0000	0.0263	7.7053	0.1222	15.7855	0.1788	15.7375	
	Inf.	1.5529	1.7593	1.5529	1.2016	0.7579	0.2588	
7	1.0000	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450
	0.9000	0.2985	0.7111	1.4043	1.4891	2.1249	1.7268	1.2961
	0.8000	0.3215	0.6057	1.5174	1.2777	2.3338	1.5461	1.6520
	0.7000	0.3571	0.5154	1.6883	1.0910	2.6177	1.3498	2.0277
	0.6000	0.4075	0.4322	1.9284	0.9170	3.0050	1.1503	2.4771
	0.5000	0.4799	0.3536	2.2726	0.7512	3.5532	0.9513	3.0640
	0.4000	0.5899	0.2782	2.7950	0.5917	4.3799	0.7542	3.9037
	0.3000	0.7745	0.2055	3.6706	0.4373	5.7612	0.5600	5.2583
	0.2000	1.1448	0.1350	5.4267	0.2874	8.5263	0.3692	7.9079
	0.1000	2.2571	0.0665	10.7004	0.1417	16.8222	0.1823	15.7480
	Inf.	1.5576	1.7988	1.6588	1.3972	1.0550	0.6560	0.2225
n	$1/R_s$	L_1	C_2	L_3	C_4	L_5	C_6	L_7

From: Williams and Taylor, p. 11.3