EE247
Lecture 4

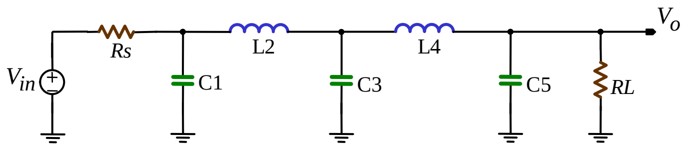
- Active ladder type filters
  - For simplicity, will start with all pole ladder type filters
    - Convert to integrator based form- example shown
  - Then will attend to high order ladder type filters incorporating zeros
    - Implement the same 7th order elliptic filter in the form of ladder RLC with zeros
      - Find level of sensitivity to component mismatch
      - Compare with cascade of biquads
    - Convert to integrator based form utilizing SFG techniques
  - Effect of integrator non-Idealities on filter frequency characteristics

Summary Lecture 3

- Active Filters
  - Active biquads
    - Integrator-based filters
      - Signal flowgraph concept
      - First order integrator-based filter
      - Second order integrator-based filter & biquads

  - High order & high Q filters
    - Cascaded biquads & first order filters
      - Cascaded biquad sensitivity to component mismatch
    - Ladder type filters

RLC Ladder Filters
Example: 5th Order Lowpass Filter



- Made of resistors, inductors, and capacitors
- Doubly terminated or singly terminated (with or w/o R_L)

Doubly terminated LC ladder filters → Lowest sensitivity to component mismatch

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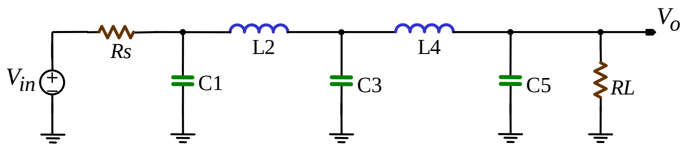
LC Ladder Filters



- First step in the design process is to find values for Ls and Cs based on specifications:
  - Filter graphs & tables found in:
    - A. Zverev, *Handbook of filter synthesis*, Wiley, 1967.
    - A. B. Williams and F. J. Taylor, *Electronic filter design*, 3rd edition, McGraw-Hill, 1995.
  - CAD tools
    - Matlab,
    - Agilent ADS (includes Filter package→ does the job of the tables)
    - Spice

LC Ladder Filter Design Example

Design a LPF with maximally flat passband:

f-3dB = 10MHz, fstop = 20MHz

Rs >27dB @ fstop

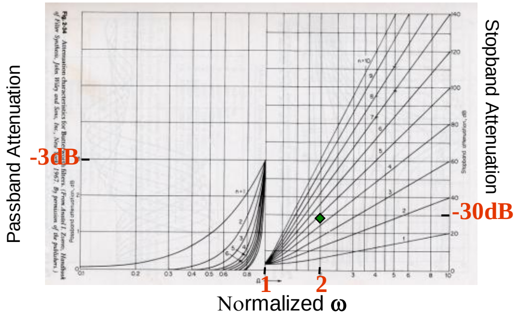
- Maximally flat passband → Butterworth
  - Find minimum filter order :
  - Here standard graphs from filter books are used

fstop / f-3dB = 2

Rs >27dB

Minimum Filter Order

⇨5th order Butterworth



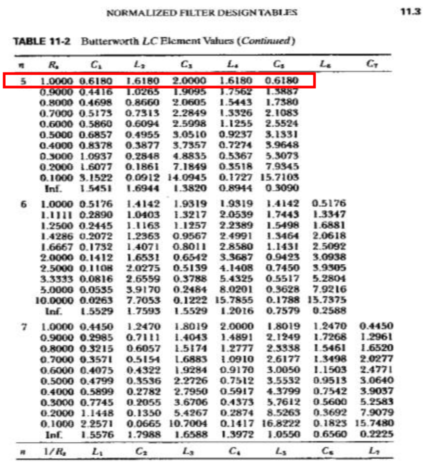
From: Williams and Taylor, p. 2-37

LC Ladder Filter Design Example

Find values for L & C from Table: ⟹

Note L &C values normalized to

$\omega_{-3dB} = 1$



NORMALIZED FILTER DESIGN TABLES 11.3

TABLE 11-2 Butterworth *LC* Element Values (*Continued*)

n	R_s	C_1	L_2	C_3	L_4	C_5	L_6	C_7
5	1.0000	0.6180	1.6180	2.0000	1.6180	0.6180		
	0.9000	0.4416	1.0265	1.9095	1.7562	1.3887		
	0.8000	0.4698	0.8660	2.0605	1.5443	1.7380		
	0.7000	0.5173	0.7313	2.2849	1.3326	2.1083		
	0.6000	0.5860	0.6094	2.5998	1.1255	2.5524		
	0.5000	0.6857	0.4955	3.0510	0.9237	3.1331		
	0.4000	0.8378	0.3877	3.7357	0.7274	3.9648		
	0.3000	1.0937	0.2848	4.8835	0.5367	5.3073		
	0.2000	1.6077	0.1861	7.1849	0.3518	7.9345		
	0.1000	3.1522	0.0912	14.0945	0.1727	15.7103		
	Inf.	1.5451	1.6944	1.3820	0.8944	0.3090		
6	1.0000	0.5176	1.4142	1.9319	1.9319	1.4142	0.5176	
	1.1111	0.2890	1.0403	1.3217	2.0539	1.7443	1.3347	
	1.2500	0.2445	1.1163	1.1257	2.2389	1.5498	1.6881	
	1.4286	0.2072	1.2363	0.9567	2.4991	1.3464	2.0618	
	1.6667	0.1732	1.4071	0.8011	2.8580	1.1431	2.5092	
	2.0000	0.1412	1.6531	0.6542	3.3687	0.9423	3.0938	
	2.5000	0.1108	2.0275	0.5139	4.1408	0.7450	3.9305	
	3.3333	0.0816	2.6559	0.3788	5.4325	0.5517	5.2804	
	5.0000	0.0535	3.9170	0.2484	8.0201	0.3628	7.9216	
	10.0000	0.0263	7.7053	0.1222	15.7855	0.1788	15.7375	
	Inf.	1.5529	1.7593	1.5529	1.2016	0.7579	0.2588	
7	1.0000	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450
	0.9000	0.2985	0.7111	1.4043	1.4891	2.1249	1.7268	1.2961
	0.8000	0.3215	0.6057	1.5174	1.2777	2.3338	1.5461	1.6520
	0.7000	0.3571	0.5154	1.6883	1.0910	2.6177	1.3498	2.0277
	0.6000	0.4075	0.4322	1.9284	0.9170	3.0050	1.1503	2.4771
	0.5000	0.4799	0.3536	2.2726	0.7512	3.5532	0.9513	3.0640
	0.4000	0.5899	0.2782	2.7950	0.5917	4.3799	0.7542	3.9037
	0.3000	0.7745	0.2055	3.6706	0.4373	5.7612	0.5600	5.2583
	0.2000	1.1448	0.1350	5.4267	0.2874	8.5263	0.3692	7.9079
	0.1000	2.2571	0.0665	10.7004	0.1417	16.8222	0.1823	15.7480
	Inf.	1.5576	1.7988	1.6588	1.3972	1.0550	0.6560	0.2225
n	$1/R_s$	L_1	C_2	L_3	C_4	L_5	C_6	L_7

From: Williams and Taylor, p. 11.3

Denormalization:

Multiply all L_{Norm}, C_{Norm} by:

$L_r = R/\omega_{-3dB}$

$C_r = 1/(R X \omega_{-3dB})$

R is the value of the source and termination resistor (choose both 1Ω for now)

Then: $L = L_r \, x L_{Norm}$

$C = C_r \, x C_{Norm}$

LC Ladder Filter Design Example

Find values for L & C from Table:

Normalized values:

$C1_{Norm} = C5_{Norm} = 0.618$

$C3_{Norm} = 2.0$

$L2_{Norm} = L4_{Norm} = 1.618$

Denormalization:

Since $\omega_{-3dB} = 2\pi \times 10MHz$

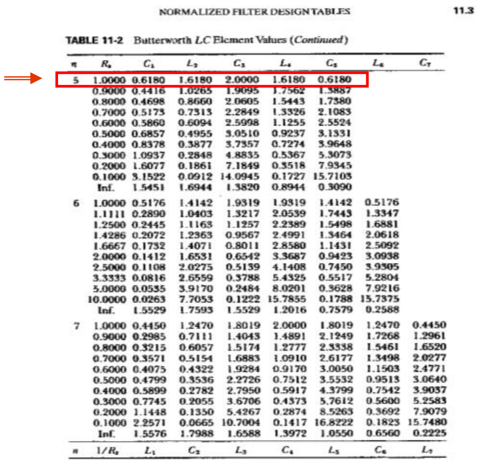
$L_r = R/\omega_{-3dB} = 15.9$ nH

$C_r = 1/(R \times \omega_{-3dB}) = 15.9$ nF

R =1

⇨C1=C5=9.836nF, C3=31.83nF

⇨L2=L4=25.75nH



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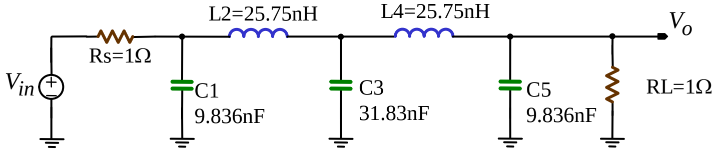
n	$1/R_s$	L_1	C_2	L_3	C_4	L_5	C_6	L_7

From: Williams and Taylor, p. 11.3

Last Lecture:
Example: 5th Order Butterworth Filter



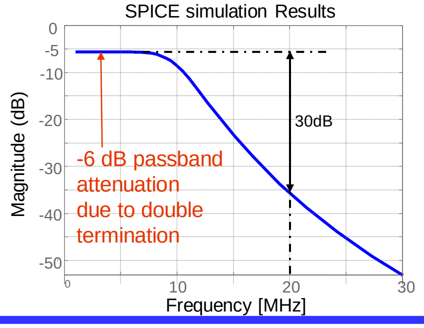
Specifications:

f-3dB = 10MHz,

fstop = 20MHz

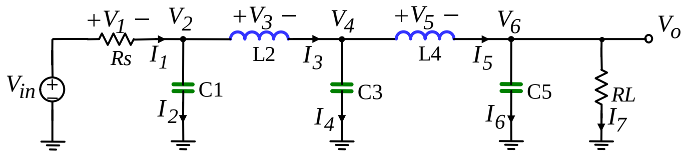
Rs >27dB

Used filter tables to obtain Ls & Cs



-6 dB passband attenuation due to double termination

Low-Pass RLC Ladder Filter
Conversion to Integrator Based Active Filter



- To convert RLC ladder prototype to integrator based filer:
  - ❑Use Signal Flowgraph technique
    - ✓Name currents and voltages for all components
    - ✓Use KCL & KVL to derive equations
    - ✓Make sure reactive elements expressed as 1/s term
    - → *V(C) =f(I) & I(L)=f(V)*
    - ✓Use state-space description to derive the SFG
    - ✓Modify & simply the SFG for implementation with integrators e.g. convert all current nodes to voltage

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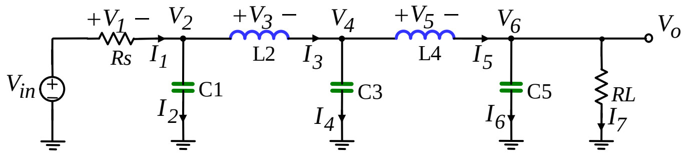
Low-Pass RLC Ladder Filter
Conversion to Integrator Based Active Filter



- Use KCL & KVL to derive equations:

$V_1 = V_{in} - V_2$, $V_2 = \frac{I_2}{sC_1}$, $V_3 = V_2 - V_4$

$V_4 = \frac{I_4}{sC_3}$, $V_5 = V_4 - V_6$, $V_6 = \frac{I_6}{sC_5}$ $V_o = V_6$

$I_1 = \frac{V_1}{Rs}$, $I_2 = I_1 - I_3$, $I_3 = \frac{V_3}{sL_2}$

$I_4 = I_3 - I_5$, $I_5 = \frac{V_5}{sL_4}$, $I_6 = I_5 - I_7$, $I_7 = \frac{V_6}{RL}$

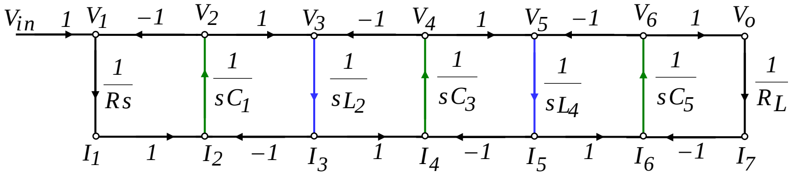
Low-Pass RLC Ladder Filter
Signal Flowgraph

$V_1 = V_{in} - V_2$, $V_2 = \frac{I_2}{sC_1}$, $V_3 = V_2 - V_4$

$V_4 = \frac{I_4}{sC_3}$, $V_5 = V_4 - V_6$, $V_6 = \frac{I_6}{sC_5}$ $V_o = V_6$

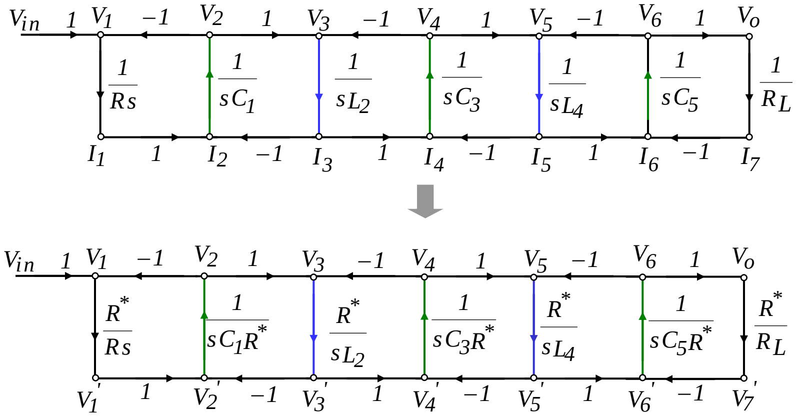
$I_1 = \frac{V_1}{Rs}$, $I_2 = I_1 - I_3$, $I_3 = \frac{V_3}{sL_2}$

$I_4 = I_3 - I_5$, $I_5 = \frac{V_5}{sL_4}$, $I_6 = I_5 - I_7$, $I_7 = \frac{V_6}{RL}$

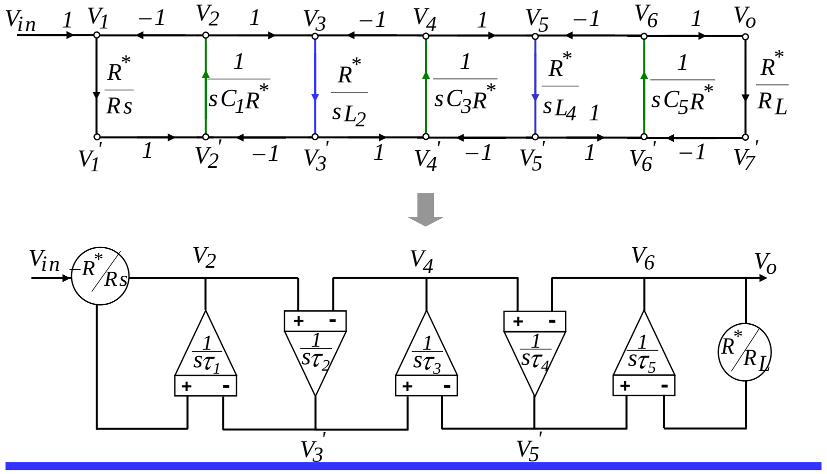


SFG

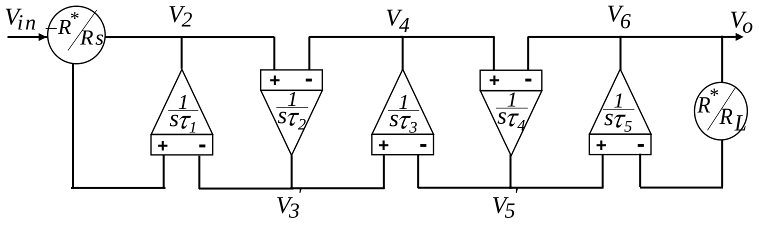
Low-Pass RLC Ladder Filter
Normalize



Low-Pass RLC Ladder Filter
Synthesize



Low-Pass RLC Ladder Filter
Integrator Based Implementation

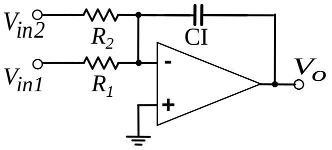


$$\tau_1 = C_1.R^* \quad , \quad \tau_2 = \frac{L_2}{R^*} = C_2.R^* \quad , \quad \tau_3 = C_3.R^* \quad , \quad \tau_4 = \frac{L4}{R^*} = C_4.R^* \quad , \quad \tau_5 = C_5.R^*$$

Main building block: Integrator
Let us start to build the filter with RC& Opamp type integrator

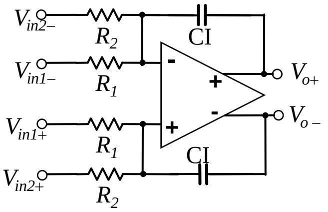
Opamp-RC Integrator

Single-Ended



$$V_o = -V_{in1} \times \frac{1}{sR_1CI} - V_{in2} \times \frac{1}{sR_2CI}$$

Differential



$$V_{o+} - V_{o-} = (V_{in1+} - V_{in1-}) \times \frac{1}{sR_1CI} + (V_{in2+} - V_{in2-}) \times \frac{1}{sR_2CI}$$

Note: Implementation with single-ended integrator requires extra circuitry for sign inversion whereas in differential case both signal polarities are available

Differential topologies→ additional advantage of immunity to parasitic signal injection & superior power-supply rejection

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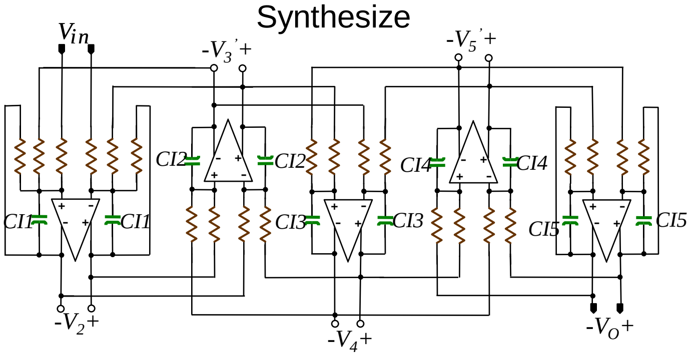
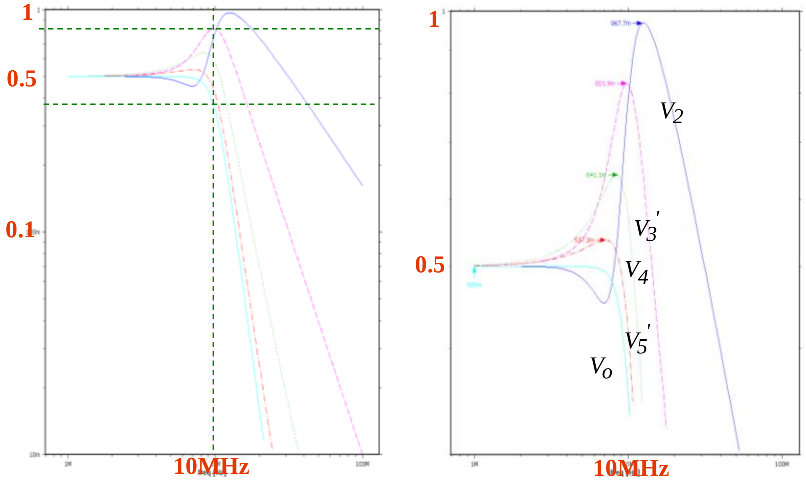
Differential Integrator Based LP Ladder Filter Synthesize



- First iteration:
  - All resistors are chosen=1Ω
  - Values for $\tau_x = R_xCI_x$ found from RLC analysis
  - Integrating capacitor values: *CI1=CI5=9.836nF, CI2=CI4=25.45nF, CI3=31.83nF*

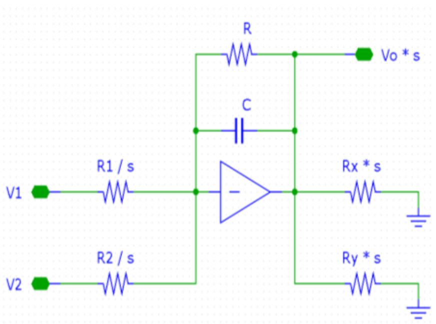
First Iteration
Simulated Magnitude Response



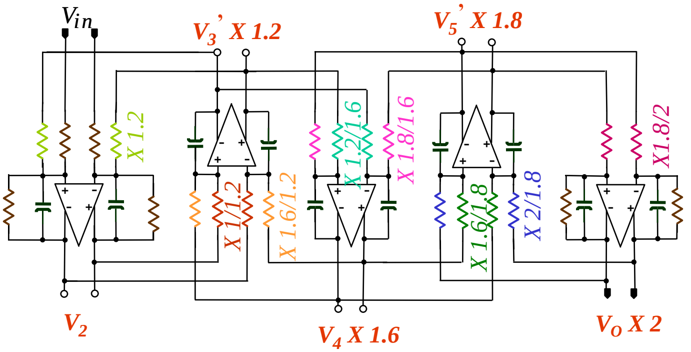
Scale Node Voltages

To maximize dynamic range
→ scale node voltages

Scale V_o by factor "s"

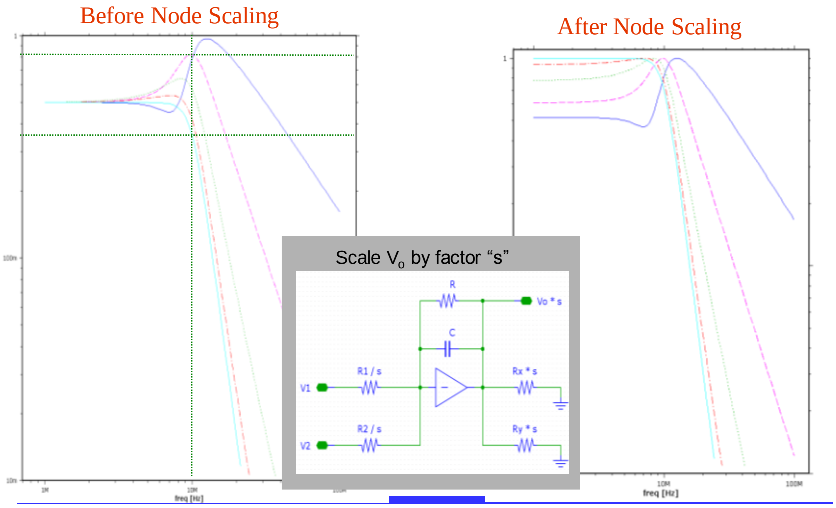


Differential Integrator Based LP Ladder Filter Node Scaling



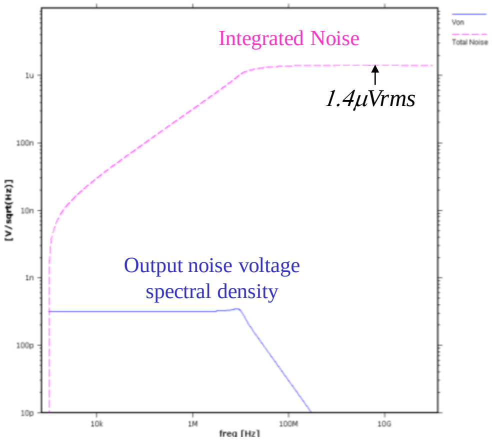
- Second iteration:
  - Nodes scaled, note output node x2
  - Resistor values scaled according to scaling of nodes
  - Capacitors the same : *C1=C5=9.836nF, C2=C4=25.45nF, C3=31.83nF*

Second Interation Maximizing Signal Handling by Node Voltage Scaling



Scale V_o by factor “s”

Filter Noise



Total noise @ the output:
1.4 μV rms
(noiseless opamps)

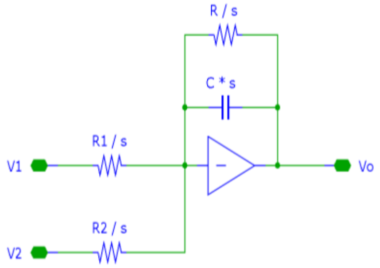
That's excellent, but:

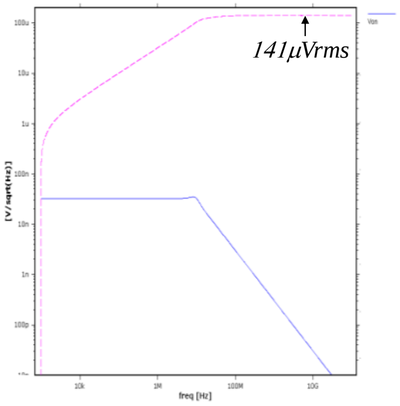
- Capacitors too large for integration → Unrealistically large Si area
- Resistors too small → high power dissipation

Typical applications allow higher noise, assuming tolerable noise in the order of 140 μV rms …

Scale to Meet Noise Target

Scale capacitors and resistors to meet noise objective

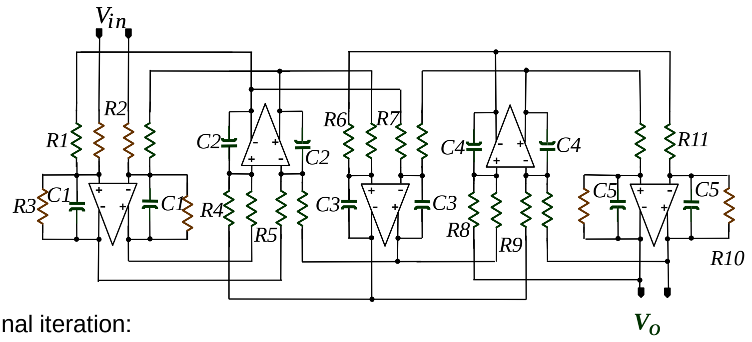




$s = 10^{-4}$ → $(V_{n1}/V_{n2})^2$

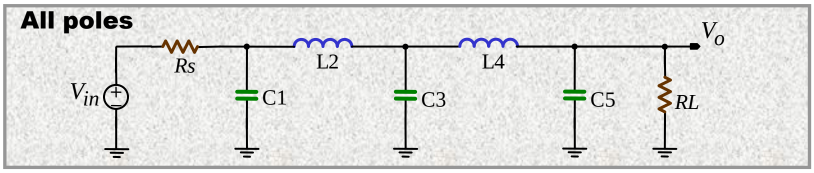
Noise after scaling: 141 μV rms (assuming noiseless opamps)

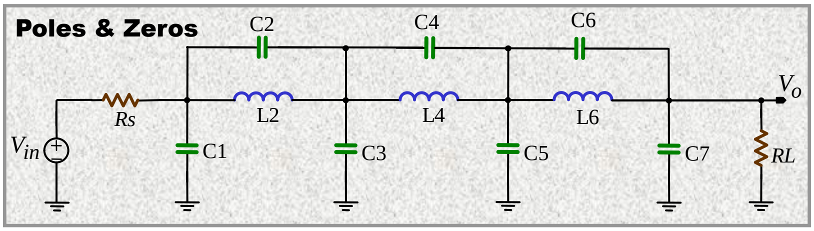
Differential Integrator Based LP Ladder Filter Final Design



- Final iteration:
  - Based on scaled nodes and noise considerations
  - Capacitors: *C1=C5=0.9836pF, C2=C4=2.545pF, C3=3.183pF*
  - *Resistors: R1=11.77K, R2=9.677K, R3=10K, R4=12.82K, R5=8.493K, R6=11.93K, R7=7.8K, R8=10.75K, R9=8.381K, R11=10K, R11=9.306K*

RLC Ladder Filters Including Transmission Zeros





RLC Ladder Filter Design Example

- Design a baseband filter for CDMA IS95 cellular phone receive path with the following specs.
  - Filter frequency mask shown on the next page
  - Allow enough margin for manufacturing variations
    - Assume overall tolerable pass-band magnitude variation of 1.8dB
    - Assume the -3dB frequency can vary by +-8% due to manufacturing tolerances & circuit inaccuracies
  - Assume any phase impairment can be compensated in the digital domain

* Note this is the same example as for cascade of biquad while the specifications are given closer to a real product case

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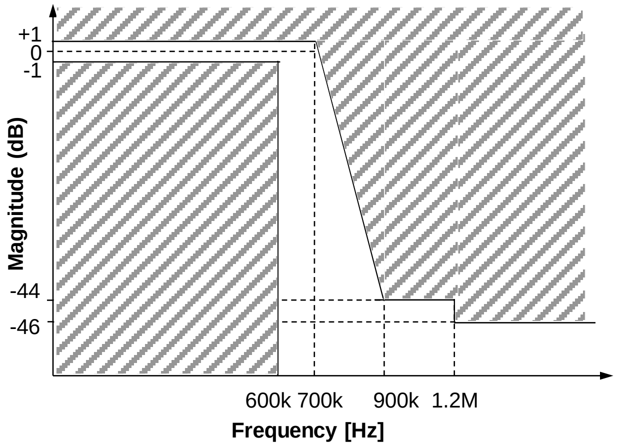
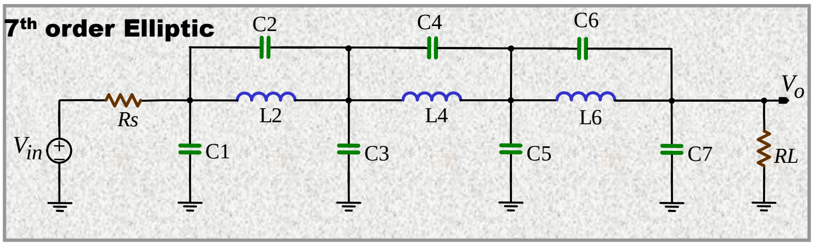
RLC Ladder Filter Design Example
CDMA IS95 Receive Filter Frequency Mask



RLC Ladder Filter Design
Example: CDMA IS95 Receive Filter

- Since phase impairment can be corrected for, use filter type with max. roll-off slope/pole
  - → Filter type → Elliptic
- Design filter freq. response to fall well within the freq. mask
  - Allow margin for component variations & mismatches
- For the passband ripple, allow enough margin for ripple change due to component & temperature variations
  - → Design nominal passband ripple of 0.2dB
- For stopband rejection add a few dB margin 44+5=49dB
- Final design specifications:
  - fpass = 650 kHz Rpass = 0.2 dB
  - fstop = 750 kHz Rstop = 49 dB
- Use Matlab or ADS or filter tables to decide the min. order for the filter (same as cascaded biquad example)
  - 7th Order Elliptic

RLC Low-Pass Ladder Filter Design
Example: CDMA IS95 Receive Filter



- Use filter tables & charts to determine LC values
- Can use the CAD tool: Agilent ADS

RLC Ladder Filter Design
Example: CDMA IS95 Receive Filter

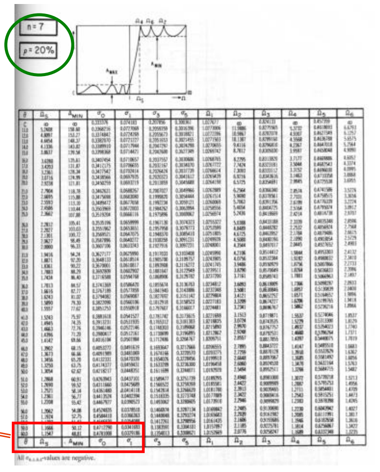
- Specifications
  - – fpass = 650 kHz　　Rpass = 0.2 dB
  - – fstop = 750 kHz　　Rstop = 49 dB
- Use filter tables to determine LC values
  - – Table from: A. Zverev, *Handbook of filter synthesis*, Wiley, 1967
  - – Elliptic filters tabulated wrt "reflection coefficient ρ"

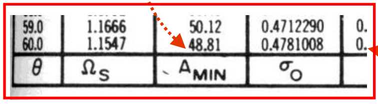
$$Rpass = -10 \times \log\left(1-\rho^2\right)$$

  - – Since *Rpass=0.2dB* → ρ =20%
  - – Use table accordingly

RLC Ladder Filter Design
Example: CDMA IS95 Receive Filter

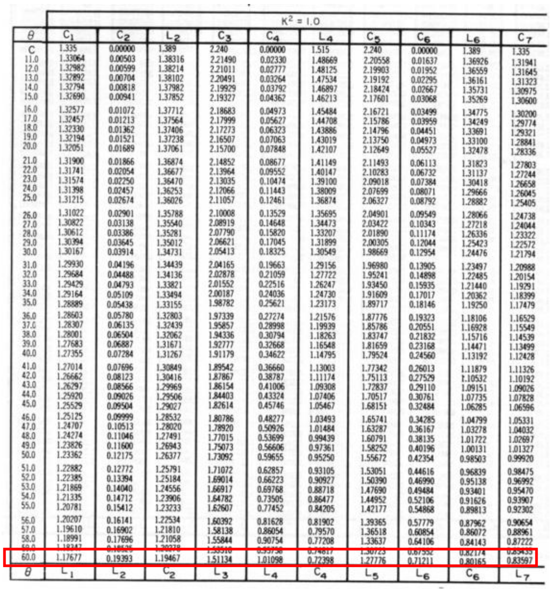
- Table from Zverev book page #281 & 282:
- Since our spec. is Amin=44dB add 5dB margin & design for Amin=49dB





59.0	1.1666	50.12	0.4712290	0.
60.0	1.1547	48.81	0.4781008	0.
θ	Ω_S	A_{MIN}	σ_O	

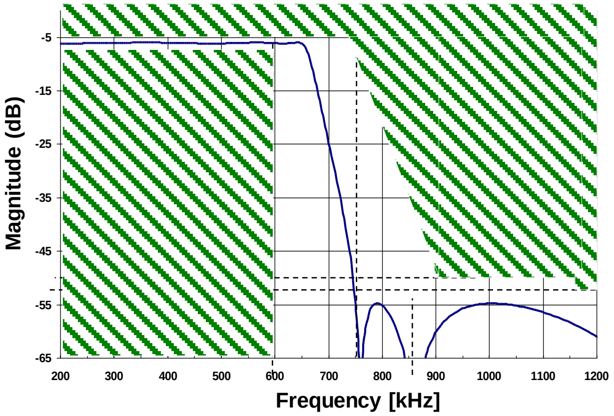
- Table from Zverev page #281 & 282:
- Normalized component values:
  - C1=1.17677
  - C2=0.19393
  - L2=1.19467
  - C3=1.51134
  - C4=1.01098
  - L4=0.72398
  - C5=1.27776
  - C6=0.71211
  - L6=0.80165
  - C7=0.83597



	$K^2 = 1.0$									
θ	C_1	C_2	L_2	C_3	C_4	L_4	C_5	C_6	L_6	C_7
C	1.335	0.00000	1.389	2.240	0.00000	1.515	2.240	0.00000	1.389	1.335
11.0	1.33064	0.00503	1.38316	2.21490	0.02330	1.48669	2.20558	0.01637	1.36926	1.31941
12.0	1.32982	0.00599	1.38214	2.21011	0.02777	1.48125	2.19903	0.01952	1.36559	1.31645
13.0	1.32892	0.00704	1.38102	2.20491	0.03264	1.47534	2.19192	0.02295	1.36161	1.31323
14.0	1.32794	0.00818	1.37982	2.19929	0.03792	1.46897	2.18424	0.02667	1.35731	1.30975
15.0	1.32690	0.00941	1.37852	2.19327	0.04362	1.46213	2.17601	0.03068	1.35269	1.30600
16.0	1.32577	0.01072	1.37712	2.18683	0.04973	1.45484	2.16721	0.03499	1.34775	1.30200
17.0	1.32457	0.01213	1.37564	2.17999	0.05627	1.44708	2.15786	0.03959	1.34249	1.29774
18.0	1.32330	0.01362	1.37406	2.17273	0.06323	1.43886	2.14796	0.04451	1.33691	1.29321
19.0	1.32194	0.01521	1.37238	2.16507	0.07063	1.43019	2.13750	0.04973	1.33100	1.28841
20.0	1.32051	0.01689	1.37061	2.15700	0.07848	1.42107	2.12649	0.05527	1.32478	1.28336
21.0	1.31900	0.01866	1.36874	2.14852	0.08677	1.41149	2.11493	0.06113	1.31823	1.27803
22.0	1.31741	0.02054	1.36677	2.13964	0.09552	1.40147	2.10283	0.06732	1.31137	1.27244
23.0	1.31574	0.02250	1.36470	2.13035	0.10474	1.39100	2.09018	0.07384	1.30418	1.26658
24.0	1.31398	0.02457	1.36253	2.12066	0.11443	1.38009	2.07699	0.08071	1.29666	1.26045
25.0	1.31215	0.02674	1.36026	2.11057	0.12461	1.36874	2.06327	0.08792	1.28882	1.25405
26.0	1.31022	0.02901	1.35788	2.10008	0.13529	1.35695	2.04901	0.09549	1.28066	1.24738
27.0	1.30822	0.03138	1.35540	2.08919	0.14648	1.34473	2.03422	0.10343	1.27218	1.24044
28.0	1.30612	0.03386	1.35281	2.07790	0.15820	1.33207	2.01890	0.11174	1.26336	1.23322
29.0	1.30394	0.03645	1.35012	2.06621	0.17045	1.31899	2.00305	0.12044	1.25423	1.22572
30.0	1.30167	0.03914	1.34731	2.05413	0.18325	1.30549	1.98669	0.12954	1.24476	1.21794
31.0	1.29930	0.04196	1.34439	2.04165	0.19663	1.29156	1.96980	0.13905	1.23497	1.20988
32.0	1.29684	0.04488	1.34136	2.02878	0.21059	1.27722	1.95241	0.14898	1.22485	1.20154
33.0	1.29429	0.04793	1.33821	2.01552	0.22516	1.26247	1.93450	0.15935	1.21440	1.19291
34.0	1.29164	0.05109	1.33494	2.00187	0.24036	1.24730	1.91609	0.17017	1.20362	1.18399
35.0	1.28889	0.05438	1.33155	1.98782	0.25621	1.23173	1.89717	0.18146	1.19250	1.17479
36.0	1.28603	0.05780	1.32803	1.97339	0.27274	1.21576	1.87776	0.19323	1.18106	1.16529
37.0	1.28307	0.06135	1.32439	1.95857	0.28998	1.19939	1.85786	0.20551	1.16928	1.15549
38.0	1.28001	0.06504	1.32062	1.94336	0.30794	1.18263	1.83747	0.21832	1.15716	1.14539
39.0	1.27683	0.06887	1.31671	1.92777	0.32668	1.16548	1.81659	0.23168	1.14471	1.13499
40.0	1.27355	0.07284	1.31267	1.91179	0.34622	1.14795	1.79524	0.24560	1.13192	1.12428
41.0	1.27014	0.07696	1.30849	1.89542	0.36660	1.13003	1.77342	0.26013	1.11879	1.11326
42.0	1.26662	0.08123	1.30416	1.87867	0.38787	1.11174	1.75113	0.27529	1.10532	1.10192
43.0	1.26297	0.08566	1.29969	1.86154	0.41006	1.09308	1.72837	0.29110	1.09151	1.09026
44.0	1.25920	0.09026	1.29506	1.84403	0.43324	1.07406	1.70517	0.30761	1.07735	1.07828
45.0	1.25529	0.09504	1.29027	1.82614	0.45746	1.05467	1.68151	0.32484	1.06285	1.06596
46.0	1.25125	0.09999	1.28532	1.80786	0.48277	1.03493	1.65741	0.34285	1.04799	1.05331
47.0	1.24707	0.10513	1.28020	1.78920	0.50926	1.01484	1.63287	0.36167	1.03278	1.04032
48.0	1.24274	0.11046	1.27491	1.77015	0.53699	0.99439	1.60791	0.38135	1.01722	1.02697
49.0	1.23826	0.11600	1.26943	1.75073	0.56606	0.97361	1.58252	0.40196	1.00131	1.01327
50.0	1.23362	0.12175	1.26377	1.73092	0.59655	0.95250	1.55672	0.42354	0.98503	0.99920
51.0	1.22882	0.12772	1.25791	1.71072	0.62857	0.93105	1.53051	0.44616	0.96839	0.98475
52.0	1.22385	0.13394	1.25184	1.69014	0.66223	0.90927	1.50390	0.46990	0.95138	0.96992
53.0	1.21869	0.14040	1.24556	1.66917	0.69768	0.88718	1.47690	0.49484	0.93401	0.95470
54.0	1.21335	0.14712	1.23906	1.64782	0.73505	0.86477	1.44952	0.52106	0.91626	0.93907
55.0	1.20781	0.15412	1.23233	1.62607	0.77452	0.84205	1.42177	0.54868	0.89813	0.92302
56.0	1.20207	0.16141	1.22534	1.60392	0.81628	0.81902	1.39365	0.57779	0.87962	0.90654
57.0	1.19610	0.16902	1.21810	1.58138	0.86054	0.79570	1.36518	0.60854	0.86072	0.88961
58.0	1.18991	0.17696	1.21058	1.55844	0.90754	0.77208	1.33637	0.64106	0.84143	0.87222
59.0	1.18347	0.18525	1.20278	1.53510	0.95758	0.74817	1.30723	0.67552	0.82174	0.85435
60.0	1.17677	0.19393	1.19467	1.51134	1.01098	0.72398	1.27776	0.71211	0.80165	0.83597
θ	L_1	L_2	C_2	L_3	L_4	C_4	L_5	L_6	C_6	L_7

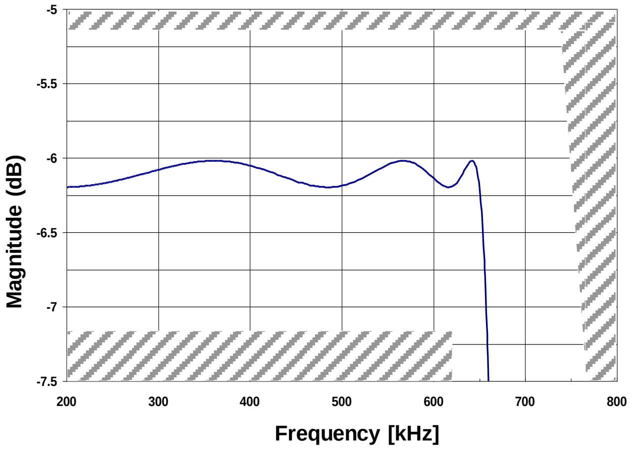
RLC Filter Frequency Response

- Component values denormalized
- Frequency response simulated
- Frequency mask superimposed
- Frequency response well within spec.



Frequency Response Passband Detail

- Passband well within spec.
- Make sure enough margin is allowed for variations due to process & temperature



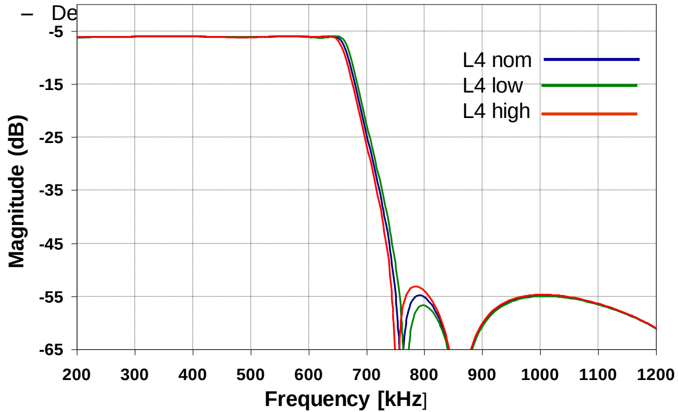
RLC Ladder Filter Sensitivity

- The design has the same specifications as the previous example implemented with cascaded biquads
- To compare the sensitivity of RLC ladder versus cascaded-biquads:
  - Changed all Ls &Cs one by one by 2% in order to change the pole/zeros by 1% (similar test as for cascaded biquad)
  - Found frequency response → most sensitive to L4 variations
  - Note that by varying L4 both poles & zeros are varied

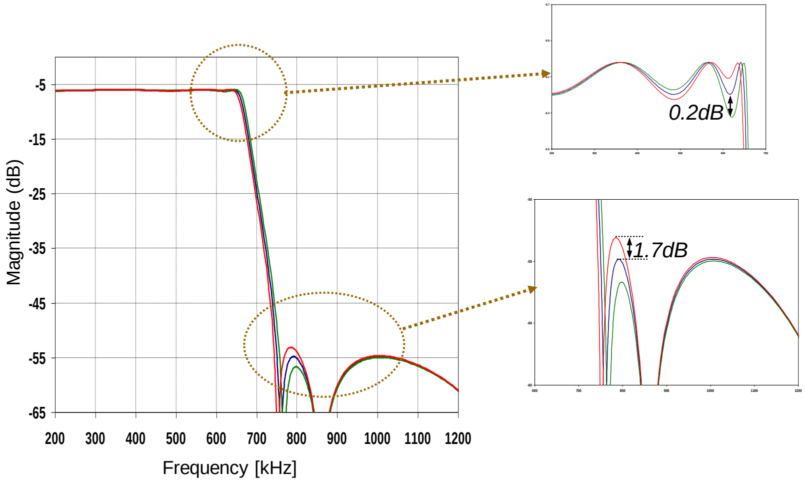
RCL Ladder Filter Sensitivity

Component mismatch in RLC filter:

- Increase L4 from its nominal value by 2%
- De



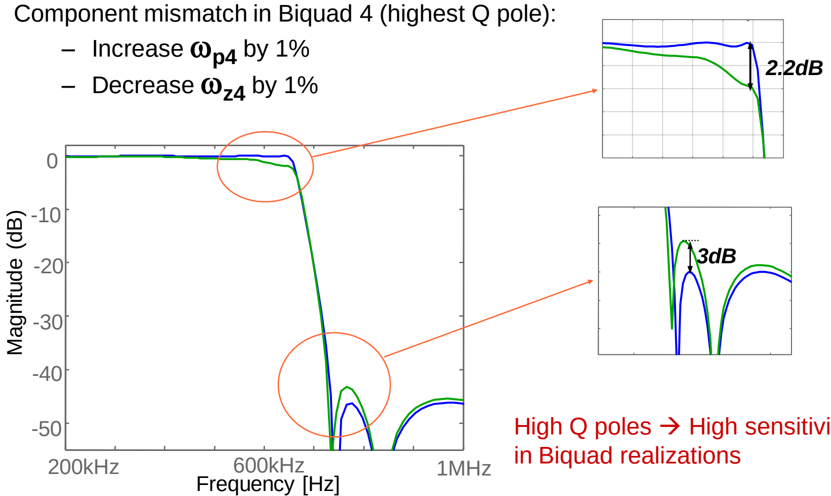
RCL Ladder Filter Sensitivity



Sensitivity of Cascade of Biquads

Component mismatch in Biquad 4 (highest Q pole):

- Increase ω_{p4} by 1%
- Decrease ω_{z4} by 1%



High Q poles → High sensitivity in Biquad realizations

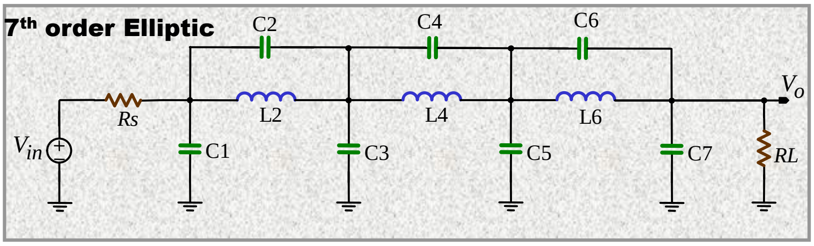
Sensitivity Comparison for Cascaded-Biquads versus RLC Ladder

- 7th Order elliptic filter
  - 1% change in pole & zero pair

	Cascaded Biquad	**RLC Ladder**
Passband deviation	2.2dB (29%)	0.2dB (2%)
Stopband deviation	3dB (40%)	1.7dB (21%)

Doubly terminated LC ladder filters ⇨ Significantly lower sensitivity compared to cascaded-biquads particularly within the passband

RLC Ladder Filter Design
Example: CDMA IS95 Receive Filter



- Previously learned to design integrator based ladder filters without transmission zeros
  - → Question:
    - o How do we implement the transmission zeros in the integrator-based version?
    - o Preferred method → no extra power dissipation → no extra active elements

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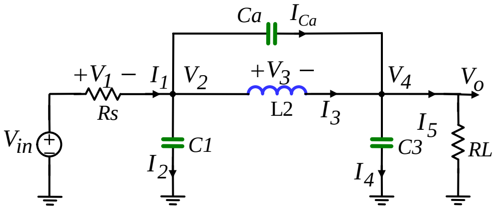
Integrator Based Ladder Filters
How Do to Implement Transmission zeros?

- Use KCL & KVL to derive :

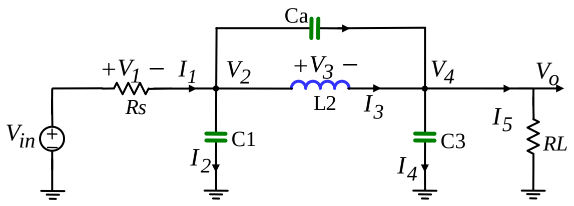


$$I_2 = I_1 - I_3 - I_{C_a}, \qquad I_{C_a} = (V_2 - V_4)sC_a, \qquad V_2 = \frac{I_2}{sC_1}$$

Substituting for I_2 $\quad \rightarrow V_2 = \dfrac{I_1 - I_3 - I_{C_a}}{sC_1}$

Substituting for I_{C_a} *and rearranging :* $\boxed{V_2 = \dfrac{I_1 - I_3}{s(C_1 + C_a)} + V_4 \times \dfrac{C_a}{C_1 + C_a}}$

Integrator Based Ladder Filters
How Do to Implement Transmission zeros?



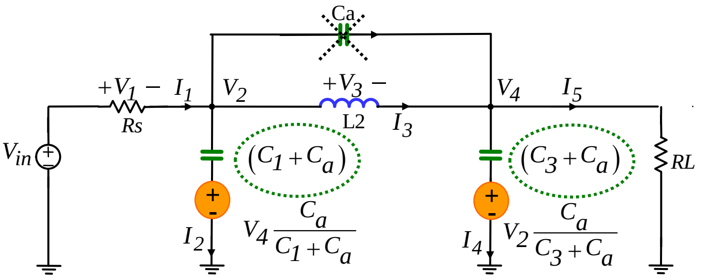
- Use KCL & KVL to derive :

$$V_2=\frac{I_1-I_3}{s\left(C_1+C_a\right)}+\ V_4\times\frac{C_a}{C_1+C_a}$$

$$V_4=\frac{I_3-I_5}{s\left(C_3+C_a\right)}+\ V_2\times\frac{C_a}{C_3+C_a}$$

Frequency independent constants
Can be substituted by:
Voltage-Controlled Voltage Source

Integrator Based Ladder Filters
Transmission zeros



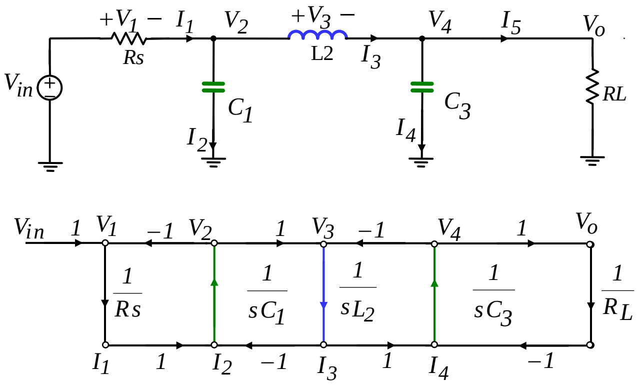
- Replace *shunt capacitors* with *voltage controlled voltage sources*:

$$V_2=\frac{I_1-I_3}{s\left(C_1+C_a\right)}+V_4\frac{C_a}{C_1+C_a}$$

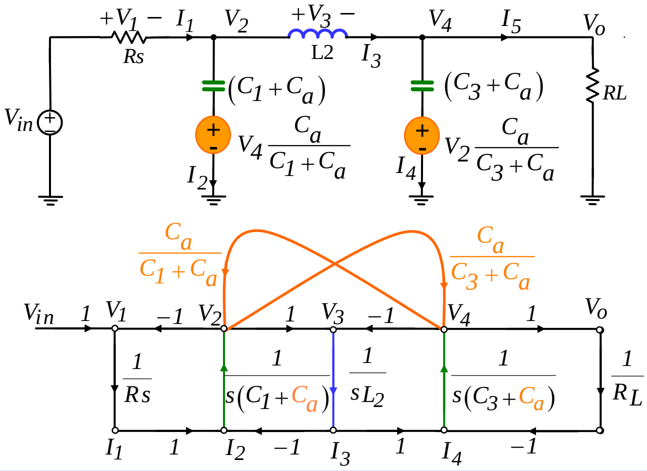
$$V_4=\frac{I_3-I_5}{s\left(C_3+C_a\right)}+V_2\frac{C_a}{C_3+C_a}$$

Exact same expressions
as with Ca present

3rd Order Lowpass Filter
All Poles & No Zeros

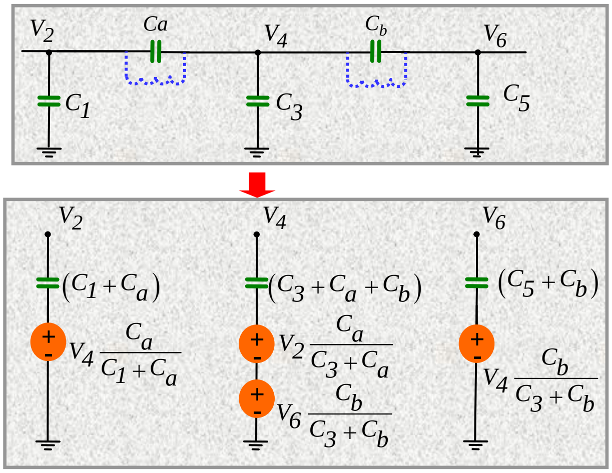


Implementation of Zeros in Active Ladder Filters Without Use of Active Elements



Integrator Based Ladder Filters

Higher Order Transmission zeros



Convert zero generating Cs in C loops to voltage-controlled voltage sources

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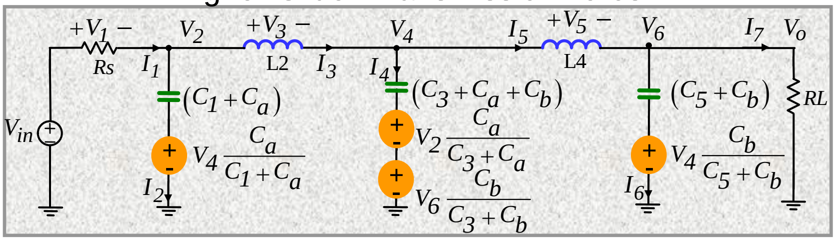
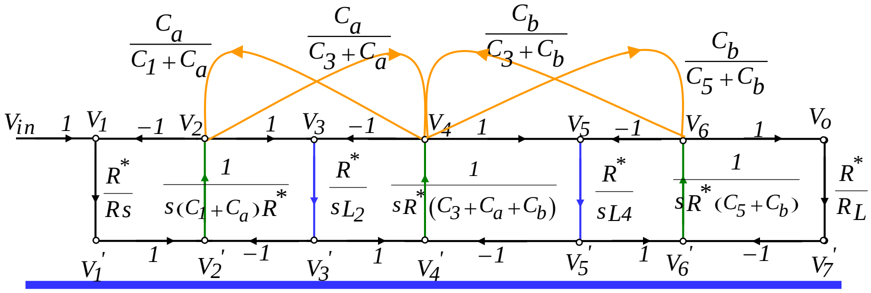
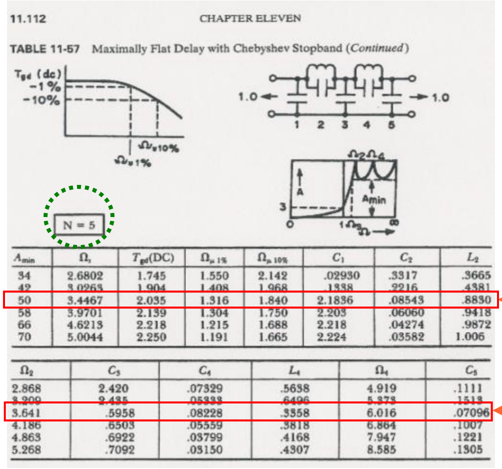
Higher Order Transmission zeros





Example:
5th Order Chebyshev II Filter



11.112 CHAPTER ELEVEN

TABLE 11-57 Maximally Flat Delay with Chebyshev Stopband (*Continued*)

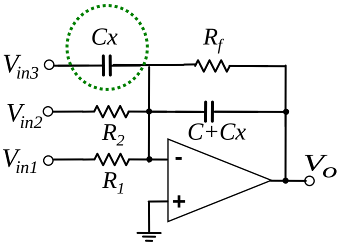
N = 5

A_{min}	Ω_s	T_{gd}(DC)	$\Omega_{\mu\,1\%}$	$\Omega_{\mu\,10\%}$	C_1	C_2	L_2
34	2.6802	1.745	1.550	2.142	.02930	.3317	.3665
42	3.0263	1.904	1.408	1.968	.1338	.2216	.4381
50	3.4467	2.035	1.316	1.840	2.1836	.08543	.8830
58	3.9701	2.139	1.304	1.750	2.203	.06060	.9418
66	4.6213	2.218	1.215	1.688	2.218	.04274	.9872
70	5.0044	2.250	1.191	1.665	2.224	.03582	1.006

Ω_2	C_3	C_4	L_4	Ω_4	C_5
2.868	2.420	.07329	.5638	4.919	.1111
3.209	2.435	.05333	.6496	5.373	.1513
3.641	.5958	.08228	.3358	6.016	.07096
4.186	.6503	.05559	.3818	6.864	.1007
4.863	.6922	.03799	.4168	7.947	.1221
5.268	.7092	.03150	.4307	8.585	.1305

- 5th order Chebyshev II
- Table from: Williams & Taylor book, p. 11.112
- 50dB stopband attenuation
- f_{-3dB} =10MHz

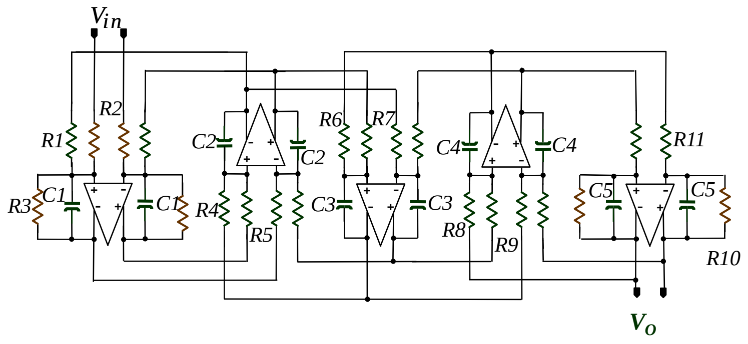
Transmission Zero Generation
Opamp-RC Integrator



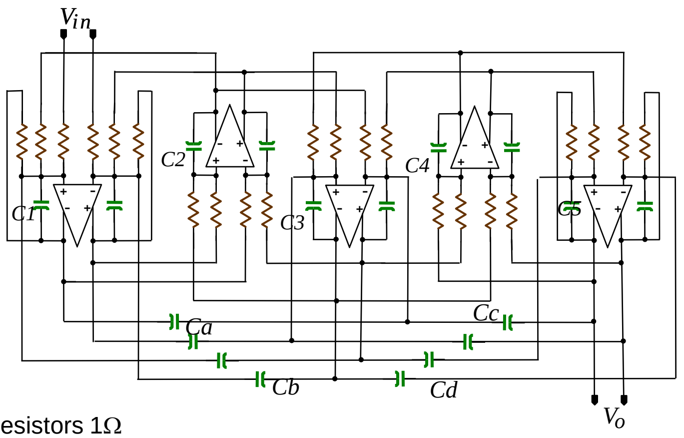
$$V_O = -\frac{1}{s(C+C_X)}\left[\frac{V_{in1}}{R_1}+\frac{V_{in2}}{R_2}+\frac{V_O}{R_f}\right]$$

$$-\,V_{in3}\times\frac{C_X}{C+C_X}$$

Differential Integrator Based LP Ladder Filter
Final Design 5th Order All-Pole

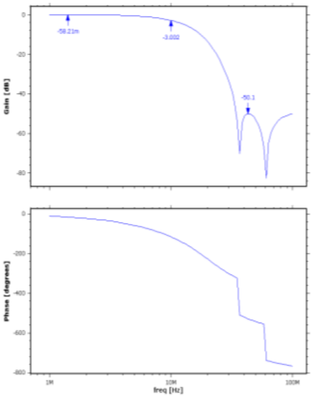


Differential 5th Order Chebychev Lowpass Filter

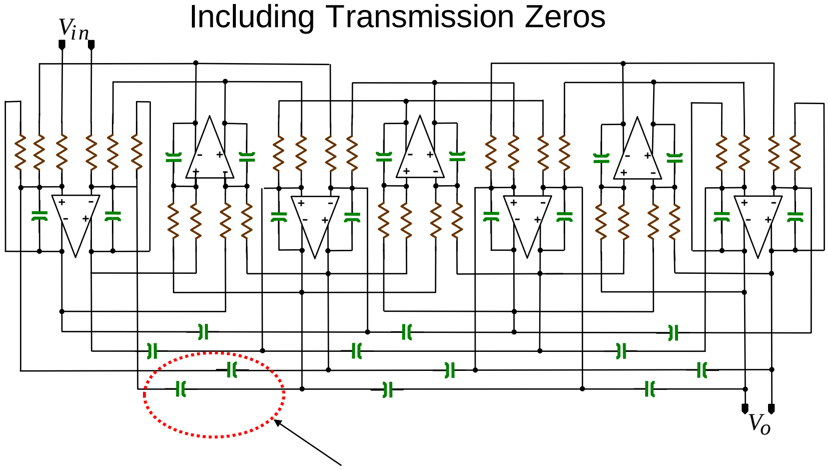


- ❑All resistors 1Ω
- ❑Capacitors: *C1=36.11nF, C2=14.05nF, C3=12.15nF,C4=5.344nF, C5=2.439nF*
- ❑*Coupling capacitors: Ca=1.36nF, Cb=1.36nF, Cc=1.31nF, Cd=1.31nF*

5^{th} Order Chebyshev II Filter Simulated Frequency Response



7th Order Differential Lowpass Filter Including Transmission Zeros



Transmission zeros implemented with pair of coupling capacitors

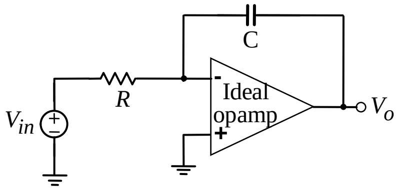
Effect of Integrator Non-Idealities on Filter Frequency Characteristics

- In the passive filter design (RLC filters) section:
  - Reactive element (L & C) non-idealities → expressed in the form of Quality Factor (Q)
  - Filter impairments due to component non-idealities explained in terms of component Q
- In the context of active filter design (integrator-based filters)
  - Integrator non-idealities → Translates to the form of Quality Factor (Q)
  - Filter impairments due to integrator non-idealities explained in terms of integrator Q

Effect of Integrator Non-Idealities on Filter Performance

- Ideal integrator characteristics
- Real integrator characteristics:
  - Effect of opamp finite DC gain
  - Effect of integrator non-dominant poles

Effect of Integrator Non-Idealities on Filter Performance
Ideal Integrator



Ideal Integrator:

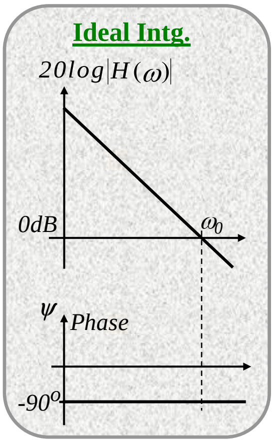
$DC\ \ gain = \infty$

$Single\ pole\ @\ DC$

$\rightarrow no\ non\text{-}dominant\ poles$

$$H(s) = \frac{-\omega_o}{s}$$

$$\omega_o = 1/RC$$



Ideal Integrator Quality Factor

Ideal intg. transfer function:

$$H(s) = \frac{-\omega_o}{s} = \frac{-\omega_o}{j\omega} = -\frac{1}{j\frac{\omega}{\omega_o}}$$

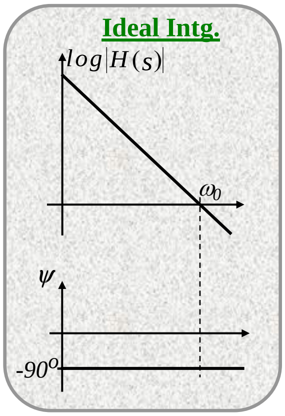
Since component Q is defined as::

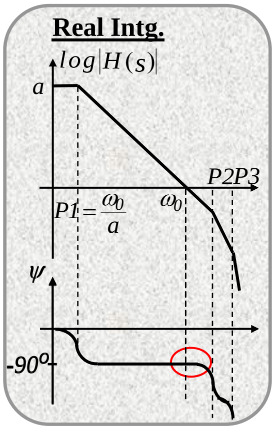
$$\begin{cases} H(j\omega) = \dfrac{1}{R(\omega) + jX(\omega)} \\ Q = \dfrac{X(\omega)}{R(\omega)} \end{cases}$$

Then Q factor at the unity-gain frequency (ω_o):

$$\boxed{Q_{ideal}^{intg.} = \infty}$$

Real Integrator Opamp Related Non-Idealities

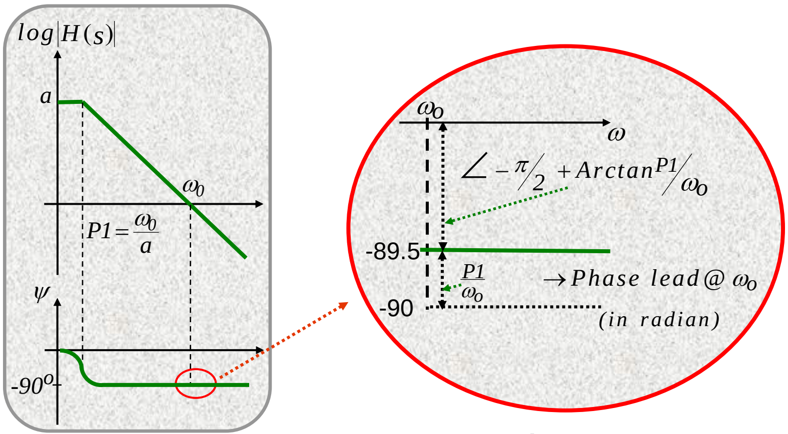




$$H(s) = \frac{-\omega_0}{s}$$

$$H(s) \approx \frac{-a}{\left(1+s\frac{a}{\omega_0}\right)\left(1+\frac{s}{p2}\right)\left(1+\frac{s}{p3}\right)\cdots}$$

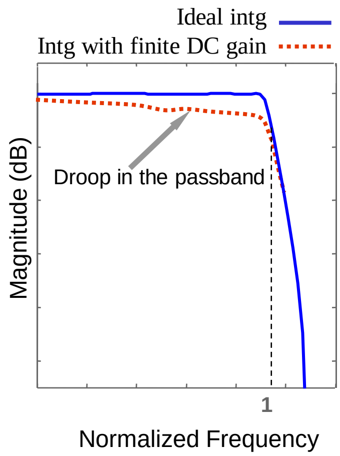
Effect of Integrator Finite DC Gain on Q



Example: $a=100 \rightarrow P1/\omega_0 = 1/100$

→ phase error ≅ $+0.5$degree

Effect of Integrator Finite DC Gain on Q
Example: Lowpass Filter



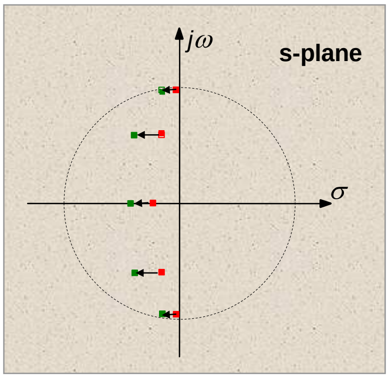
- Finite opamp DC gain

  → Phase lead @ ω_0

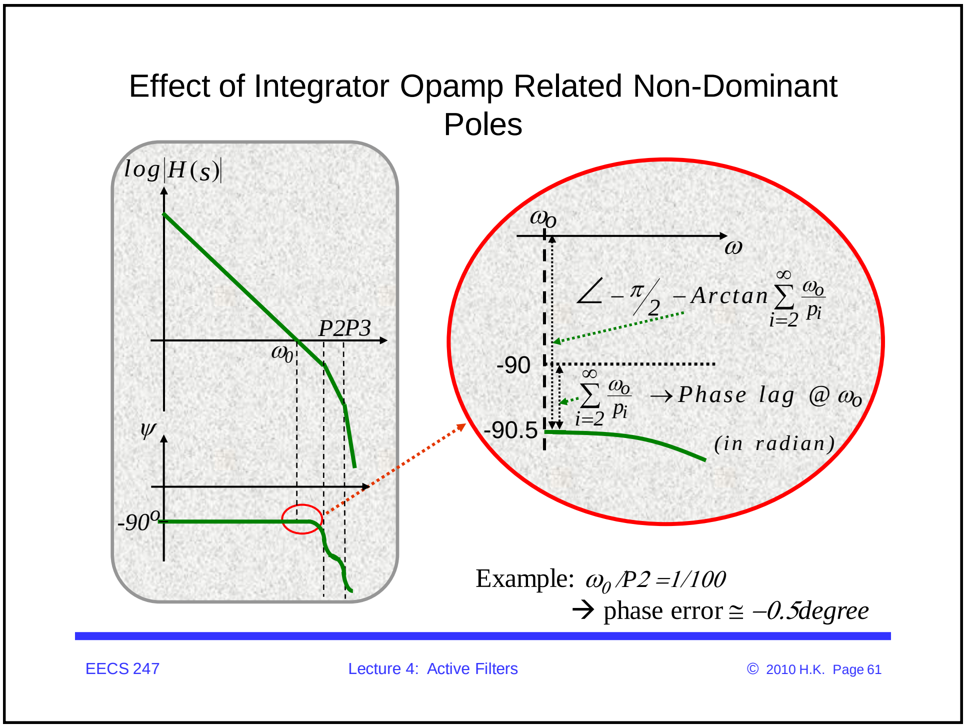
  →Droop in the passband

Effect of Integrator Finite DC Gain on Q
Example: Lowpass Filter

- Effect of opamp finite DC gain on filter singularities
- Pushes the ideal poles away from the $j\omega$ axis
- Results in Q reduction of the poles and thus droop in the passband



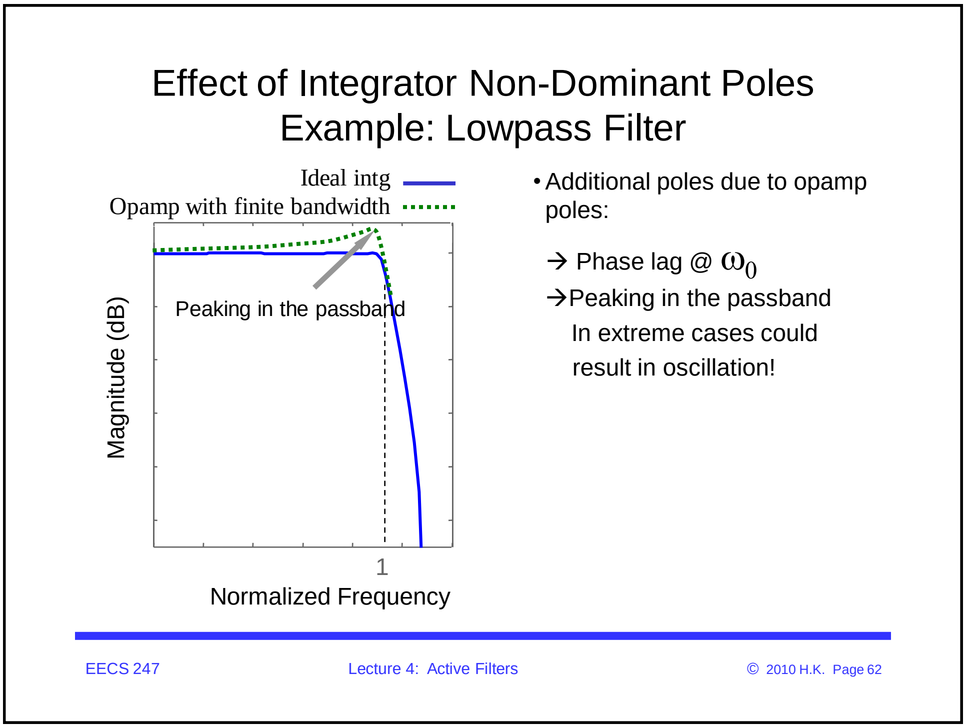
🟩 Opamp with finite DC gain
🟥 Ideal opamps & ideal filter pole locations



Effect of Integrator Opamp Related Non-Dominant Poles

Example: $\omega_0 / P2 = 1/100$

→ phase error $\cong -0.5 degree$



Effect of Integrator Non-Dominant Poles
Example: Lowpass Filter

- Additional poles due to opamp poles:

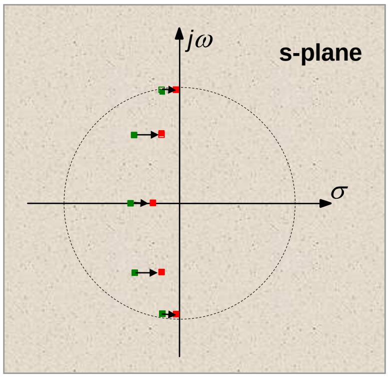
→ Phase lag @ ω_0

→Peaking in the passband

In extreme cases could result in oscillation!

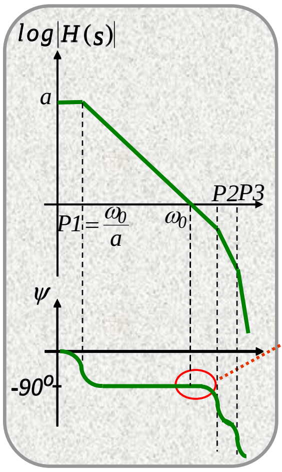
Effect of Integrator Non-Dominant Poles Example: Lowpass Filter

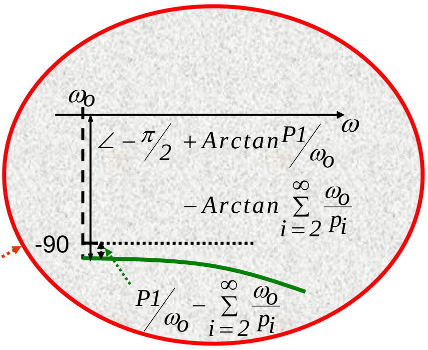
- Effect of opamp finite bandwidth on filter singularities
- Pushes the ideal poles towards $j\omega$ axis
- Results in Q enhancement of the poles and thus peaking in the passband



🟥 Opamp with finite bandwidth
🟩 Ideal opamps & ideal pole locations

Effect of Integrator Non-Dominant Poles & Finite DC Gain on Q





Note that the two terms have different signs
→ Can cancel each other's effect!

Integrator Quality Factor

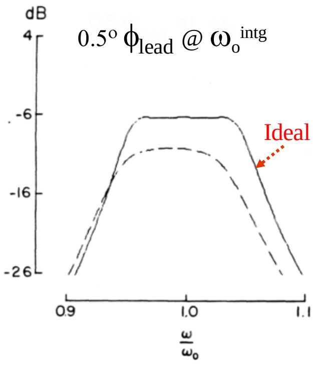
Real intg. transfer function: $H(s) \approx \frac{-a}{\left(1+s\frac{a}{\omega_o}\right)\left(1+\frac{s}{p2}\right)\left(1+\frac{s}{p3}\right)\dots}$

Based on the definition of Q and assuming that: $\frac{\omega_o}{p_{2,3,\dots\dots}} << 1 \quad \& \quad a >> 1$

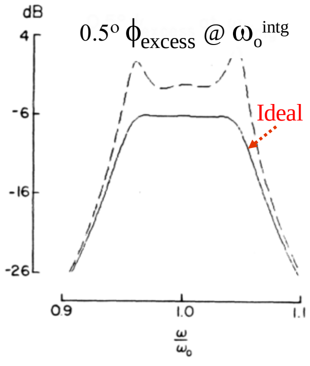
It can be shown that in the vicinity of unity-gain-frequency:

$$\boxed{Q_{real}^{intg.} \approx \frac{1}{\underbrace{\frac{1}{a}}_{\text{Phase lead @ }\omega_o} - \omega_o \underbrace{\sum_{i=2}^{\infty}\frac{1}{p_i}}_{\text{Phase lag @ }\omega_0}}}$$

Example:
Effect of Integrator Finite Q on Bandpass Filter Behavior



Integrator DC gain=*100*

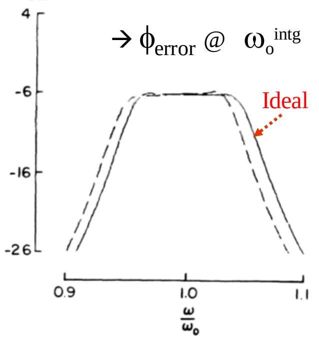


Integrator $P2\ @\ 100.\omega_o$

Example:
Effect of Integrator Q on Filter Behavior

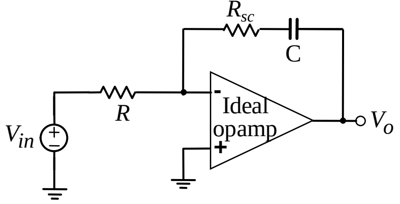
$(0.5^o\ \phi_{lead} - 0.5^o\ \phi_{excess})\ @\ \ \omega_o^{intg}$

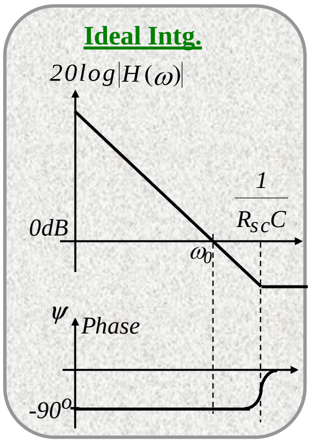
$\rightarrow \phi_{error}\ @\ \ \omega_o^{intg}\ \sim 0$



Integrator DC gain=*100* & *P2 @ 100.* ω_o

Effect of Integrating Capacitor Series Resistance on Integrator Q





Finite R_{sc} *adds LHP zero @* $\frac{1}{R_{sc}C}$

$$H(s) = \frac{-\omega_o(1+R_{sc}Cs)}{s}$$

$$\rightarrow Q_{intg} \approx \boxed{\frac{R}{R_{sc}}}$$

Typically, opamp non-idealites dominate Q_{intg}

Summary

Effect of Integrator Non-Idealities on Q

$$Q_{ideal}^{intg.} = \infty$$

$$Q_{real}^{intg.} \approx \frac{1}{\frac{1}{a} - \omega_o \sum_{i=2}^{\infty} \frac{1}{p_i}}$$

- Amplifier finite DC gain reduces the overall Q in the same manner as series/parallel resistance associated with passive elements
- Amplifier poles located above integrator unity-gain frequency enhance the *Q*!
  - If non-dominant poles close to unity-gain freq. → Oscillation
- Depending on the location of unity-gain-frequency, the two terms can cancel each other out!
- Overall quality factor of the integrator has to be much higher compared to the filter's highest pole Q