EE247
Lecture 5

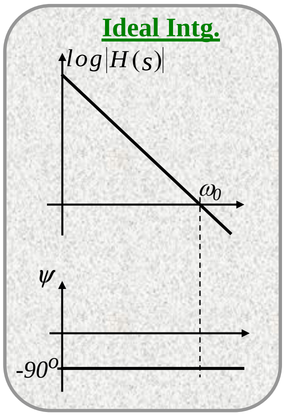
- Filters
  - Effect of integrator non-idealities on filter behavior
    - Integrator quality factor and its influence on filter frequency characteristics (brief review for last lecture)
    - Filter dynamic range limitations due to limited integrator linearity
      - Measures of linearity: Harmonic distortion, intermodulation distortion, intercept point
    - Effect of integrator component variations and mismatch on filter response
  - Various integrator topologies utilized in monolithic filters
    - Resistor + C based filters
    - Transconductance (gm) + C based filters
    - Switched-capacitor filters
  - Continuous-time filter considerations
    - Facts about monolithic Rs, gms, & Cs and its effect on integrated filter characteristics

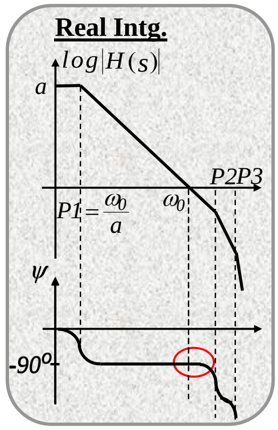
Summary of Lecture 4

- Ladder type RLC filters converted to integrator based active filters
  - All pole ladder type filters
    - Convert RLC ladder filters to integrator based form
    - Example: 5^{th} order Butterworth filter
  - High order ladder type filters incorporating zeros
    - 7th order elliptic filter in the form of ladder RLC with zeros
      - Sensitivity to component mismatch
      - Compare with cascade of biquads

    → *Doubly terminated LC ladder filters ⇨ Lowest sensitivity to component variations*
    - Convert to integrator based form utilizing SFG techniques
    - Example: Differential high order filter implementation
  - Effect of integrator non-idealities on continuous-time filter behavior
    - Effect of integrator finite DC gain & non-dominant poles on filter frequency response

Real Integrator Non-Idealities

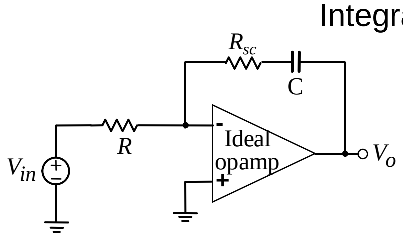


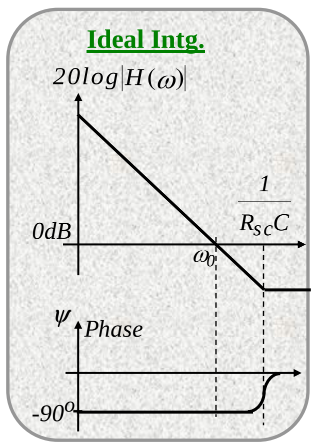


$$H(s) = \frac{-\omega_0}{s}$$

$$H(s) \approx \frac{-a}{\left(1+s\frac{a}{\omega_0}\right)\left(1+\frac{s}{p2}\right)\left(1+\frac{s}{p3}\right)\ldots}$$

Effect of Integrating Capacitor Series Resistance on Integrator Q





Finite R_{sc} adds LHP zero @ $\frac{1}{R_{sc}C}$

$$H(s) = \frac{-\omega_o(1+R_{sc}Cs)}{s}$$

$$\rightarrow Q_{intg} \approx \frac{R}{R_{sc}}$$

Typically, opamp non-idealites dominate Q_{intg}

Summary

Effect of Integrator Non-Idealities on Q

$$Q_{ideal}^{intg.} = \infty$$

$$Q_{real}^{intg.} \approx \frac{1}{\frac{1}{a} - \omega_o \sum_{i=2}^{\infty} \frac{1}{p_i}}$$

- Amplifier finite DC gain reduces the overall Q in the same manner as series/parallel resistance associated with passive elements
- Amplifier poles located above integrator unity-gain frequency enhance the *Q*!
  - If non-dominant poles close to unity-gain freq. → Oscillation
- Depending on the location of unity-gain-frequency, the two terms can cancel each other out!
- Overall quality factor of the integrator has to be much higher compared to the filter's highest pole Q

Effect of Integrator Non-Linearities on Overall Integrator-Based Filter Performance

- Dynamic range of a filter is determined by the ratio of maximum signal output with acceptable performance over total noise
- Maximum signal handling capability of a filter is determined by the non-linearities associated with its building blocks
- Integrator linearity function of opamp/R/C (or any other component used to build the integrator) linearity-
- Linearity specifications for active filters typically given in terms of :
  - Maximum allowable harmonic distortion @ the output
  - Maximum tolerable intermodulation distortion
  - Intercept points & compression point referred to output or input

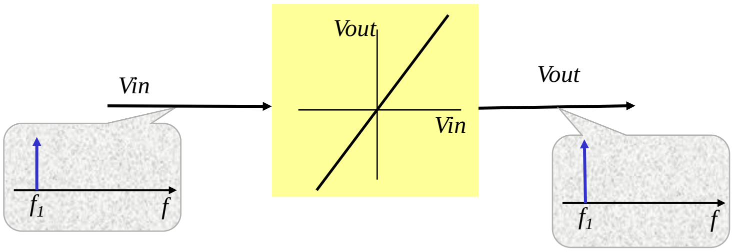
Component Linearity versus Overall Filter Performance
1- Ideal Components

Ideal DC transfer characteristics:

Perfectly linear output versus input tranfer function with no clipping

$Vout = \alpha \; Vin \quad \text{for} \; -\infty \leq Vin \leq \infty$

$If \; Vin = A\sin(\omega_1 t) \rightarrow Vout = \alpha \; A\sin(\omega_1 t)$



Component Linearity versus Overall Filter Performance
2- Semi-Ideal Components

Semi-ideal DC transfer characteristics:

Perfectly linear output versus input transfer function with clipping

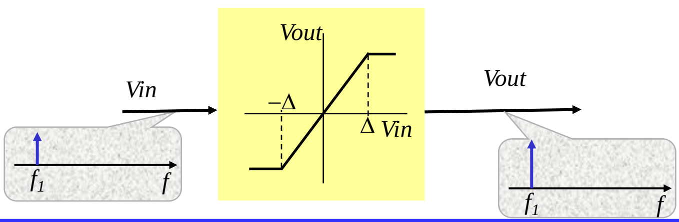
$Vout = \alpha \; Vin \quad \text{for} \; -\Delta \leq Vin \leq +\Delta$

$Vout = -\Delta\alpha \quad \text{for} \; Vin \leq -\Delta$

$Vout = \Delta\alpha \quad \text{for} \; Vin \geq \Delta$

$If \; Vin = A\sin(\omega_1 t) \rightarrow Vout = \alpha \; A\sin(\omega_1 t) \quad \text{for} \; -\Delta \leq Vin \leq +\Delta$

Otherwise clipped & distorted



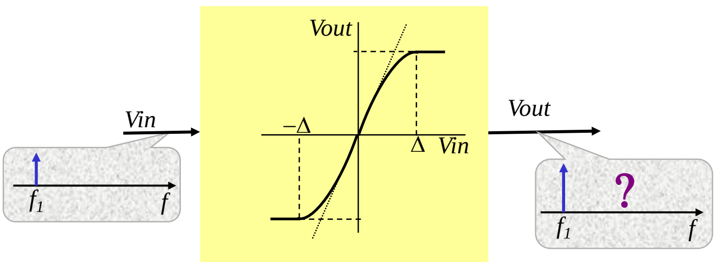
Effect of Component Non-Linearities on Overall Filter Linearity
Real Components including Non-Linearities

Real DC transfer characteristics: Both soft non-linearities & hard (clipping)

$Vout = \alpha_1 Vin + \alpha_2 Vin^2 + \alpha_3 Vin^3 + ... \text{for} \quad -\Delta \le Vin \le \Delta$

Clipped otherwise

$If \;\; Vin = A\sin(\omega_1 t)$



Effect of Component Non-Linearities on Overall Filter Linearity
Real Components including Non-Linearities

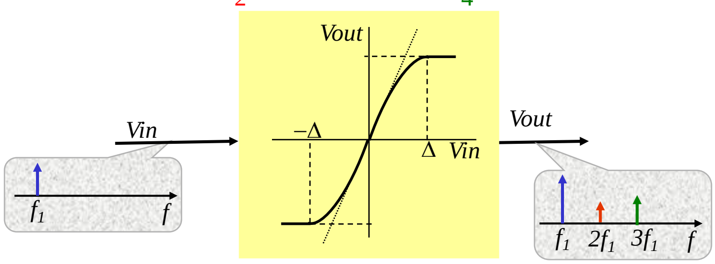
Typical real circuit DC transfer characteristics:

$Vout = \alpha_1 Vin + \alpha_2 Vin^2 + \alpha_3 Vin^3 + ... \;\; If \;\; Vin = A\sin(\omega_1 t) \;\&\; A < \Delta$

Then:

$\rightarrow Vout = \alpha_1 A\sin(\omega_1 t) + \alpha_2 A^2 \sin(\omega_1 t)^2 + \alpha_3 A^3 \sin(\omega_1 t)^3 + ...$

$or \; Vout = \alpha_1 A\sin(\omega_1 t) + \frac{\alpha_2 A^2}{2}\left(1 - \cos(2\omega_1 t)\right) + \frac{\alpha_3 A^3}{4}\left(3\sin(\omega_1 t) - \sin(3\omega_1 t)\right) + ...$



Effect of Component Non-Linearities on Overall Filter Linearity
Harmonic Distortion

$$Vout = \alpha_1 A\sin(\omega t) + \frac{\alpha_2 A^2}{2}(1-\cos(2\omega t)) + \frac{\alpha_3 A^3}{4}(3\sin(\omega t) - \sin(3\omega t)) + \ldots\ldots\ldots\ldots$$

$$HD2 = \frac{amplitude\ 2^{nd}\ harmonic\ distortion\ component}{amplitude\ fundamental}$$

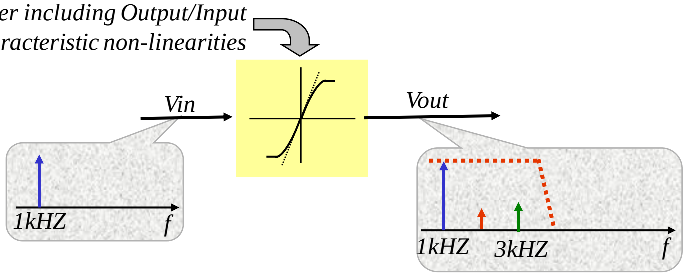
$$HD3 = \frac{amplitude\ 3^{rd}\ harmonic\ distortion\ component}{amplitude\ fundamental}$$

$$\rightarrow HD2 = \frac{1}{2}\times\frac{\alpha_2}{\alpha 1}A\ , \qquad HD3 = \frac{1}{4}\times\frac{\alpha_3}{\alpha 1}A^2$$

Example: Significance of Filter Harmonic Distortion in Voice-Band CODECs

- Voice-band CODEC filter (CODEC stands for coder-decoder, telephone circuitry includes CODECs with extensive amount of integrated active filters)
- Specifications includes limits associated with maximum allowable harmonic distortion at the output (< typically < 1% → -40dB)

CODEC Filter including Output/Input transfer characteristic non-linearities



Example: Significance of Filter Harmonic Distortion in Voice-Band CODECs

- Specifications includes limits associated with maximum allowable harmonic distortion at the output (< typically < 1% → -40dB)
- Let us assume filter output/input transfer characteristic:

$$\frac{\alpha_3}{\alpha 1} = 1/100 \quad and \quad \alpha_2 \text{ is negligible}$$

since:

$$HD3 = \frac{1}{4} \times \frac{\alpha_3}{\alpha 1} A^2$$

The requirement of $HD3 < 1/100 \rightarrow A_{\max} \leq 2V_{peak}$

- Note that with fixed HD3 requirements, larger α_3 would result in smaller acceptable maximum signal levels and therefore reduces the overall dynamic range.

→ Maximizing dynamic range requires highly linear circuit components

Effect of Component Non-Linearities on Overall Filter Linearity Intermodulation Distortion

DC transfer characteristics including nonlinear terms, input 2 sinusoidal waveforms:

$$Vout = \alpha_1 Vin + \alpha_2 Vin^2 + \alpha_3 Vin^3 + ...$$

$$If \;\; Vin = A_1 \sin(\omega_1 t) + A_2 \sin(\omega_2 t)$$

Then Vout will have the following components:

$$\alpha_1 Vin \rightarrow \alpha_1 A_1 \sin(\omega_1 t) + \alpha_1 A_2 \sin(\omega_2 t)$$

$$\alpha_2 Vin^2 \rightarrow \alpha_2 A_1^2 \sin(\omega_1 t)^2 + \alpha_2 A_2^2 \sin(\omega_2 t)^2 + 2\alpha_2 A_1 A_2 \sin(\omega_1 t)\sin(\omega_2 t) + ...$$

$$\rightarrow \frac{\alpha_2 A_1^2}{2}\left(1 - \cos(2\omega_1 t)\right) + \frac{\alpha_2 A_2^2}{2}\left(1 - \cos(2\omega_2 t)\right)$$

$$+ \alpha_2 A_1 A_2 \left[\cos((\omega_1 - \omega_2)t) - \cos((\omega_1 + \omega_2)t)\right]$$

$$\alpha_3 Vin^3 \rightarrow +\alpha_3 A_1^3 \sin(\omega_1 t)^3 + \alpha_3 A_2^3 \sin(\omega_2 t)^3$$

$$+ 3\alpha_3 A_1^2 A_2 \sin(\omega_1 t)^2 \sin(\omega_2 t) + 3\alpha_3 A_2^2 A_1 \sin(\omega_2 t)^2 \sin(\omega_1 t)$$

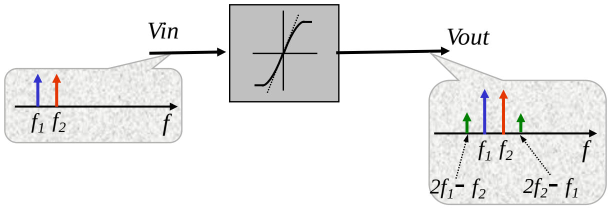
$$+ \frac{3\alpha_3 A_1^2 A_2}{4}\left[\sin(2\omega_1 + \omega_2)t - \sin(2\omega_1 - \omega_2)t\right]$$

Effect of Component Non-Linearities on Overall Filter Linearity
Intermodulation Distortion

Real DC transfer characteristics, input 2 sin waves:

$$Vout = \alpha_1 Vin + \alpha_2 Vin^2 + \alpha_3 Vin^3 + ...$$

$$If \;\; Vin = A_1 \sin(\omega_1 t) + A_2 \sin(\omega_2 t)$$



For f_1 & f_2 close in frequency → Components associated with $(2f_1\text{-}f_2)$ & $(2f_2\text{-}f_1)$ are the closest to the fundamental signals on the frequency axis and thus most harmful

Effect of Component Non-Linearities on Overall Filter Linearity
Intermodulation Distortion

Intermodulation distortion is measured in terms of IM2 and IM3:

Typically for input two sinusoids with equal amplitude ($A1 = A2 = A$)

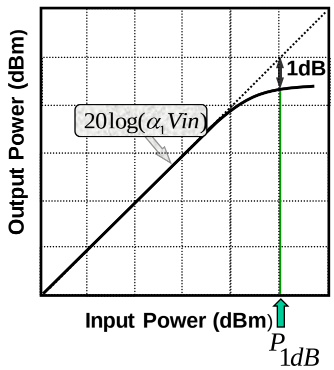
$$IM2 = \frac{amplitude\, 2^{nd}\; IM\;\; component}{amplitude\;\; fundamental}$$

$$IM3 = \frac{amplitude\, 3^{rd}\; IM\;\; component}{amplitude\;\; fundamental}$$

$$IM2 = \frac{\alpha_2}{\alpha_1} A \; + ... \qquad IM3 = \frac{3}{4}\frac{\alpha_3}{\alpha_1} A^2 + \frac{25}{8}\frac{\alpha_5}{\alpha_1} A^4 + ...$$

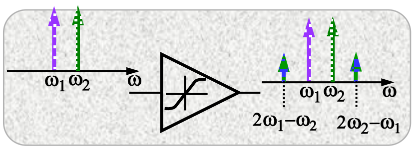
Wireless Communications Measure of Linearity

1dB Compression Point



$$Vout = \alpha_1 Vin + \alpha_2 Vin^2 + \alpha_3 Vin^3 + ...$$

Wireless Communications Measure of Linearity Third Order Intercept Point

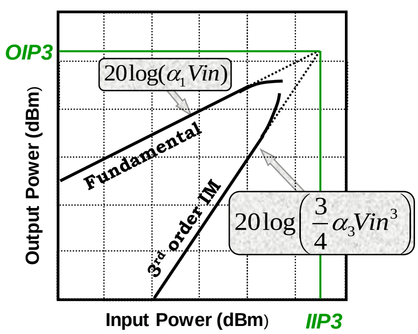


$$Vout = \alpha_1 Vin + \alpha_2 Vin^2 + \alpha_3 Vin^3 + ...$$

$$IM_3 = \frac{3rd}{1st}$$

$$= \frac{3}{4}\frac{\alpha_3}{\alpha_1} Vin^2 + \frac{25}{8}\frac{\alpha_5}{\alpha_1} Vin^4 + ...$$

$$= 1 @ IIP3$$



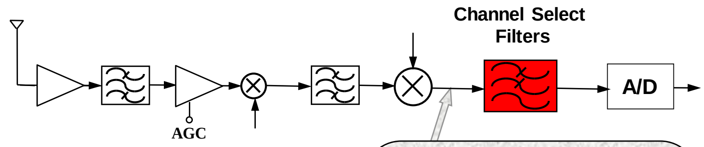
Typically:

$$IIP_3 - P_{1dB} = 9.6dB$$

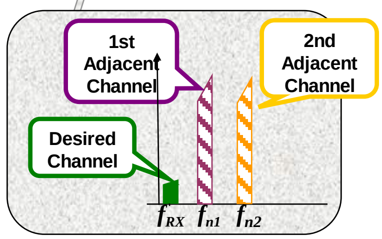
Most common measure of linearity for wireless circuits:
→ OIP3 & IIP3, Third order output/input intercept point

Example: Significance of Filter Intermodulation Distortion in Wireless Systems

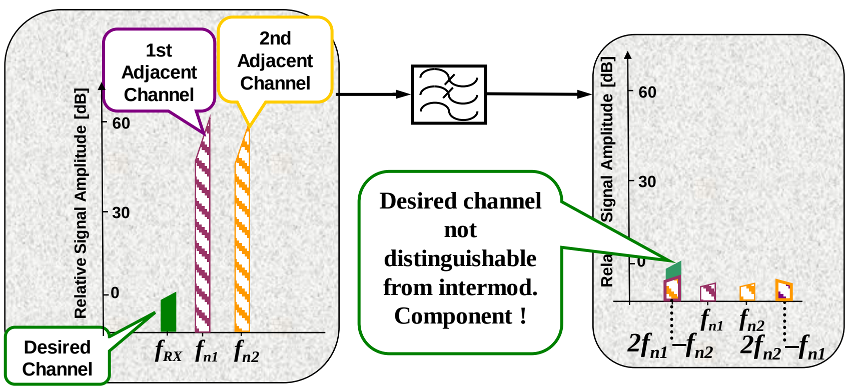
- Typical wireless receiver architecture



Worst case signal scenario wrt linearity of the building blocks
→ Two adjacent channels large compared to desired channel

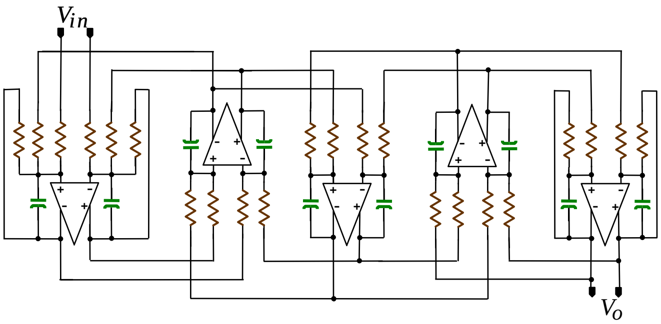


Example: Significance of Filter Intermodulation Distortion in Wireless Systems



- Adjacent channels can be as much as 60dB higher compared to the desired RX signal!
- Notice that in this example, 3rd order intermodulation component associated with the two adjacent channel, falls on the desired channel signal!

Filter Linearity

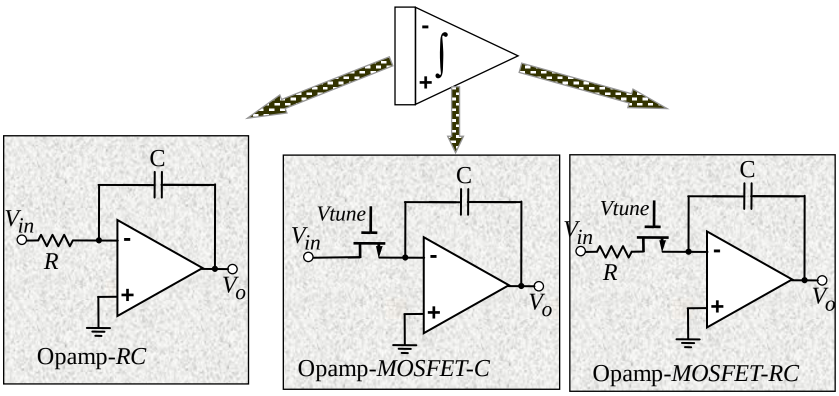


- Maximum signal handling capability is usually determined by the specifications wrt harmonic distortion and /or intermodulation distortion Distortion in a filter is a function of linearity of the components
- Example: In the above circuit linearity of the filter is mainly a function of linearity of the *opamp* voltage transfer characteristics

Various Types of Integrator Based Filter

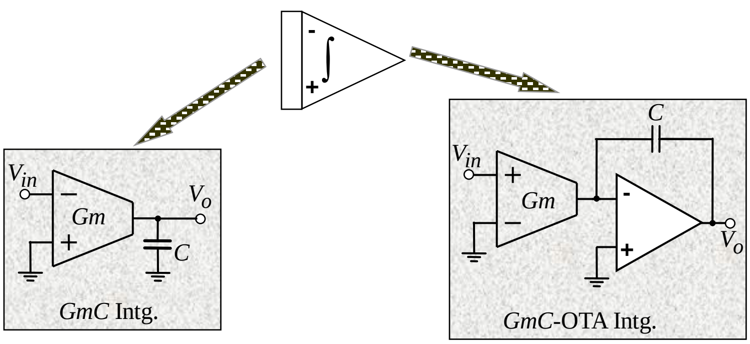
- Continuous Time
  - Resistive element based
    - Opamp-RC
    - Opamp-MOSFET-C
    - Opamp-MOSFET-RC
  - Transconductance (Gm) based
    - Gm-C
    - Opamp-Gm-C
- Sampled Data
  - Switched-capacitor Integrator

Continuous-Time Resistive Element Type Integrators
Opamp-RC & Opamp-MOSFET-C & Opamp-MOSFET-RC



Ideal transfer function: $\frac{V_o}{V_{in}} = \frac{-\omega_o}{s}$ where $\omega_o = \frac{1}{R_{eq}C}$

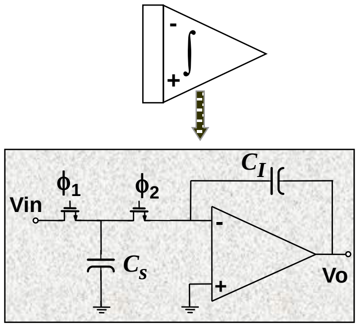
Continuous-Time Transconductance Type Integrator
Gm-C & Opamp-Gm-C

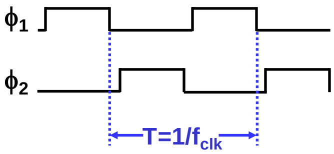


Ideal transfer function: $\frac{V_o}{V_{in}} = \frac{-\omega_o}{s}$ where $\omega_o = \frac{G_m}{C}$

Integrator Implementation

Switched-Capacitor





$$for \quad f_{signal} << f_{clk} \quad V_0 = \frac{f_{clk} \times C_S}{C_I} \int V_{in}\, dt$$

$$\omega_0 = f_{clk} \times \frac{C_S}{C_I}$$

Main advantage: Critical frequency function of <u>*ratio*</u> of caps & clock freq.

→ Critical filter frequencies (e.g. LPF -3dB freq.) very accurate

Few Facts About Monolithic *Rs & Cs & Gms*

- Monolithic continuous-time filter critical frequency set by *RxC* or *C/Gm*
- <u>Absolute value</u> of integrated *Rs & Cs & Gms* are quite variable
  - *Rs* vary due to doping and etching non-uniformities
    - Could vary by as much as ~+*-20 to 40%* due to process & temperature variations
  - *Cs* vary because of oxide thickness variations and etching inaccuracies
    - Could vary ~ +*-10 to15%*
  - *Gms* typically function of mobility, oxide thickness, current, device geometry …
    - Could vary > ~ +- *40%* or more with process & temp. & supply voltage

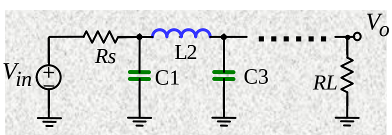
    → <u>Integrated continuous-time filter critical frequency could vary by over +*-50%*</u>

Few Facts About Monolithic Rs & Cs

- While absolute value of monolithic *Rs & Cs* and *gms* are quite variable, with special attention paid to layout, *C & R & gms* quite well-matched
  - Ratios very accurate and stable over processing, temperature, and time

- With special attention to layout (e.g. interleaving, use of dummy devices, common-centroid geometries…):
  - Capacitor mismatch << 0.1%
  - Resistor mismatch < 0.1%
  - Gm mismatch < 0.5%

Impact of Component Variations on Filter Characteristics



RLC Filters

Facts about RLC filters

- ω_{-3dB} determined by absolute value of *Ls & Cs*
- Shape of filter depends on ratios of normalized *L & C*

$$C_1^{RLC} = C_r \times C_1^{Norm} = \frac{C_1^{Norm}}{R^* \times \omega_{-3dB}}$$

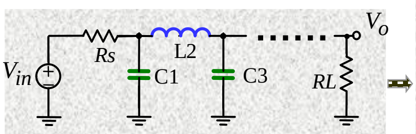
$$L_2^{RLC} = L_r \times L_2^{Norm} = \frac{L_2^{Norm} \times R^*}{\omega_{-3dB}}$$

Effect of Monolithic R & C Variations on Filter Characteristics

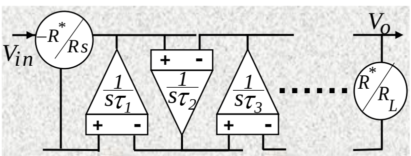
- Filter shape (whether Elliptic with 0.1dB Rpass or Butterworth..etc) is a function of *ratio* of normalized *Ls & Cs* in RLC filters
- Critical frequency (e.g. ω_{-3dB}) function of absolute value of *Ls xCs*
- Absolute value of integrated *Rs & Cs & Gms* are quite variable
- *Ratios* very accurate and stable over time and temperature

→ What is the effect of on-chip component variations on monolithic filter frequency characteristics?

Impact of Process Variations on Filter Characteristics



RLC Filters



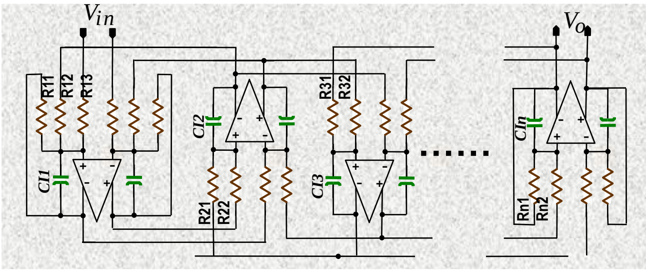
Integrator Based Filters

$$\tau_1 = C_1^{RLC}.R^* = \frac{C_1^{Norm}}{\omega_{-3dB}}$$

$$\tau_2 = \frac{L_2^{RLC}}{R^*} = \frac{L_2^{Norm}}{\omega_{-3dB}}$$

$$\frac{\tau_1}{\tau_2} = \frac{C_1^{Norm}}{L_2^{Norm}}$$

Impact of Process Variations on Filter Characteristics



$$\tau_1^{int g} = C_{I1}.R_1 = \frac{C_1^{Norm}}{\omega_{-3dB}}$$

$$\tau_2^{int g} = C_{I2}.R_2 = \frac{L_2^{Norm}}{\omega_{-3dB}}$$

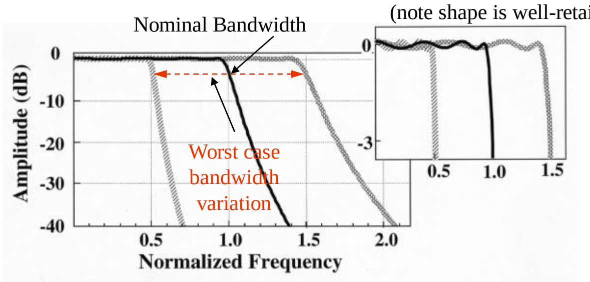
Variation in *absolute value* of integrated *Rs & Cs* → change in critical freq. (ω_{-3dB})

$$\frac{\tau_1^{int g}}{\tau_2^{int g}} = \frac{C_{I1}.R_1}{C_{I2}.R_2} = \frac{C_1^{Norm}}{L_2^{Norm}}$$

Since *ratios* of Rs & Cs very accurate

→ Continuous-time monolithic filters retain their shape due to good component matching even with variability in absolute component values

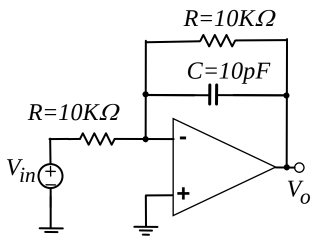
Example: LPF Worst Case Corner Frequency Variations



- While absolute value of on-chip RC (gm-C) time-constants could vary by as much as 100% (process & temp.)
- With proper precautions, excellent component matching can be achieved:
  - → Well-preserved relative amplitude & phase vs freq. characteristics
  - → Need to only adjust (tune) continuous-time filter critical frequencies

Tunable Opamp-RC Filters Example

- 1st order Opamp-RC filter is designed to have a corner frequency of *1.6MHz*
- Assuming process variations of:
  - *C* varies by *+-10%*
  - *R* varies by *+-25%*
- Build the filter in such a way that the corner frequency can be adjusted post-manufacturing.



Nominal R & C values for 1.6MHz corner frequency

Filter Corner Frequency Variations

- Assuming expected process variations of:
  - Maximum C variations by +-10%
    Cnom=10pF → Cmin=9pF, Cmax=11pF
  - Maximum R variations by +-25%
    Rnom=10K→ Rmin=7.5K, Rmax=12.5K
  - Corner frequency ranges from
    → 2.357MHz to 1.157MHz

→Corner frequency varies by +48% & -27%

Variable Resistor or Capacitor

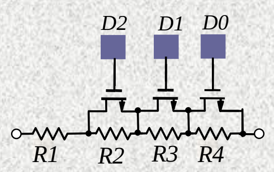
- In order to make provisions for filter to be tunable either R or C should be made adjustable (this example→ adjustable R)
- Monolithic Rs can only be made adjustable in discrete steps (not continuous)

$$\frac{R_{nom}^{max}}{R_{nom}} = \frac{f_{max}}{f_{nom}} = 1.48$$

$$\rightarrow R_{nom}^{max} = 14.8k\Omega$$

$$\frac{R_{nom}^{min}}{R_{nom}} = \frac{f_{min}}{f_{nom}} = 0.72$$

$$\rightarrow R_{nom}^{min} = 7.2k\Omega$$



Variable Resister
MOS xtors act as switches
→D_n high → switch acts as short circuit
→ D_n low → switch open circuit

Tunable Resistor

- Maximum C variations by +-10%→ Cmin=9pF, Cmax=11pF
- Maximum R variations by +-25%→ Rmin=7.5K, Rmax=12.5K
  →Corner frequency varies by +48% & -27.%
- Assuming control signal has $n = 3$bit (0 or 1) for adjustment → R2=2R3=4R4

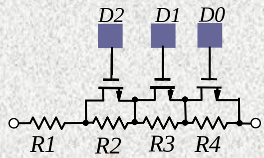
$$R_1 = R_{nom}^{min} = 7.2k\Omega$$

$$R_2 = \left(R_{nom}^{max} - R_{nom}^{min}\right) \times \frac{2^{n-1}}{2^n - 1} = (14.8k - 7.2k)\,4/7 = 4.34k\Omega$$

$$R_3 = \left(R_{nom}^{max} - R_{nom}^{min}\right) \times \frac{2^{n-2}}{2^n - 1} = (14.8k - 7.2k)\,2/7 = 2.17k\Omega$$

$$R_4 = \left(R_{nom}^{max} - R_{nom}^{min}\right) \times \frac{2^{n-3}}{2^n - 1} = (14.8k - 7.2k)\,1/7 = 1.08k\Omega$$

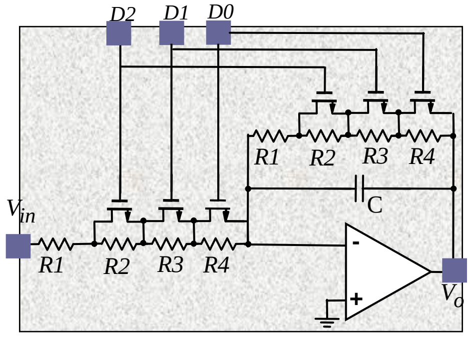
Tuning resolution $\approx 1.08k/10k \approx 10\%$



Variable Resister
MOS xtors act as switches

Tunable Opamp-RC Filter

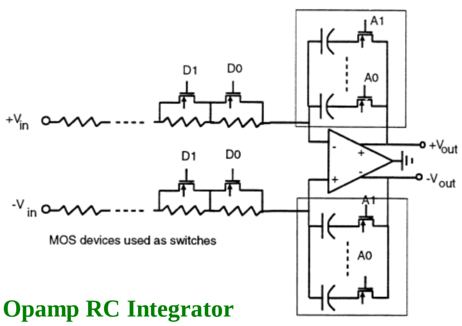
D2	D1	D0	Rnom
1	1	1	7.2K
1	1	0	8.28K
1	0	1	9.37K
…	…	…	
…	…	…	
…	…	…	
0	0	0	14.8K



Post manufacturing:
- Set all Dx to 100 (mid point)
- Measure -3dB frequency
  - If frequency too high decrement D to D-1
  - If frequency too low increment D to D+1
  - If frequency within 10% of the desired corner frequency → stop
  - else

For higher order filters, all filter integrators tuned simultaneously

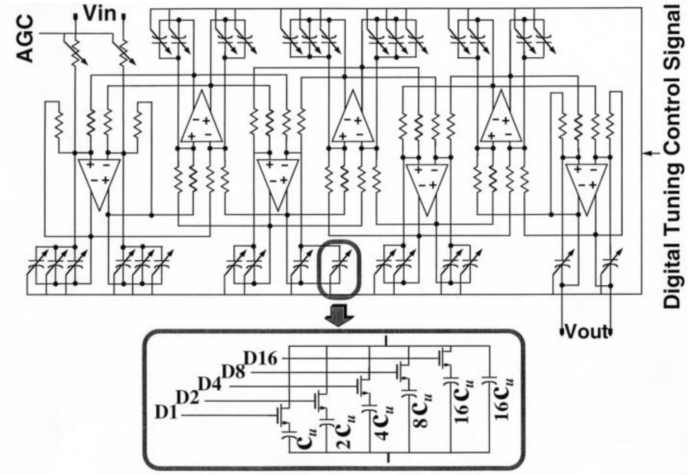
Tunable Opamp-RC Filters Summary



Tunable Opamp RC Integrator

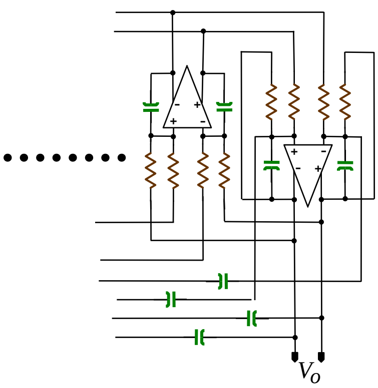
- Program Cs and/or Rs to freq. tune the filter
- All filter integrators tuned simultaneously
- Tuning in discrete steps & not continuous
- Tuning resolution limited
- Switch parasitic C & series R can affect the freq. response of the filter

Example: Tunable Low-Pass Opamp-RC Filter
Adjustable Capacitors

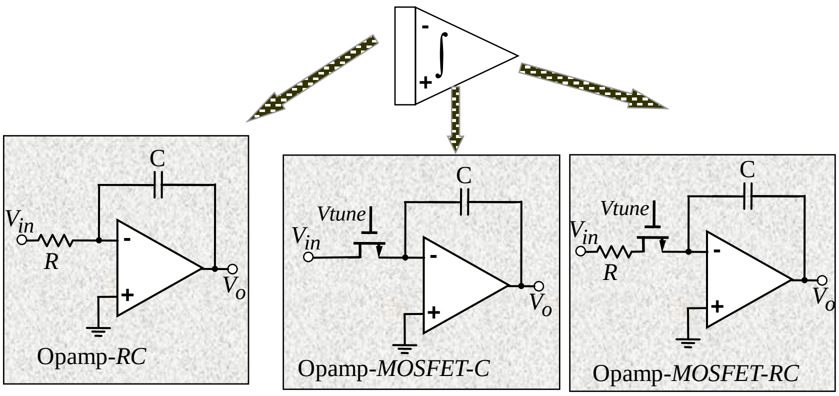


Opamp RC Filters

- Advantages
  - Since resistors are quite linear, linearity only a function of opamp linearity
    - → good linearity
- Disadvantages
  - Opamps have to drive resistive load, low output impedance is required
    - → High power consumption
  - Continuous tuning not possible- tuning only in discrete steps
  - Tuning requires programmable Rs and/or Cs

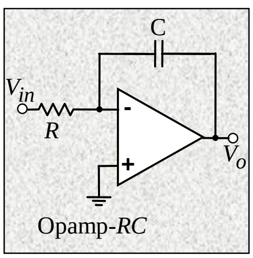


Integrator Implementation
Opamp-RC & Opamp-MOSFET-C & Opamp-MOSFET-RC



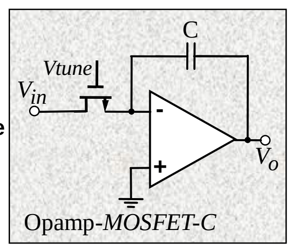
$$\frac{V_o}{V_{in}} = \frac{-\omega_o}{s} \quad \text{where} \quad \omega_o = \frac{1}{R_{eq}C}$$

Use of MOSFETs as Variable Resistors

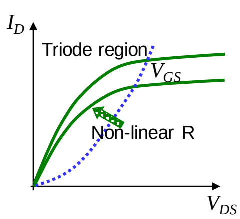


R replaced by MOSFET Operating in triode mode

→Continuously variable resistor:



MOSFET IV characteristic:

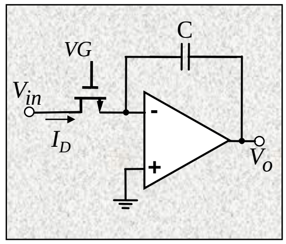


Opamp MOSFET-C Integrator
Single-Ended Integrator

$$I_D = \mu C_{ox} \frac{W}{L}\left[(V_{gs} - V_{th})V_{ds} - \frac{V_{ds}^2}{2}\right]$$

$$I_D = \mu C_{ox} \frac{W}{L}\left[(V_{gs} - V_{th})V_i - \frac{V_i^2}{2}\right]$$

$$G = \frac{\partial I_D}{\partial V_i} = \mu C_{ox} \frac{W}{L}(V_{gs} - V_{th} - V_i)$$



→Tunable by varying *VG*:

By varying *VG* effective admittance is tuned
→ Tunable integrator time constant

Problem: Single-ended MOSFET-C Integrator→ Effective R non-linear
Note that the non-linearity is mainly 2nd order type

Use of MOSFETs as Resistors
Differential Integrator

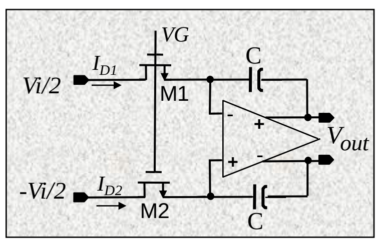
$$I_D = \mu C_{ox} \frac{W}{L}\left(V_{gs} - V_{th} - \frac{V_{ds}}{2}\right)V_{ds}$$

$$I_{D1} = \mu C_{ox} \frac{W}{L}\left(V_{gs} - V_{th} - \frac{V_i}{4}\right)\frac{V_i}{2}$$

$$I_{D2} = -\mu C_{ox} \frac{W}{L}\left(V_{gs} - V_{th} + \frac{V_i}{4}\right)\frac{V_i}{2}$$

$$I_{D1} - I_{D2} = \mu C_{ox} \frac{W}{L}(V_{gs} - V_{th})V_i$$

$$G = \frac{\partial(I_{D1} - I_{D2})}{\partial V_i} = \mu C_{ox} \frac{W}{L}(V_{gs} - V_{th})$$



Opamp-*MOSFET-C*

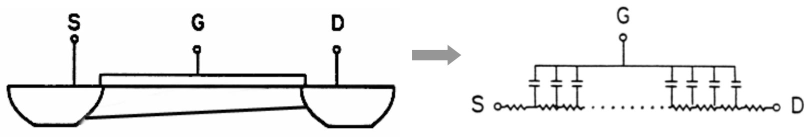
- **Non-linear term is of even order & cancelled!**
- **Admittance independent of Vi**

Problem: Threshold voltage dependence

Use of MOSFET as Resistor Issues

MOS xtor operating in triode region
Cross section view

Distributed channel resistance &
gate capacitance



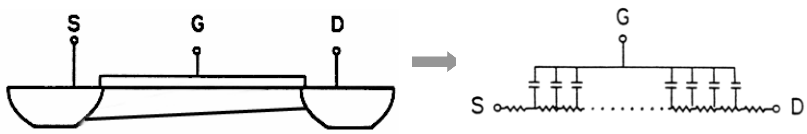
- Distributed nature of gate capacitance & channel resistance results in infinite no. of high-frequency poles:

  → Excess phase @ the unity-gain frequency of the integrator
  → Enhanced integrator Q
  → Enhanced filter Q,
  →Peaking in the filter passband

Use of MOSFET as Resistor Issues

MOS xtor operating in triode region
Cross section view

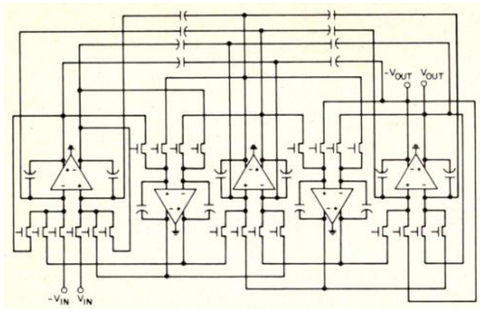
Distributed channel resistance &
gate capacitance



- Tradeoffs affecting the choice of device channel length:
  – Filter performance mandates well-matched MOSFETs → long channel devices desirable
  – Excess phase increases with L^2 → Q enhancement and potential for oscillation!
    →Tradeoff between device matching and integrator *Q*
    →This type of filter limited to low frequencies

Example:
Opamp MOSFET-C Filter

- Suitable for low frequency applications
- Issues with linearity
- Linearity achieved ~40-50dB
- Needs tuning
- Continuously tunable



5th Order Elliptic MOSFET-C LPF with 4kHz Bandwidth

Ref: Y. Tsividis, M.Banu, and J. Khoury, "Continuous-Time MOSFET-C Filters in VLSI", *IEEE Journal of Solid State Circuits* Vol. SC-21, No.1 Feb. 1986, pp. 15-30

Improved MOSFET-C Integrator

$$I_D = \mu C_{ox} \frac{W}{L}\left(V_{gs} - V_{th} - \frac{V_{ds}}{2}\right)V_{ds}$$

$$I_{D1} = \mu C_{ox} \frac{W}{L}\left(V_{gs1} - V_{th} - \frac{V_i}{4}\right)\frac{V_i}{2}$$

$$I_{D3} = -\mu C_{ox} \frac{W}{L}\left(V_{gs3} - V_{th} + \frac{V_i}{4}\right)\frac{V_i}{2}$$

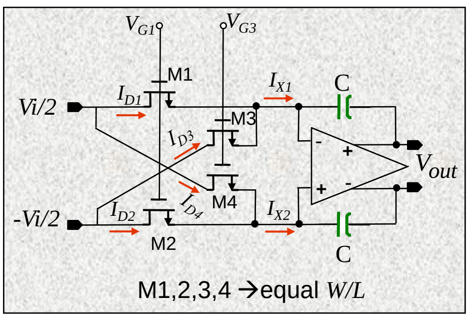
$$I_{X1} = I_{D1} + I_{D3}$$

$$= \mu C_{ox} \frac{W}{L}\left(V_{gs1} - V_{gs3} - \frac{V_i}{2}\right)\frac{V_i}{2}$$

$$I_{X2} = \mu C_{ox} \frac{W}{L}\left(V_{gs3} - V_{gs1} - \frac{V_i}{2}\right)\frac{V_i}{2}$$

$$I_{X1} - I_{X2} = \mu C_{ox} \frac{W}{L}\left(V_{gs1} - V_{gs3}\right)V_i$$

$$G = \frac{\partial\left(I_{X1} - I_{X2}\right)}{\partial V_i} = \mu C_{ox} \frac{W}{L}\left(V_{gs1} - V_{gs3}\right)$$

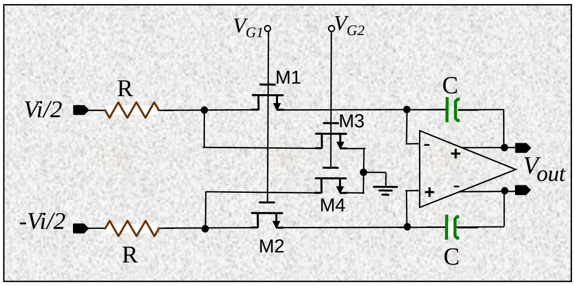


No threshold voltage dependence

Linearity achieved in the order of 50-70dB

Ref: Z. Czarnul, "Modification of the Banu-Tsividis Continuous-Time Integrator Structure," *IEEE Transactions on Circuits and Systems*, Vol. CAS-33, No. 7, pp. 714-716, July 1986.

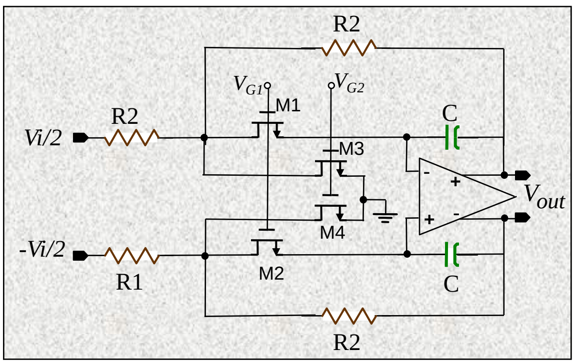
R-MOSFET-C Integrator



- Improvement over MOSFET-C by adding resistor in series with MOSFET
- Voltage drop primarily across fixed resistor → small MOSFET Vds → improved linearity & reduced tuning range
- Generally low frequency applications

Ref: U-K Moon, and B-S Song, "Design of a Low-Distortion 22-kHz Fifth Order Bessel Filter," *IEEE Journal of Solid State Circuits*, Vol. 28, No. 12, pp. 1254-1264, Dec. 1993.

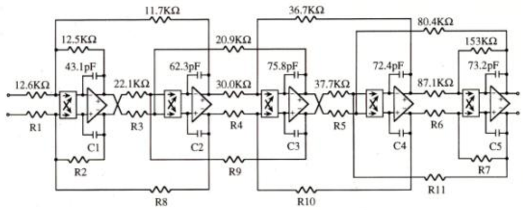
R-MOSFET-C Lossy Integrator



Negative feedback around the non-linear MOSFETs improves linearity but compromises frequency response accuracy

Ref: U-K Moon, and B-S Song, "Design of a Low-Distortion 22-kHz Fifth Order Bessel Filter," *IEEE Journal of Solid State Circuits*, Vol. 28, No. 12, pp. 1254-1264, Dec. 1993.

Example:
Opamp MOSFET-RC Filter



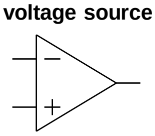
5th Order Bessel MOSFET-RC LPF 22kHz bandwidth
THD → -90dB for 4Vp-p , 2kHz input signal

- Suitable for low frequency, low Q applications
- Significant improvement in linearity compared to MOSFET-C
- Needs tuning

Ref: U-K Moon, and B-S Song, "Design of a Low-Distortion 22-kHz Fifth Order Bessel Filter," *IEEE Journal of Solid State Circuits*, Vol. 28, No. 12, pp. 1254-1264, Dec. 1993.

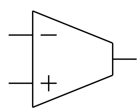
Operational Amplifiers (Opamps) versus Operational Transconductance Amplifiers (OTA)

<u>Opamp</u>
Voltage controlled voltage source



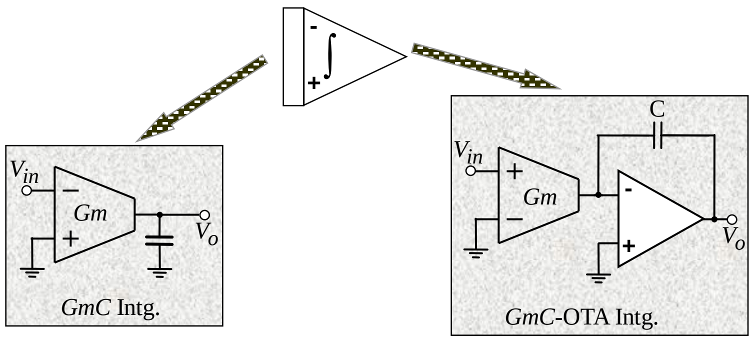
- Output in the form of voltage
- Low output impedance
- Can drive R-loads
- Good for RC filters, OK for SC filters
- Extra buffer adds complexity, power dissipation

<u>OTA</u>
Voltage controlled current source



- Output in the form of current
- High output impedance
- In the context of filter design called *gm-cells*
- Cannot drive R-loads
- Good for SC & gm-C filters
- Typically, less complex compared to opamp→ higher freq. potential
- Typically lower power

Integrator Implementation
Transconductance-C & Opamp-Transconductance-C

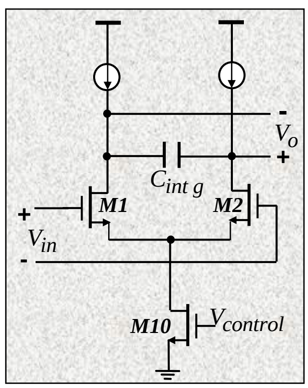


$$\frac{V_o}{V_{in}} = \frac{-\omega_o}{s} \quad \text{where} \quad \omega_o = \frac{G_m}{C}$$

Gm-C Filters
Simplest Form of CMOS Gm-C Integrator

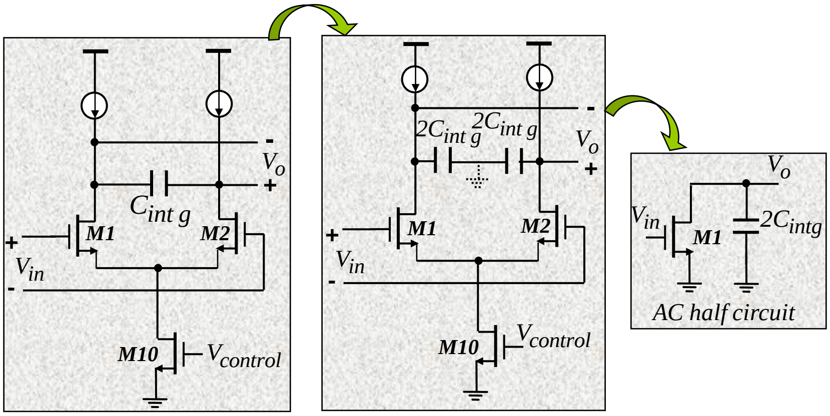
- Transconductance element formed by the source-coupled pair *M1 & M2*
- All MOSFETs operating in saturation region
- Current in *M1& M2* can be varied by changing $V_{control}$

  → Transconductance of *M1& M2* varied through $V_{control}$



Ref: H. Khorramabadi and P.R. Gray, "High Frequency CMOS continuous-time filters," IEEE Journal of Solid-State Circuits, Vol.-SC-19, No. 6, pp.939-948, Dec. 1984.

Simplest Form of CMOS Gm-C Integrator
AC Half Circuit



Gm-C Filters
Simplest Form of CMOS Gm-C Integrator

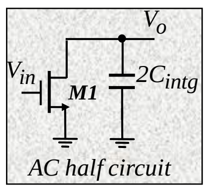
- Use ac half circuit & small signal model to derive transfer function:

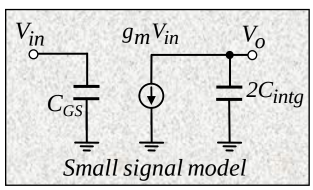
$$V_o = -g_m^{M1,2} \times V_{in} \times 2C_{int\,g}\, s$$

$$\frac{V_o}{V_{in}} = -\frac{g_m^{M1,2}}{2C_{int\,g}\, s}$$

$$\frac{V_o}{V_{in}} = \frac{-\omega_o}{s}$$

$$\rightarrow \omega_o = \frac{g_m^{M1,2}}{2 \times C_{int\,g}}$$





Gm-C Filters
Simplest Form of CMOS Gm-C Integrator

- MOSFET in saturation region:

$$I_d = \frac{\mu C_{ox}}{2}\frac{W}{L}\left(V_{gs} - V_{th}\right)^2$$

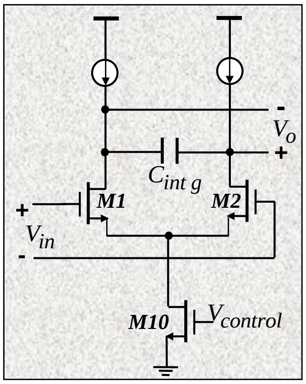
- Gm is given by:

$$g_m^{M1\&M2} = \frac{\partial I_d}{\partial V_{gs}} = \mu C_{ox}\frac{W}{L}\left(V_{gs} - V_{th}\right)$$

$$= 2\frac{I_d}{\left(V_{gs} - V_{th}\right)}$$

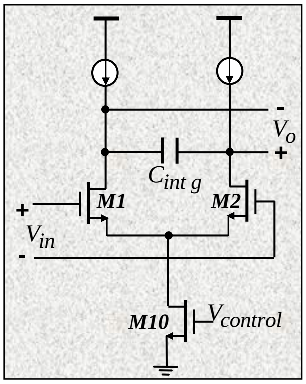
$$= 2\left(\frac{1}{2}\mu C_{ox}\frac{W}{L} I_d\right)^{1/2}$$

Id varied via Vcontrol
→ *gm* tunable via *Vcontrol*

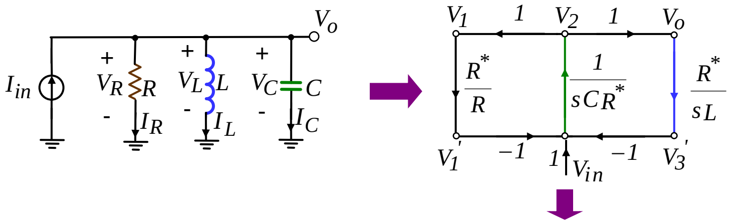


Gm-C Filters
2nd Order Gm-C Filter

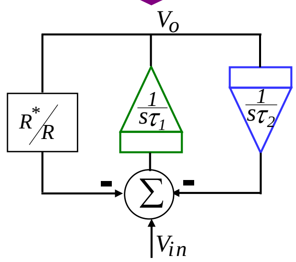
- Use the Gm-cell to build a 2nd order bandpass filter



2^{nd} Order Bandpass Filter



$\tau_1 = R^* \times C \qquad \tau_2 = L \big/ R^*$



2nd Order Integrator-Based Bandpass Filter

$$\frac{V_{BP}}{V_{in}} = \frac{\tau_2 s}{\tau_1 \tau_2 s^2 + \beta \tau_2 s + 1}$$

$$\tau_1 = R^* \times C \qquad \tau_2 = L \big/ R^*$$

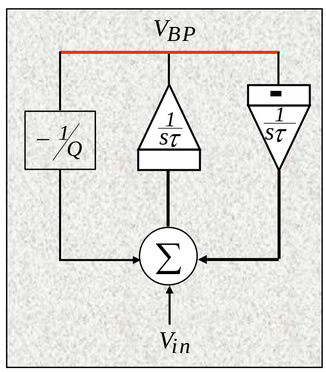
$$\beta = R^* \big/ R$$

$$\omega_0 = 1 \big/ \sqrt{\tau_1 \tau_2} = 1 \big/ \sqrt{L\,C}$$

$$Q = 1/\beta \times \sqrt{\tau_1 \big/ \tau_2}$$

From matching point of view desirable:

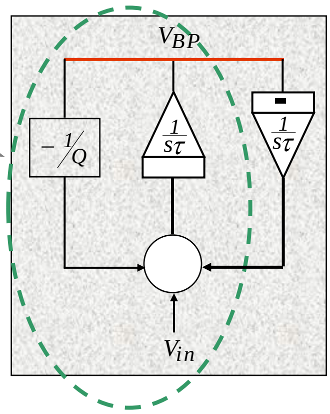
$$\tau_1 = \tau_2 = \tau = \frac{1}{\omega_0} \quad \rightarrow \quad Q = R \big/ R^*$$



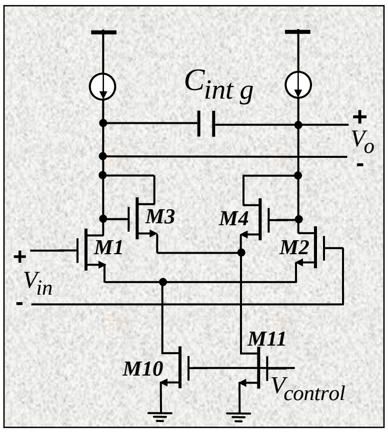
2nd Order Integrator-Based Bandpass Filter

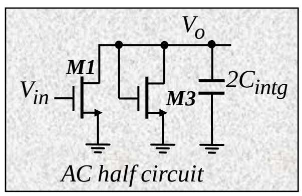
First implement this part
With transfer function:

$$\frac{V_0}{V_{in}} = \frac{-1}{\frac{s}{\omega_0} + \frac{1}{Q}}$$

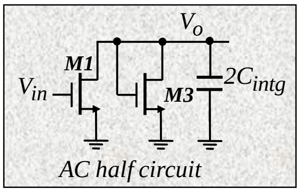


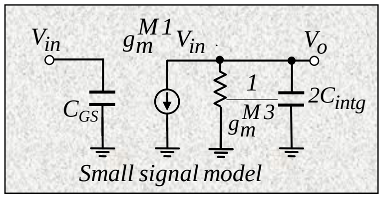
Terminated Gm-C Integrator





Terminated Gm-C Integrator

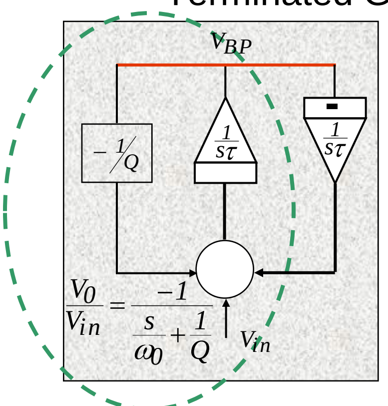


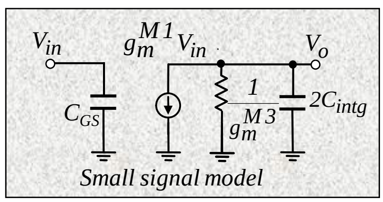


$$\frac{V_o}{V_{in}} = \frac{-1}{s\frac{2C_{int\,g}}{g_m^{M1}} + \frac{g_m^{M3}}{g_m^{M1}}}$$

Compare to: $\frac{V_0}{V_{in}} = \frac{-1}{\frac{s}{\omega_0} + \frac{1}{Q}}$

Terminated Gm-C Integrator





$$\frac{V_o}{V_{in}} = \frac{-1}{\frac{s \times 2C_{int\,g}}{g_m^{M1}} + \frac{g_m^{M3}}{g_m^{M1}}}$$

$$\rightarrow \omega_0 = \frac{g_m^{M1}}{2C_{int\,g}} \quad \& \quad Q = \frac{g_m^{M1}}{g_m^{M3}}$$

Question: How to define Q accurately?

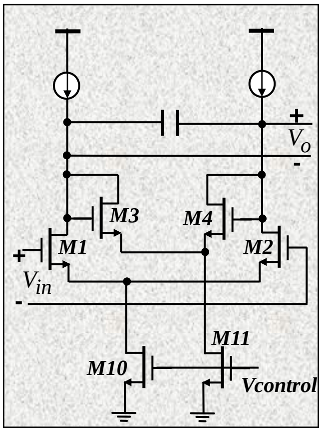
Terminated Gm-C Integrator

$$g_m^{M1} = 2\left(\frac{1}{2}\mu C_{ox}\frac{W_{M1}}{L_{M1}}I_d^{M1}\right)^{1/2}$$

$$g_m^{M3} = 2\left(\frac{1}{2}\mu C_{ox}\frac{W_{M3}}{L_{M3}}I_d^{M3}\right)^{1/2}$$

Let us assume equal channel lengths for M1, M3 then:

$$\frac{g_m^{M1}}{g_m^{M3}} = \left(\frac{I_d^{M1}}{I_d^{M3}} \times \frac{W_{M1}}{W_{M3}}\right)^{1/2}$$



Terminated Gm-C Integrator

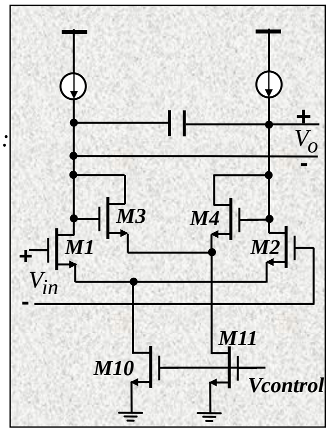
Note that:

$$\frac{I_d^{M1}}{I_d^{M3}} = \frac{I_d^{M10}}{I_d^{M11}}$$

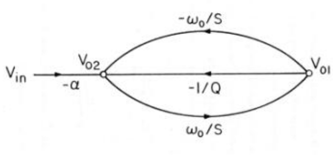
Assuming equal channel lengths for M10, M11:

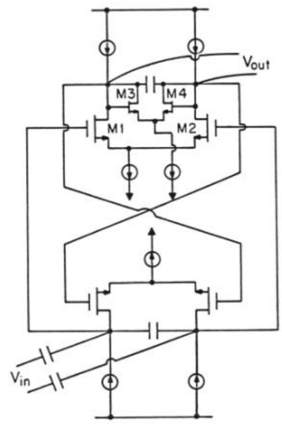
$$\frac{I_d^{M10}}{I_d^{M11}} = \frac{W_{M10}}{W_{M11}}$$

$$\rightarrow \frac{g_m^{M1}}{g_m^{M3}} = \left(\frac{W_{M10}}{W_{M11}} \times \frac{W_{M1}}{W_{M3}}\right)^{1/2}$$



2nd Order Gm-C Filter





- Simple design
- Tunable
- Q function of device ratios:

$$Q = \frac{g_m^{M1,2}}{g_m^{M3,4}}$$