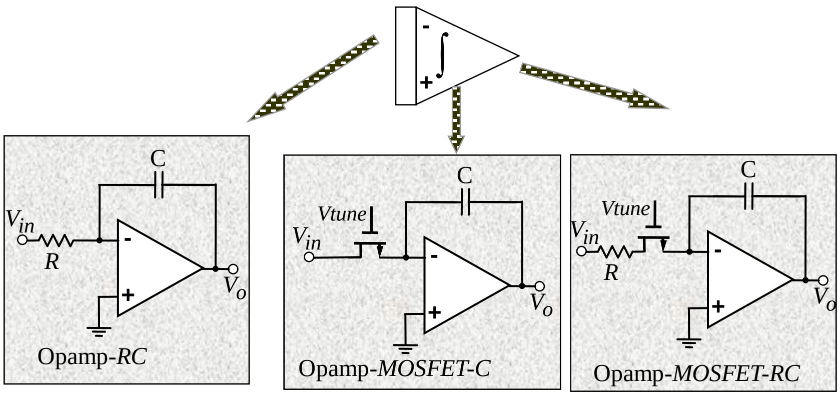
EE247
Lecture 6

- Continuous-time filters (continued)
  - Opamp MOSFET-C filters
  - Opamp MOSFET-RC filters
  - Gm-C filters
- Frequency tuning for continuous-time filters
  - Trimming via fuses or laser
  - Automatic on-chip filter tuning
    - Continuous tuning
      - Master-slave tuning
    - Periodic off-line tuning
      - Systems where filter is followed by ADC & DSP, existing hardware can be used to periodically update filter freq. response

Summary
Lecture 5

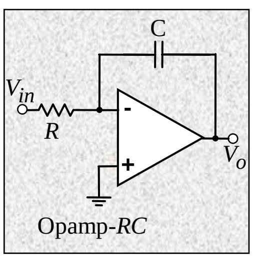
- Continuous-time filters
  - Effect of integrator non-idealities on integrated continuous-time filter behavior
    - Effect of integrator finite DC gain & non-dominant poles on filter frequency response
    - Integrator non-linearities affecting filter maximum signal handling capability (harmonic distortion and intermodulation distortion)
    - Effect of integrator component variations and mismatch on filter response & need for frequency tuning
- Frequency tuning for continuous-time filters
  - Frequency adjustment by making provisions to have variable R or C
- Various integrator topologies used in filters
  - Opamp MOSFET-C filters (to be continued)

Integrator Implementation
Opamp-RC & Opamp-MOSFET-C & Opamp-MOSFET-RC



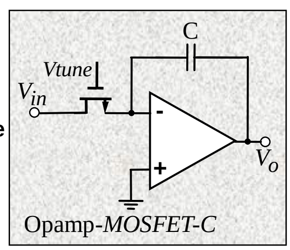
$$\frac{V_o}{V_{in}} = \frac{-\omega_o}{s} \quad \text{where} \quad \omega_o = \frac{1}{R_{eq}C}$$

Use of MOSFETs as Variable Resistors

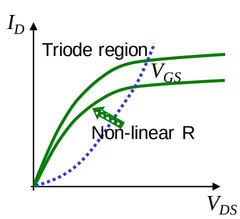


R replaced by MOSFET Operating in triode mode

→Continuously variable resistor:



MOSFET IV characteristic:

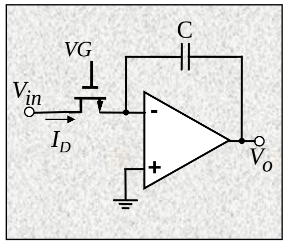


Opamp MOSFET-C Integrator
Single-Ended Integrator

$$I_D = \mu C_{ox} \frac{W}{L}\left[(V_{gs} - V_{th})V_{ds} - \frac{V_{ds}^2}{2}\right]$$

$$I_D = \mu C_{ox} \frac{W}{L}\left[(V_{gs} - V_{th})V_i - \frac{V_i^2}{2}\right]$$

$$G = \frac{\partial I_D}{\partial V_i} = \mu C_{ox} \frac{W}{L}(V_{gs} - V_{th} - V_i)$$



→Tunable by varying *VG*:

By varying *VG* effective admittance is tuned
→ Tunable integrator time constant

Problem: Single-ended MOSFET-C Integrator→ Effective R non-linear
Note that the non-linearity is mainly 2nd order type

Use of MOSFETs as Resistors
Differential Integrator

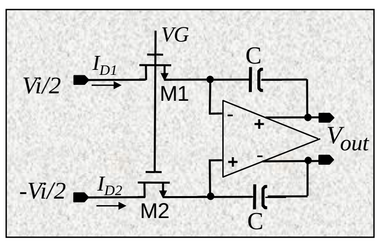
$$I_D = \mu C_{ox} \frac{W}{L}\left(V_{gs} - V_{th} - \frac{V_{ds}}{2}\right)V_{ds}$$

$$I_{D1} = \mu C_{ox} \frac{W}{L}\left(V_{gs} - V_{th} - \frac{V_i}{4}\right)\frac{V_i}{2}$$

$$I_{D2} = -\mu C_{ox} \frac{W}{L}\left(V_{gs} - V_{th} + \frac{V_i}{4}\right)\frac{V_i}{2}$$

$$I_{D1} - I_{D2} = \mu C_{ox} \frac{W}{L}(V_{gs} - V_{th})V_i$$

$$G = \frac{\partial(I_{D1} - I_{D2})}{\partial V_i} = \mu C_{ox} \frac{W}{L}(V_{gs} - V_{th})$$



Opamp-*MOSFET-C*

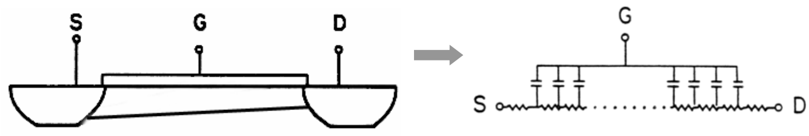
- **Non-linear term is of even order & cancelled!**
- **Admittance independent of Vi**

Problem: Threshold voltage dependence

Use of MOSFET as Resistor Issues

MOS xtor operating in triode region
Cross section view

Distributed channel resistance &
gate capacitance



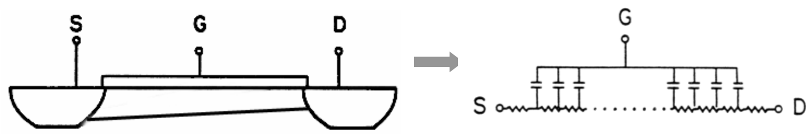
- Distributed nature of gate capacitance & channel resistance results in infinite no. of high-frequency poles:

  → Excess phase @ the unity-gain frequency of the integrator
  → Enhanced integrator Q
  → Enhanced filter Q,
  →Peaking in the filter passband

Use of MOSFET as Resistor Issues

MOS xtor operating in triode region
Cross section view

Distributed channel resistance &
gate capacitance

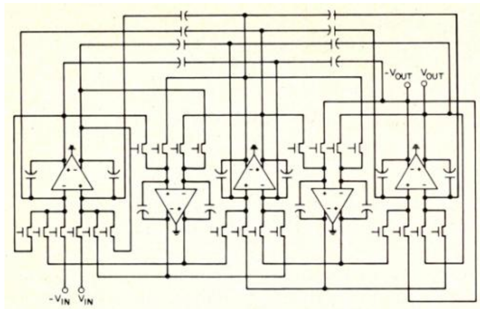


- Tradeoffs affecting the choice of device channel length:
  - Filter performance mandates well-matched MOSFETs → long channel devices desirable
  - Excess phase increases with L^2 → Q enhancement and potential for oscillation!

    →Tradeoff between device matching and integrator *Q*
    →This type of filter limited to low frequencies

Example: Opamp MOSFET-C Filter

- Suitable for low frequency applications
- Issues with linearity
- Linearity achieved ~40-50dB
- Needs tuning
- Continuously tunable



5th Order Elliptic MOSFET-C LPF with 4kHz Bandwidth

Ref: Y. Tsividis, M.Banu, and J. Khoury, "Continuous-Time MOSFET-C Filters in VLSI", *IEEE Journal of Solid State Circuits* Vol. SC-21, No.1 Feb. 1986, pp. 15-30

Improved MOSFET-C Integrator

$$I_D = \mu C_{ox}\frac{W}{L}\left(V_{gs} - V_{th} - \frac{V_{ds}}{2}\right)V_{ds}$$

$$I_{D1} = \mu C_{ox}\frac{W}{L}\left(V_{gs1} - V_{th} - \frac{V_i}{4}\right)\frac{V_i}{2}$$

$$I_{D3} = -\mu C_{ox}\frac{W}{L}\left(V_{gs3} - V_{th} + \frac{V_i}{4}\right)\frac{V_i}{2}$$

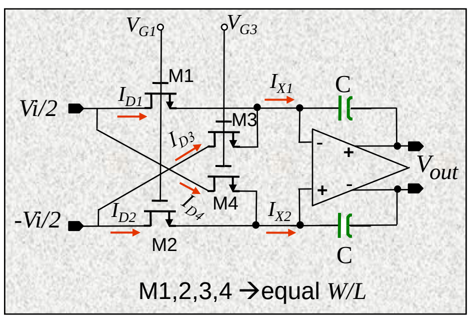
$$I_{X1} = I_{D1} + I_{D3}$$

$$= \mu C_{ox}\frac{W}{L}\left(V_{gs1} - V_{gs3} - \frac{V_i}{2}\right)\frac{V_i}{2}$$

$$I_{X2} = \mu C_{ox}\frac{W}{L}\left(V_{gs3} - V_{gs1} - \frac{V_i}{2}\right)\frac{V_i}{2}$$

$$I_{X1} - I_{X2} = \mu C_{ox}\frac{W}{L}\left(V_{gs1} - V_{gs3}\right)V_i$$

$$G = \frac{\partial\left(I_{X1} - I_{X2}\right)}{\partial V_i} = \mu C_{ox}\frac{W}{L}\left(V_{gs1} - V_{gs3}\right)$$

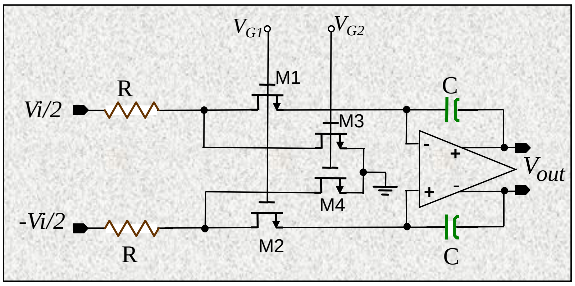


No threshold voltage dependence

Linearity achieved in the order of 50-70dB

Ref: Z. Czarnul, "Modification of the Banu-Tsividis Continuous-Time Integrator Structure," *IEEE Transactions on Circuits and Systems,* Vol. CAS-33, No. 7, pp. 714-716, July 1986.

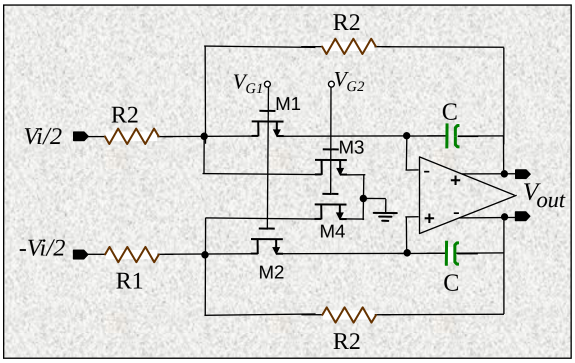
R-MOSFET-C Integrator



- Improvement over MOSFET-C by adding resistor in series with MOSFET
- Voltage drop primarily across fixed resistor → small MOSFET Vds → improved linearity & reduced tuning range
- Generally low frequency & low Q applications

Ref: U-K Moon, and B-S Song, "Design of a Low-Distortion 22-kHz Fifth Order Bessel Filter," *IEEE Journal of Solid State Circuits*, Vol. 28, No. 12, pp. 1254-1264, Dec. 1993.

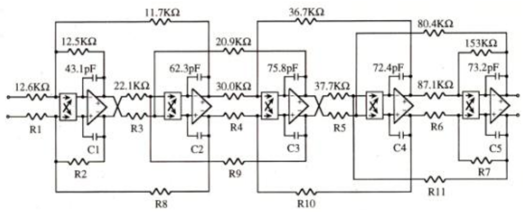
R-MOSFET-C Lossy Integrator



Negative feedback around the non-linear MOSFETs improves linearity but compromises frequency response accuracy

Ref: U-K Moon, and B-S Song, "Design of a Low-Distortion 22-kHz Fifth Order Bessel Filter," *IEEE Journal of Solid State Circuits*, Vol. 28, No. 12, pp. 1254-1264, Dec. 1993.

Example:
Opamp MOSFET-RC Filter



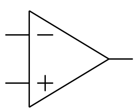
5th Order Bessel MOSFET-RC LPF 22kHz bandwidth
THD → -90dB for 4Vp-p , 2kHz input signal

- Suitable for low frequency, low Q applications
- Significant improvement in linearity compared to MOSFET-C
- Needs tuning

Ref: U-K Moon, and B-S Song, "Design of a Low-Distortion 22-kHz Fifth Order Bessel Filter," *IEEE Journal of Solid State Circuits*, Vol. 28, No. 12, pp. 1254-1264, Dec. 1993.

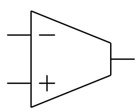
Operational Amplifiers (Opamps) versus Operational Transconductance Amplifiers (OTA)

Opamp
Voltage controlled voltage source



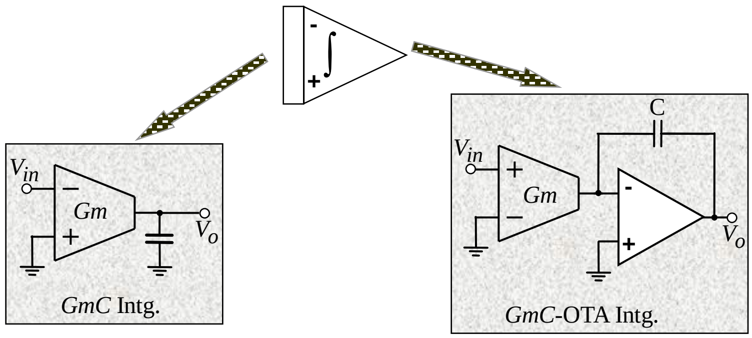
- Output in the form of voltage
- Low output impedance
- Can drive R-loads
- Good for RC filters, OK for SC filters
- Extra buffer adds complexity, power dissipation

OTA
Voltage controlled current source



- Output in the form of current
- High output impedance
- In the context of filter design called *gm-cells*
- Cannot drive R-loads
- Good for SC & gm-C filters
- Typically, less complex compared to opamp→ higher freq. potential
- Typically lower power

Integrator Implementation
Transconductance-C & Opamp-Transconductance-C



$$\frac{V_o}{V_{in}} = \frac{-\omega_o}{s} \quad \text{where} \quad \omega_o = \frac{G_m}{C}$$

Gm-C Filters
Simplest Form of CMOS Gm-C Integrator

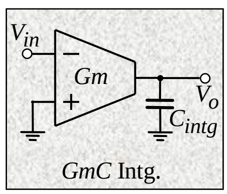
- Use small signal model to derive transfer function:

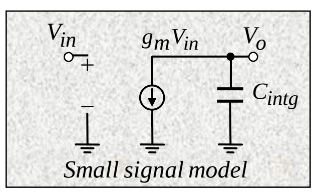
$$V_o = -g_m \times V_{in} \times C_{int\,g}\,s$$

$$\frac{V_o}{V_{in}} = -\frac{g_m}{C_{int\,g}\,s}$$

$$\frac{V_o}{V_{in}} = \frac{-\omega_o}{s}$$

$$\rightarrow \omega_o = \frac{g_m}{C_{int\,g}\,s}$$

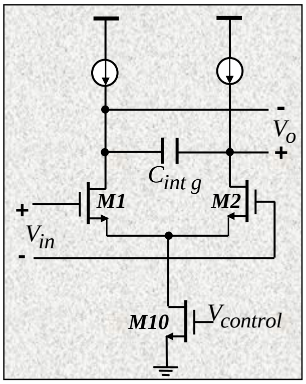




- Issue: Design is parasitic sensitive

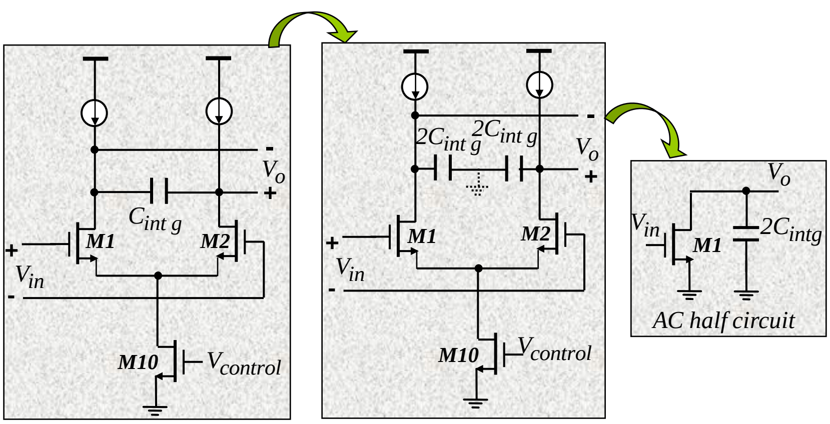
Gm-C Filters
Simplest Form of CMOS Gm-C Integrator

- Transconductance element formed by the source-coupled pair *M1 & M2*
- All MOSFETs operating in saturation region
- Current in *M1& M2* can be varied by changing $V_{control}$
- Find transfer function by drawing ac small-signal half circuit



Ref: H. Khorramabadi and P.R. Gray, "High Frequency CMOS continuous-time filters," IEEE Journal of Solid-State Circuits, Vol.-SC-19, No. 6, pp.939-948, Dec. 1984.

Simplest Form of CMOS Gm-C Integrator
AC Half Circuit



Gm-C Filters
Simplest Form of CMOS Gm-C Integrator

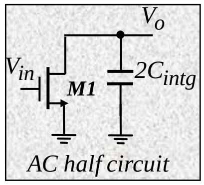
- Use ac half circuit & small signal model to derive transfer function:

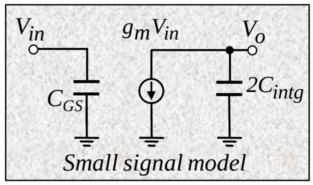
$$V_o = -g_m^{M1,2} \times V_{in} \times 2C_{intg}\, s$$

$$\frac{V_o}{V_{in}} = -\frac{g_m^{M1,2}}{2C_{intg}\, s}$$

$$\frac{V_o}{V_{in}} = \frac{-\omega_o}{s}$$

$$\rightarrow \omega_o = \frac{g_m^{M1,2}}{2 \times C_{intg}}$$





Gm-C Filters
Simplest Form of CMOS Gm-C Integrator

- MOSFET in saturation region:

$$I_d = \frac{\mu C_{ox}}{2}\frac{W}{L}\left(V_{gs} - V_{th}\right)^2$$

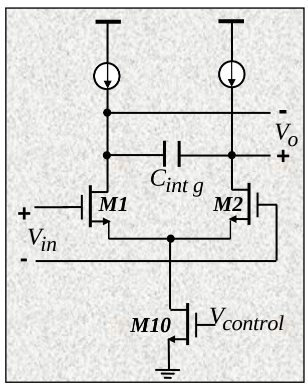
- Gm is given by:

$$g_m^{M1\&M2} = \frac{\partial I_d}{\partial V_{gs}} = \mu C_{ox}\frac{W}{L}\left(V_{gs} - V_{th}\right)$$

$$= 2\frac{I_d}{\left(V_{gs} - V_{th}\right)}$$

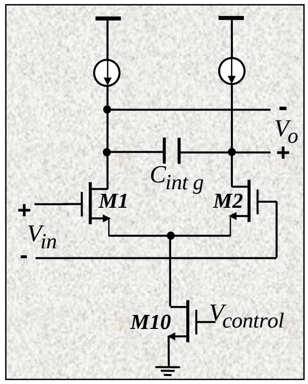
$$= 2\left(\frac{1}{2}\mu C_{ox}\frac{W}{L} I_d\right)^{1/2}$$

Id varied via Vcontrol
→ *gm* tunable via *Vcontrol*

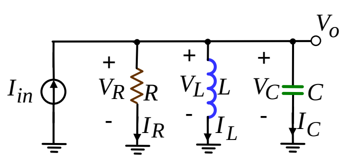


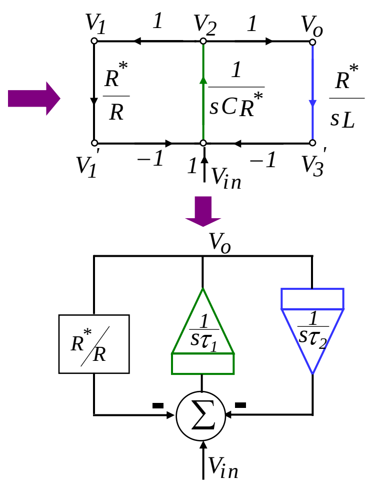
Gm-C Filters
2nd Order Gm-C Filter

- Use the Gm-cell to build a 2nd order bandpass filter



2nd Order Bandpass Filter





$$\tau_1 = R^* \times C \qquad \tau_2 = L \big/ R^*$$

2nd Order Integrator-Based Bandpass Filter

$$\frac{V_{BP}}{V_{in}} = \frac{\tau_2 s}{\tau_1 \tau_2 s^2 + \beta \tau_2 s + 1}$$

$$\tau_1 = R^* \times C \qquad \tau_2 = L \big/ R^*$$

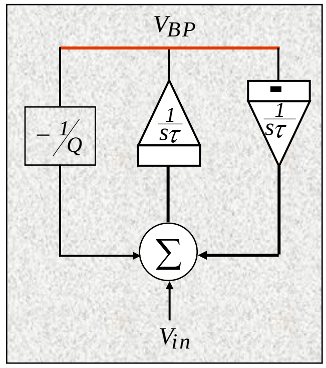
$$\beta = R^* \big/ R$$

$$\omega_0 = 1 \big/ \sqrt{\tau_1 \tau_2} = 1 \big/ \sqrt{L\,C}$$

$$Q = 1/\beta \times \sqrt{\tau_1 \big/ \tau_2}$$

From matching point of view desirable:

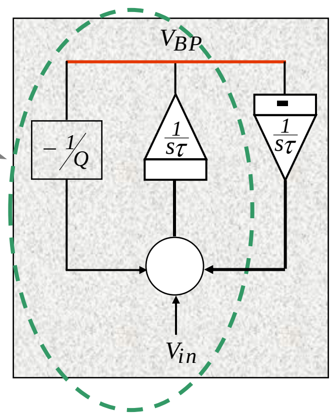
$$\tau_1 = \tau_2 = \tau = \frac{1}{\omega_0} \quad \rightarrow \quad Q = R \big/ R^*$$



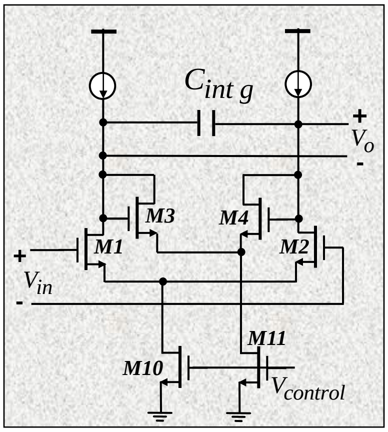
2nd Order Integrator-Based Bandpass Filter

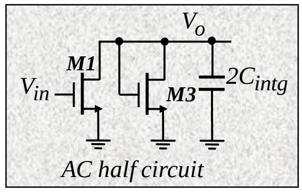
First implement this part
With transfer function:

$$\frac{V_0}{V_{in}} = \frac{-1}{\frac{s}{\omega_0} + \frac{1}{Q}}$$

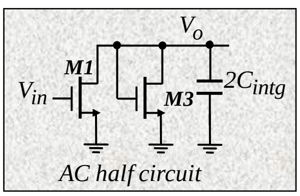


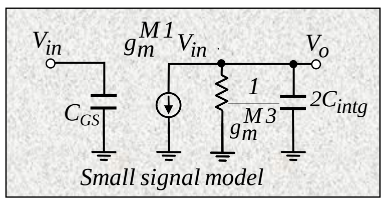
Terminated Gm-C Integrator





Terminated Gm-C Integrator

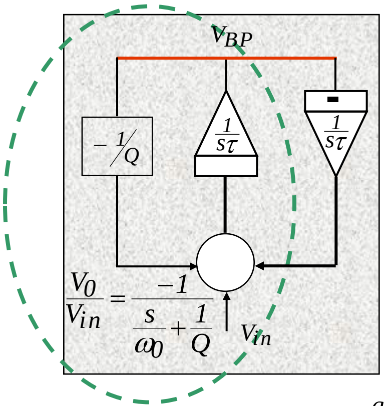


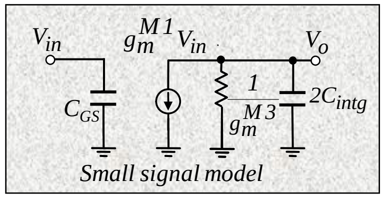


$$\boxed{\frac{V_o}{V_{in}} = \frac{-1}{s\dfrac{2C_{int\,g}}{g_m^{M1}} + \dfrac{g_m^{M3}}{g_m^{M1}}}}$$

Compare to: $\dfrac{V_0}{V_{in}} = \dfrac{-1}{\dfrac{s}{\omega_0} + \dfrac{1}{Q}}$

Terminated Gm-C Integrator





$$\frac{V_o}{V_{in}} = \frac{-1}{\frac{s \times 2C_{intg}}{g_m^{M1}} + \frac{g_m^{M3}}{g_m^{M1}}}$$

$$\rightarrow \omega_0 = \frac{g_m^{M1}}{2C_{intg}} \quad \& \quad Q = \frac{g_m^{M1}}{g_m^{M3}}$$

Question: How to define Q accurately?

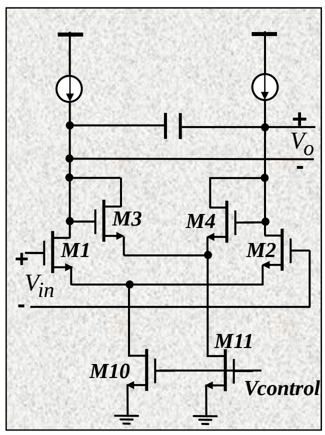
Terminated Gm-C Integrator

$$g_m^{M1} = 2\left(\frac{1}{2}\mu C_{ox}\frac{W_{M1}}{L_{M1}}I_d^{M1}\right)^{1/2}$$

$$g_m^{M3} = 2\left(\frac{1}{2}\mu C_{ox}\frac{W_{M3}}{L_{M3}}I_d^{M3}\right)^{1/2}$$

Let us assume equal channel lengths for M1, M3 then:

$$\frac{g_m^{M1}}{g_m^{M3}} = \left(\frac{I_d^{M1}}{I_d^{M3}} \times \frac{W_{M1}}{W_{M3}}\right)^{1/2}$$



Terminated Gm-C Integrator

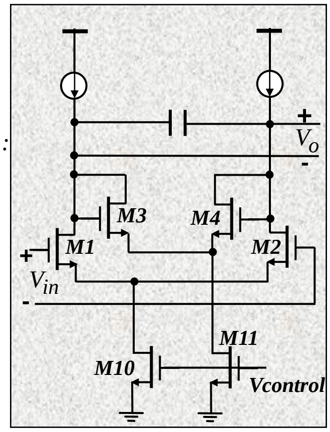
Note that:

$$\frac{I_d^{M1}}{I_d^{M3}} = \frac{I_d^{M10}}{I_d^{M11}}$$

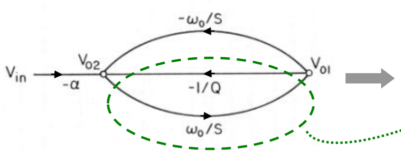
Assuming equal channel lengths for M10, M11:

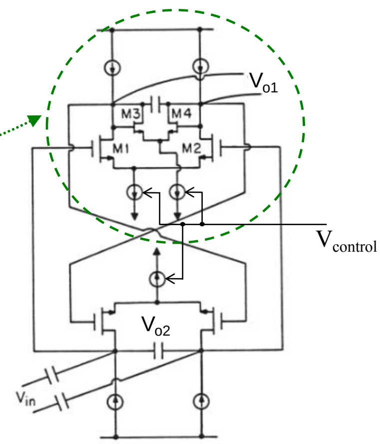
$$\frac{I_d^{M10}}{I_d^{M11}} = \frac{W_{M10}}{W_{M11}}$$

$$\rightarrow \boxed{\frac{g_m^{M1}}{g_m^{M3}} = \left(\frac{W_{M10}}{W_{M11}} \times \frac{W_{M1}}{W_{M3}}\right)^{1/2}}$$



2nd Order Gm-C Filter





- Simple design
- Tunable, f_{center} tuned via a single control signal ($V_{control}$)
- Q function of device ratios:

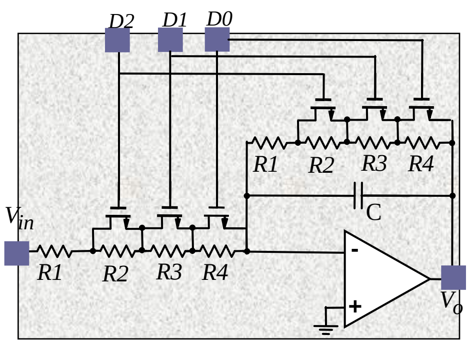
$$Q = \frac{g_m^{M1,2}}{g_m^{M3,4}}$$

Continuous-Time Filter Frequency Tuning Techniques

- Component trimming
- Automatic on-chip filter tuning
  - Continuous tuning
    - Master-slave tuning
  - Periodic off-line tuning
    - Systems where filter is followed by ADC & DSP, existing hardware can be used to periodically update filter freq. response

Example: Tunable Opamp-RC Filter

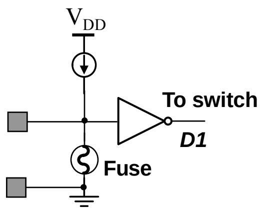
Post manufacturing:
- Usually at wafer-sort tuning performed
- Measure -3dB frequency
  - If frequency too high decrement D to D-1
  - If frequency too low increment D to D+1
  - If frequency within 10% of the desired corner freq. stop



Not practical to require end-user to tune the filter
→ Need to fix the adjustment at the factory

Factory Trimming

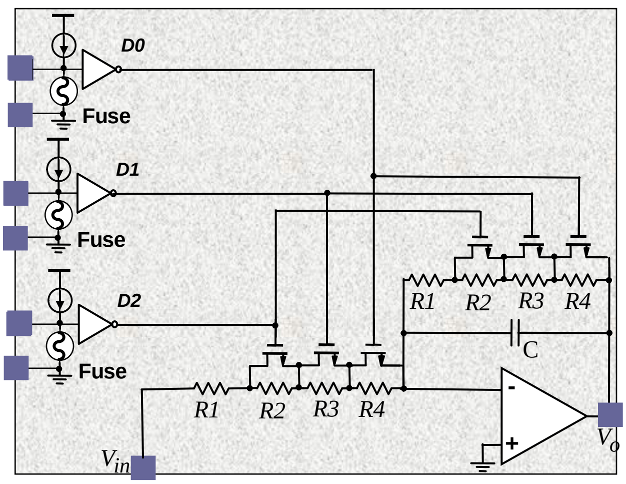
- Factory component trimming
  - Build fuses on-chip
    - Based on measurements @ wafer-sort blow fuses selectively by applying high current to the fuse
      - Expensive
      - Fuse regrowth problems!
      - Does not account for temp. variations & aging
  - Laser trimming
    - Trim components or cut fuses by laser
      - Even more expensive
      - Does not account for temp. variations & aging



Fuse not blown → D1=1
Fuse blown → D1=0

Example:Tunable/Trimmable Opamp-RC Filter

D2	D1	D0	Rnom
1	1	1	7.2K
1	1	0	8.28K
1	0	1	9.37K
0	0	0	14.8K



Automatic Frequency Tuning

- By adding additional circuitry to the main filter circuit
  - Have the filter critical frequency automatically tuned
    - ☺Expensive trimming avoided
    - ☺Accounts for critical frequency variations due to temperature, supply voltage, and effect of aging
    - 😐Additional hardware, increased Si area & power dissipation & reference signal feed-thru

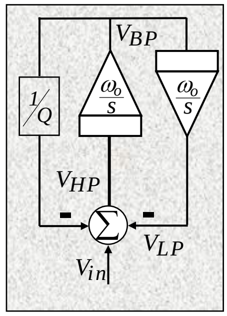
Master-Slave Automatic Frequency Tuning

- Following facts used in this scheme:
  - Use a replica of the main filter or its main building block in the tuning circuitry
  - The replica is called the ***master*** and the main filter is named the ***slave***
  - Place the replica in close proximity of the main filter to ensure good matching between master & slave
  - Use the tuning signal generated to tune the replica, to also tune the main filter
  - In the literature, this scheme is called *master-slave* tuning!

Master-Slave Frequency Tuning
1-Reference Filter (VCF)

- Use a biquad built with replica of main filter integrator for master filter (VCF)
- Utilize the fact that @ the frequency f_o, the lowpass (or highpass) outputs signal should be 90 degree out of phase wrt to input

$$\frac{V_{LP}}{V_{in}} = \left.\frac{1}{\frac{s^2}{\omega_o^2} + \frac{s}{Q\omega_o} + 1}\right|_{\omega=\omega_o} = -jQ \quad \rightarrow \quad \phi = -90^o$$



- Apply a sinusoid at the desired $f_o^{desired}$
- Compare the phase of LP output versus input
- Based on the phase difference:
  - –Increase or decrease filter critical freq.

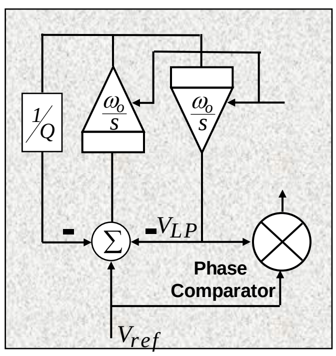
Master-Slave Frequency Tuning
1-Reference Filter (VCF)

$$V_{ref} = A\,sin(\omega t)$$

$$V_{LP} = A_2\ sin(\omega t + \phi)$$

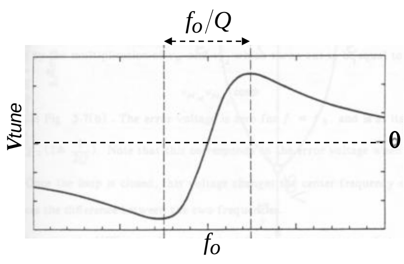
$$V_{ref} \times V_{LP} = A_2 A\ sin(\omega t) sin(\omega t + \phi)$$

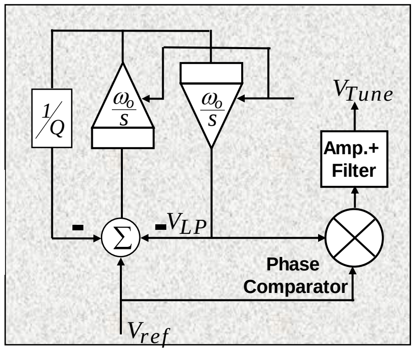
$$V_{ref} \times V_{LP} = \underbrace{\frac{A_2 A}{2} cos\phi}_{\text{Note that this term is=0 only when the incoming signal is at exactly the filter -3dB frequency}} - \underbrace{\frac{A_2 A}{2} cos(2\omega t + \phi)}_{\text{Filter Out}}$$



Master-Slave Frequency Tuning
1-Reference Filter (VCF)

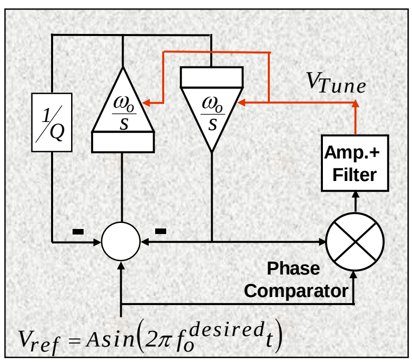
$$V_{tune} \approx -K \times V_{ref}^{rms} \times V_{LP}^{rms} \times \cos\phi$$



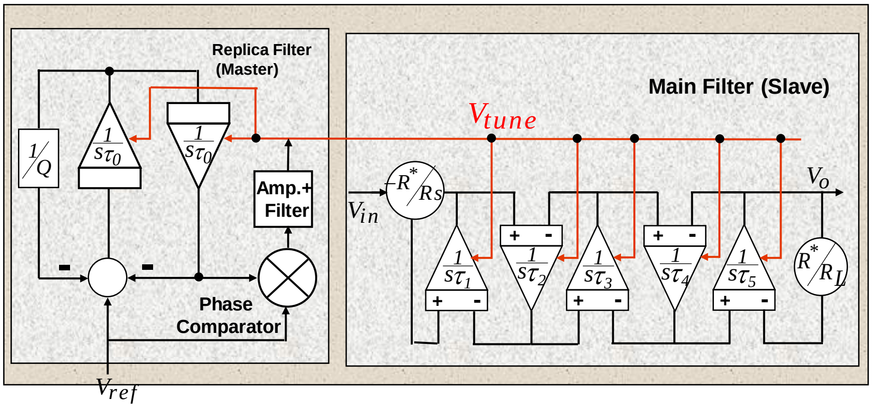


Master-Slave Frequency Tuning
1-Reference Filter (VCF)

- By closing the loop, feedback tends to drive the error voltage (V_{Tune}) to zero.
  - → Locks f_o to $f_o^{desired}$, the critical frequency of the filter to the accurate reference frequency
- Typically the reference frequency is provided by a crystal oscillator with accuracies in the order of few ppm



Master-Slave Frequency Tuning
1-Reference Filter (VCF)



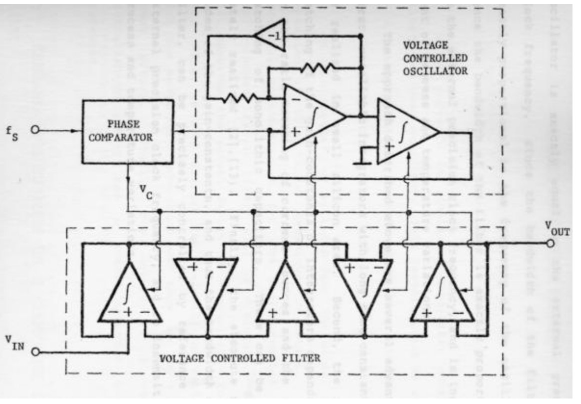
Ref: H. Khorramabadi and P.R. Gray, "High Frequency CMOS continuous-time filters," IEEE Journal of Solid-State Circuits, Vol.-SC-19, No. 6, pp.939-948, Dec. 1984.

Master-Slave Frequency Tuning
1- Reference Filter (VCF)

- Issues to be aware of:
  - Input reference tuning signal needs to be sinusoid → Disadvantage since clocks are usually available as square waveform
  - Reference signal feed-through via parasitic coupling to the output of the filter can limit filter dynamic range (reported levels of about 100μVrms)
  - Ref. signal feed-through is a function of:
    - Reference signal frequency with respect to filter passband
    - Filter topology
    - Care in the layout
    - Fully differential topologies beneficial

Master-Slave Frequency Tuning
2- Reference Voltage-Controlled-Oscillator (VCO)

- Instead of VCF a voltage-controlled-oscillator (VCO) is used
- VCO made of replica integrator used in main filter
- Tuning circuit operates exactly as a conventional phase-locked loop (PLL)
- Tuning signal used to tune main filter

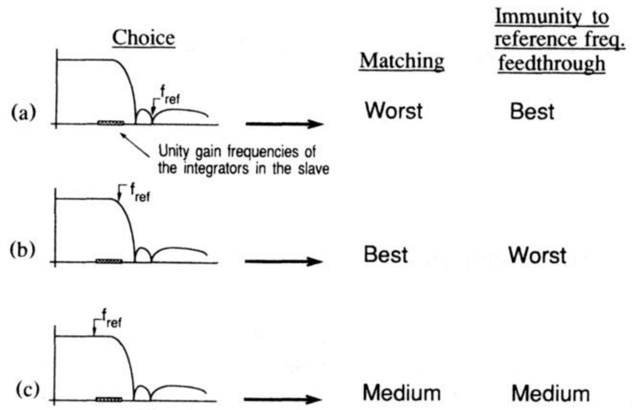


Ref: K.S. Tan and P.R. Gray, "Fully integrated analog filters using bipolar FET technology," IEEE, J. Solid-State Circuits, vol. SC-13, no.6, pp. 814-821, December 1978..

Master-Slave Frequency Tuning
2- Reference Voltage-Controlled-Oscillator (VCO)

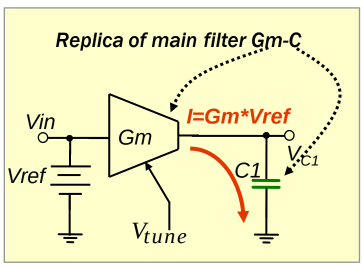
- Issues to be aware of:
  - Design of stable & repeatable oscillator challenging
  - VCO operation should be limited to the linear region of the integrator or else the operation loses accuracy (e.g. large signal transconductance versus small signal in a gm-C filter)
  - Limiting the VCO signal range to the linear region not a trivial design issue
  - In the case of VCF based tuning ckt, there was only ref. signal feedthrough. In this case, there is also the feedthrough of the VCO signal!!
  - Advantage over VCF based tuning → Reference input signal square wave (not sin.)

Master-Slave Frequency Tuning
Choice of Ref. Frequency wrt Feedthrough Immunity



Ref: V. Gopinathan, et. al, "Design Considerations for High-Frequency Continuous-Time Filters and Implementation of an Antialiasing Filter for Digital Video," *IEEE JSSC*, Vol. SC-25, no. 6 pp. 1368-1378, Dec. 1990.

Master-Slave Frequency Tuning
3-Reference Integrator Locked to Reference Frequency

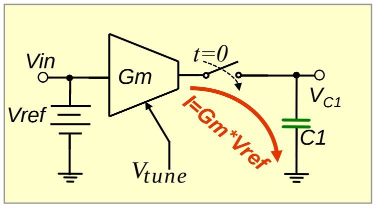


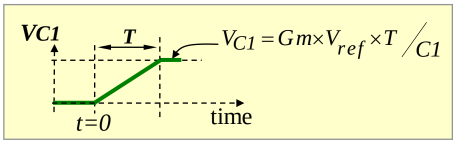
- Replica of main filter integrator e.g. Gm-C building block used
- Utilizes the fact that a DC voltage source connected to the input of the Gm cell generates a constant current at the output proportional to the transconductance and the voltage reference

  I = Gm.Vref

Reference Integrator Locked to Reference Frequency

- Consider the following sequence:
  - Integrating capacitor is fully discharged @ $t=0$
  - At $t=0$ the capacitor is connected to the output of the Gm cell then:





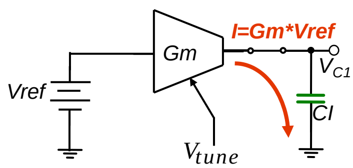
$$Q_{C1}=V_{C1}\times C1=Gm\times V_{ref}\times T$$
$$\rightarrow V_{C1}=Gm\times V_{ref}\times T\,/\,C1$$

Reference Integrator Locked to Reference Frequency

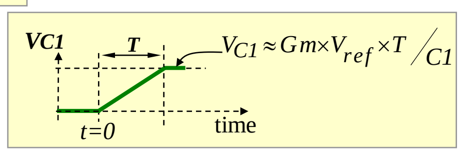
Since at the end of the period T:

$$V_{C1}\approx Gm\times V_{ref}\times T\,/\,C1$$

If V_{C1} is forced to be equal to V_{ref} then:



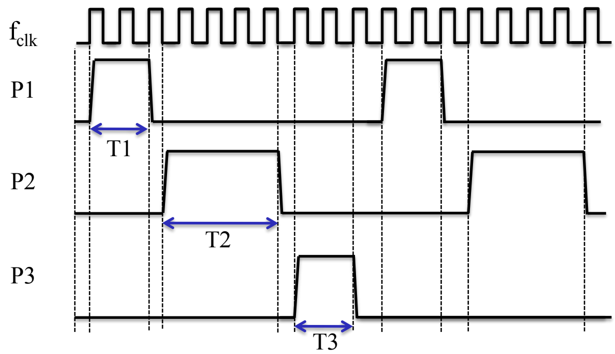
$$\frac{C1}{Gm}=T=\frac{N}{f_{clk}}$$



How do we manage to force $V_{C1}=V_{ref}$?

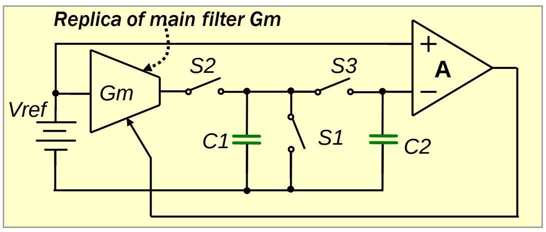
→ **Use feedback!!**

Reference Integrator Locked to Reference Frequency Clocking Scheme

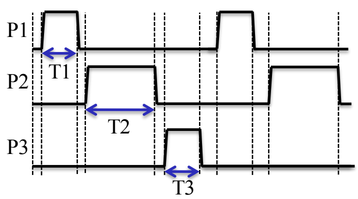


- Three clock phase operation
- Non-overlaping signals P1, P2, P3 derived from a master clock (f_{clk})
- Note: $T2=4/f_{clk}$

Reference Integrator Locked to Reference Frequency

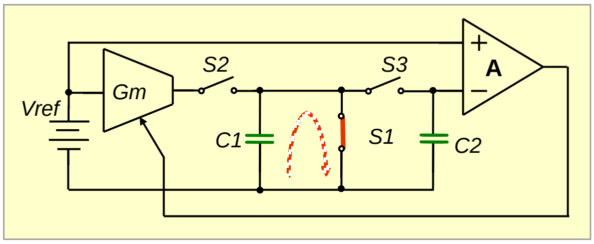


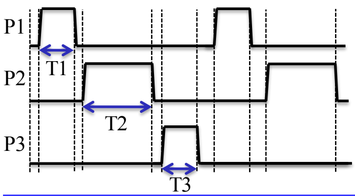
- Three clock phase operation
- To analyze → study one phase at a time



Ref: A. Durham, J. Hughes, and W. Redman- White, "Circuit Architectures for High Linearity Monolithic Continuous-Time Filtering," *IEEE Transactions on Circuits and Systems*, pp. 651-657, Sept. 1992.

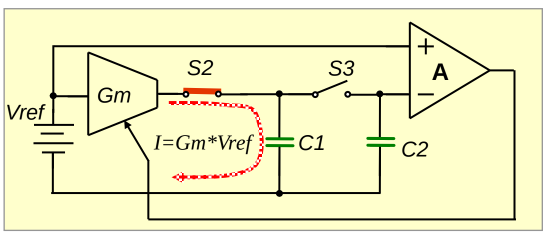
Reference Integrator Locked to Reference Frequency
P1 high→ S1 closed



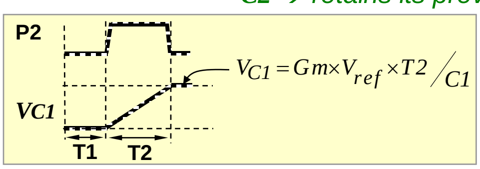


C1 → discharged → $V_{C1}=0$
C2 → *retains its previous charge*

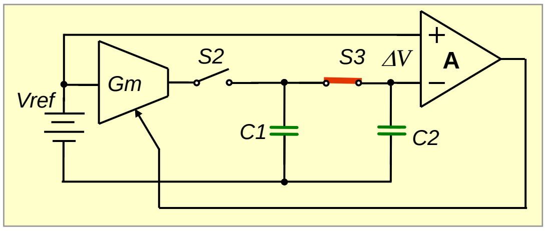
Reference Integrator Locked to Reference Frequency
P2 high → S2 closed



C1 → charged with constant current: $I=Gm*Vref$
C2 → *retains its previous charge*



Reference Integrator Locked to Reference Frequency
P3 high → S3 closed



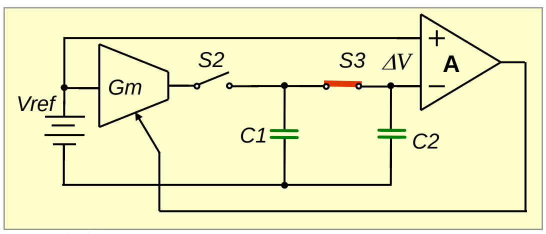
C1 charge shares with C2

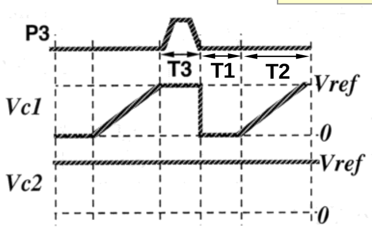
$$V_{C1}^{T2}C1 + V_{C2}^{T2}C2 = (C1+C2)V_{C1,2}^{T3}$$
$$V_{C1,2}^{T3} = V_{C1}^{T2}\frac{C1}{C1+C2} + V_{C2}^{T2}\frac{C2}{C1+C2}$$

Few cycles following startup system approaches steady state:

$$V_{C1,2}^{T3} = V_{C1}^{T2} = V_{C2}^{T2}$$

Reference Integrator Locked to Reference Frequency
P3 high → S3 closed



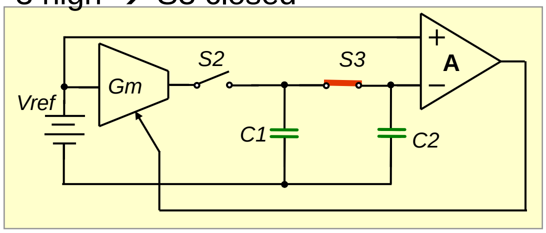


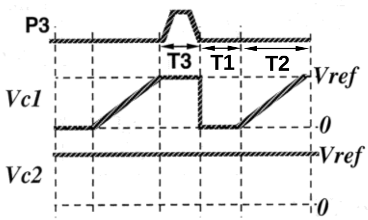
C1 charge shares with C2
Few cycles following startup
Assuming A is large, feedback forces:

$\Delta V \rightarrow 0$

$\rightarrow V_{C2} = V_{ref}$

Reference Integrator Locked to Reference Frequency
P3 high → S3 closed





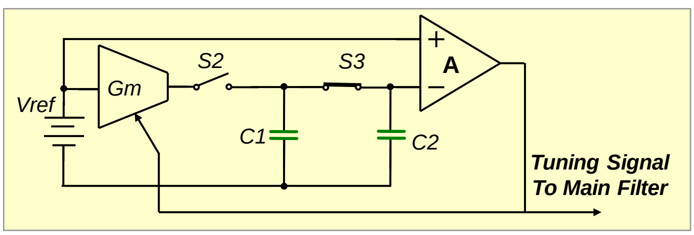
$V_{C1} = V_{C2} = Vref$

$since:\ V_{C1} = Gm \times V_{ref} \times T2 \big/ C1$

$then:\ \ V_{ref} = Gm \times V_{ref} \times T2 \big/ C1$

$or:\ \ \boxed{C1 \big/ Gm = T2 = N / fclk}$

Summary
Replica Integrator Locked to Reference Frequency

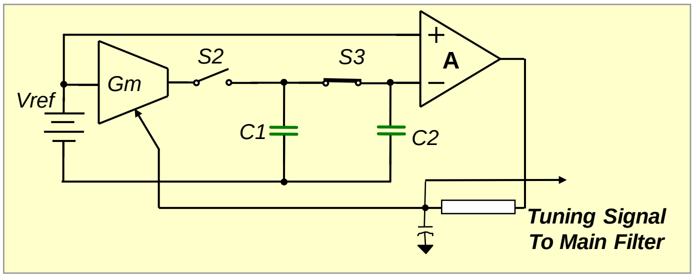


Feedback forces Gm to assume a value so that :

- Integrator time constant locked to an accurate frequency
- Tuning signal used to adjust the time constant of the main filter integrators

$$\boxed{\begin{array}{l} \tau_{intg} = C1 \big/ Gm = N / fclk \\ or \\ \omega_0^{intg} = Gm \big/ C1 = fclk / N \end{array}}$$

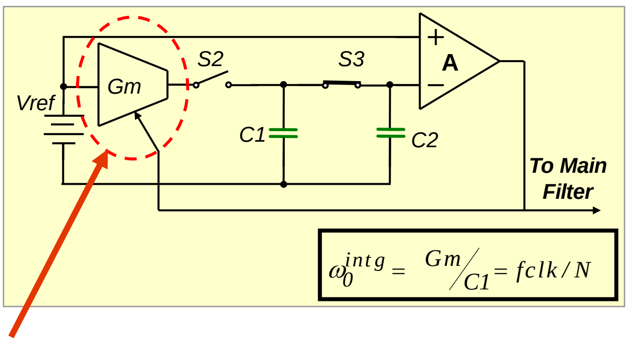
Issues
1- Loop Stability



- Note: Need to pay attention to loop stability
  - ✓ C1 chosen to be smaller than C2 – tradeoff between stability and speed of lock acquisition
  - ✓ Lowpass filter at the output of amplifier (A) helps stabilize the loop

Issues
2- GM-Cell DC Offset Induced Error

Problems to be aware of:



→ Tuning error due to master integrator DC offset

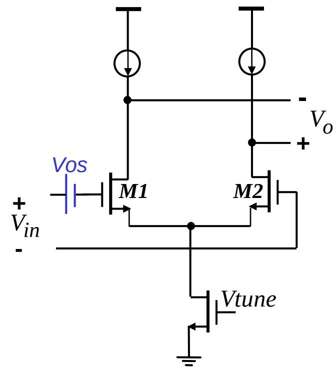
Issues
Gm Cell DC Offset

What is DC offset?

Simple example:

For the differential pair shown here, mismatch in input device or load characteristics would cause DC offset:
→ ***Vo*** = ***0*** requires a non-zero input voltage

Offset could be modeled as a small DC voltage source at the input for which with shorted inputs→ ***Vo*** = ***0***



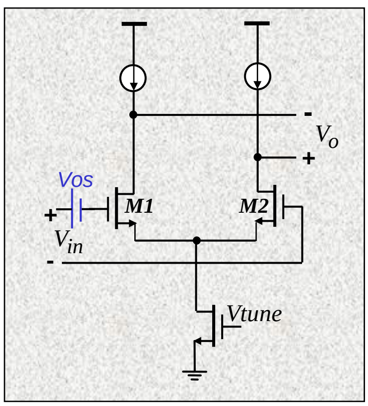
Example: Differential Pair

Simple Gm-Cell DC Offset

Mismatch associated with the diff. pair: M1 & M2
→ DC offset

$$V_{os} = (V_{th1} - V_{th2}) - \frac{1}{2} V_{ov1,2} \frac{\Delta (W/L)_{M1,2}}{(W/L)_{M1,2}}$$

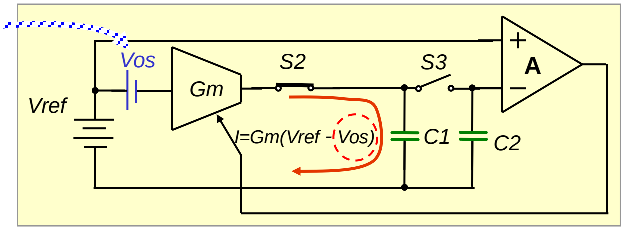
Assuming offset due to load device mismatch is negligible



Ref: Gray, Hurst, Lewis, Meyer, *Analysis & Design of Analog Integrated Circuits*, Wiley 2001, page 335

Gm-Cell Offset Induced Error

Voltage source representing DC offset



- Effect of Gm-cell DC offset:

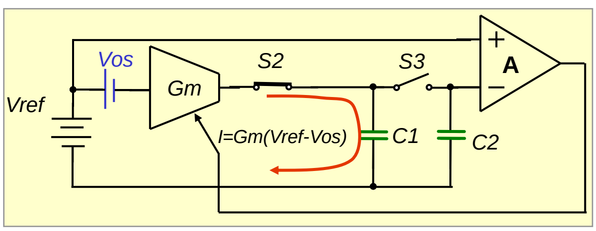
$$V_{C1} = V_{C2} = Vref$$

$$Ideal: V_{C1} = Gm \times V_{ref} \times T2 \big/ C1$$

$$with\ offset: \quad V_{C1} = Gm \times \left(V_{ref} - V_{os}\right) \times T2 \big/ C1$$

$$or: \quad \boxed{C1 \big/ Gm = T2 \left(1 - \frac{V_{os}}{V_{ref}}\right)}$$

Gm-Cell Offset Induced Error



- Example:

$$C1 \big/ Gm = T2 \left(1 - \frac{V_{os}}{V_{ref}}\right) \qquad f_{critical} \propto Gm \big/ C1$$

$$for \ \frac{V_{os}}{V_{ref}} = 1/10$$

$$10\% \ \ error\ in\ tuning!$$