EE247
Lecture 9

- Switched-capacitor filters
  - Introduction to S.C. filters
    - Issue of aliasing mandating use of anti-aliasing prefilters
    - Example of anti-aliasing prefilter for S.C. filters
  - Switched-capacitor network electronic noise
  - Switched-capacitor integrators
    - DDI integrators
    - LDI integrators

EE247
Summary of Last Lecture

- Continuous-time filter design considerations
  - Monolithic highpass filters
  - Active bandpass filter design
    - Lowpass to bandpass transformation
    - Example: 6^{th} order bandpass filter
    - Gm-C bandpass filter using simple diff. pair
  - Various Gm-C filter implementations
- Performance comparison of various continuous-time filter topologies
- Introduction to switched-capacitor filters

Switched-Capacitor Resistors

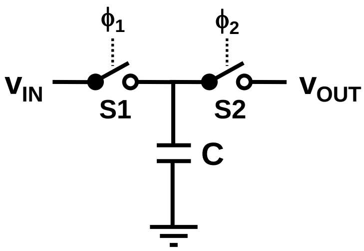
$$i = f_S C(v_{IN} - v_{OUT})$$

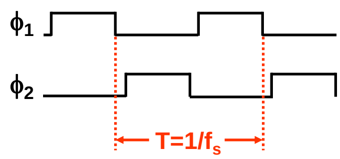
With the current through the switched-capacitor resistor proportional to the voltage across it, the equivalent "switched capacitor resistance" is:

$$R_{eq} = \frac{V_{IN} - V_{OUT}}{i} = \frac{1}{f_S C}$$

Example:
$f_S = 100KHz, C = 0.1pF$
$\rightarrow R_{eq} = 100Mega\Omega$

Note: Can build large time-constant in small area



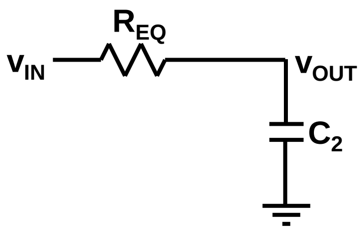


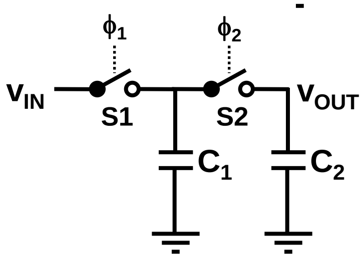
Switched-Capacitor Filter

- Let's build a "switched- capacitor " filter …
- Start with a simple RC LPF
- Replace the physical resistor by an equivalent switched-capacitor resistor
- 3-dB bandwidth:

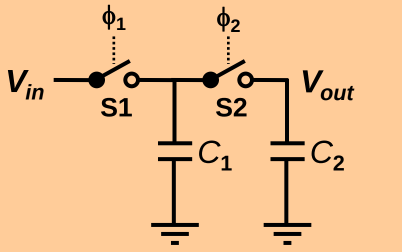
$$\omega_{-3dB} = \frac{1}{R_{eq} C_2} = f_s \times \frac{C_1}{C_2}$$

$$f_{-3dB} = \frac{1}{2\pi} f_s \times \frac{C_1}{C_2}$$



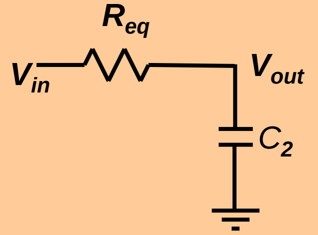


Switched-Capacitor Filter Advantage versus Continuous-Time Filter



$$f_{-3dB} = \frac{1}{2\pi} f_s \times \frac{C_1}{C_2}$$

- Corner freq. proportional to:
  System clock (accurate to few ppm)
  C ratio accurate → < 0.1%



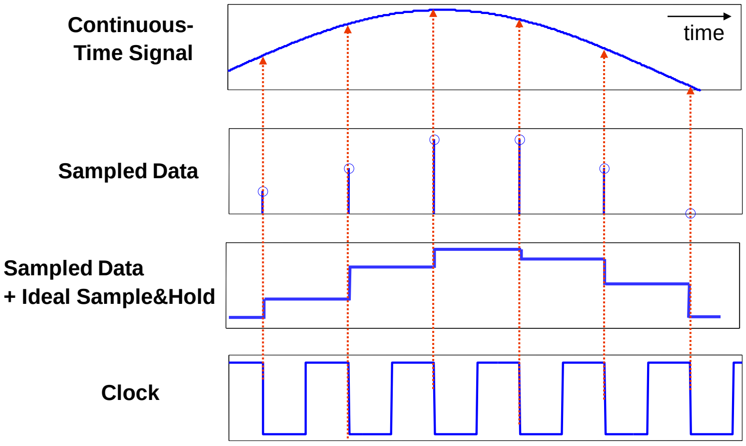
$$f_{-3dB} = \frac{1}{2\pi} \times \frac{1}{R_{eq} C_2}$$

- Corner freq. proportional to:
  Absolute value of Rs & Cs
  Poor accuracy → 20 to 50%

Main advantage of SC filters → inherent critical frequency accuracy

Typical Sampling Process

Continuous-Time(CT) ⇒ Sampled Data (SD)

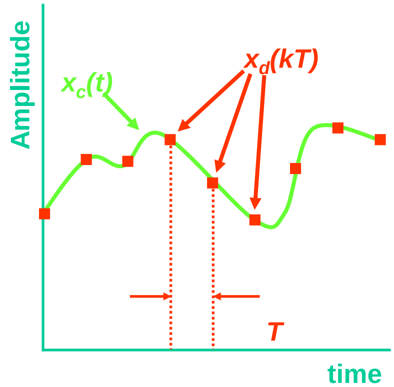


Uniform Sampling

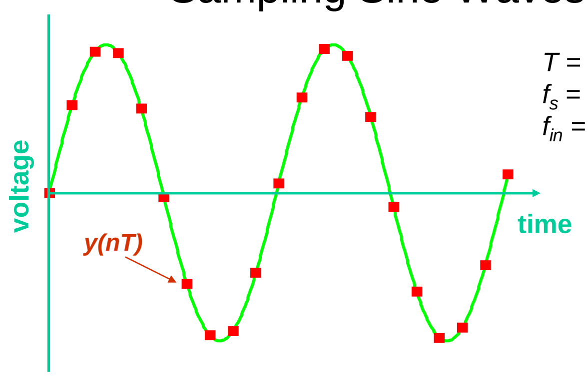
<u>Nomenclature:</u>

Continuous time signal	$x_c(t)$
Sampling interval	T
Sampling frequency	$f_s = 1/T$
Sampled signal	$x_d(kT) = x(k)$

- Samples are the waveform values at kT instances and <u>undefined</u> in between
- Problem: Multiple continuous time signals can yield exactly the same discrete time signal
- Let's examine samples taken at 1μs intervals of several sinusoidal waveforms …



Sampling Sine Waves



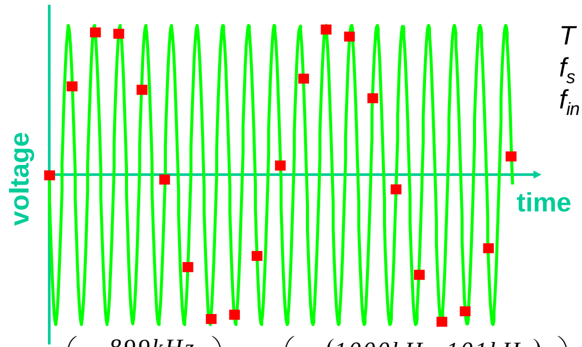
$T = 1\mu s$
$f_s = 1/T = 1MHz$
$f_{in} = 101kHz$

$v(t) = cos\ (2\pi.f_{in}.t)$

Sampled-data domian $\rightarrow t \rightarrow n.T$ or $t \rightarrow n/f_s$ (n integer)

$$v(n) = cos\left(2\pi.\frac{f_{in}}{f_s}.n\right) = cos\left(2\pi.\frac{101kHz}{1MHz}.n\right)$$

Sampling Sine Waves
Aliasing

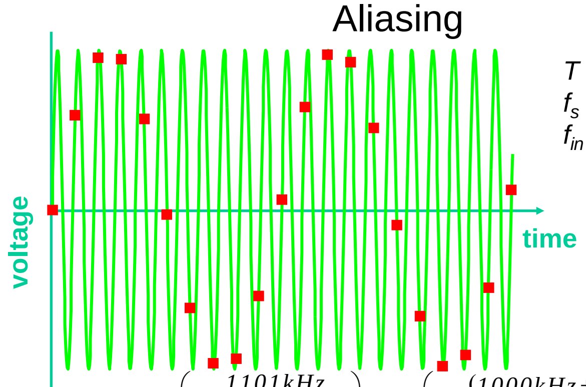


$T = 1\mu s$
$f_s = 1/T = 1MHz$
$f_{in} = 899kHz$

$$v(n)=\cos\left(2\pi\frac{899kHz}{1MHz}n\right)=\cos\left(2\pi\frac{(1000kHz\text{-}101kHz)}{1MHz}n\right)=\cos\left(2\pi-2\pi\frac{101kHz}{1MHz}n\right)$$

$$=\cos\left(2\pi\frac{101kHz}{1MHz}n\right)$$

Sampling Sine Waves
Aliasing



$T = 1\mu s$
$f_s = 1/T = 1MHz$
$f_{in} = 1101kHz$

$$v(n)=\cos\left(2\pi.\frac{1101kHz}{1MHz}.n\right)=\cos\left(2\pi.\frac{(1000kHz+101kHz)}{1MHz}.n\right)$$

$$=\cos\left(2\pi.\frac{101kHz}{1MHz}.n\right)$$

Sampling Sine Waves

Problem:

Sampled data domain → identical samples for:

$v(t) = \cos[2\pi f_{in} t]$

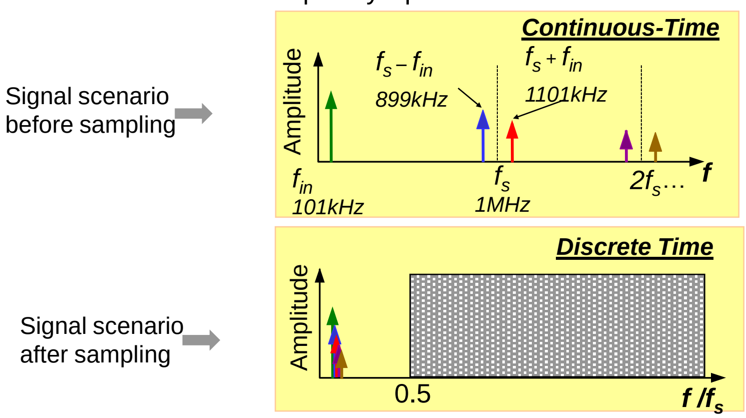
$v(t) = \cos[2\pi (f_{in} + n.f_s) t]$

$v(t) = \cos[2\pi (f_{in} - n.f_s) t]$

* (n-integer)

→Multiple continuous time signals can yield exactly the same discrete time signal

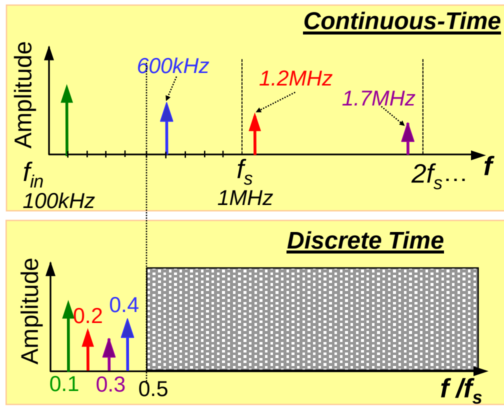
Sampling Sine Waves
Frequency Spectrum



Key point: Signals @ $nf_S \pm f_{max\_\_signal}$ fold back into band of interest→**Aliasing**

Sampling Sine Waves
Frequency Spectrum

Signal scenario before sampling



Signal scenario after sampling

Key point: Signals @ $nf_S \pm f_{max\_\_signal}$ fold back into band of interest→**Aliasing**

Aliasing

- Multiple continuous time signals can produce identical series of samples
- The folding back of signals from $nf_s \pm f_{sig}$ (n integer) down to the band f_{fin} is called aliasing
  – Sampling theorem: $f_s > 2f_{max\_Signal}$
- If aliasing occurs, no signal processing operation downstream of the sampling process can recover the original continuous time signal

How to Avoid Aliasing?

- Must obey sampling theorem:

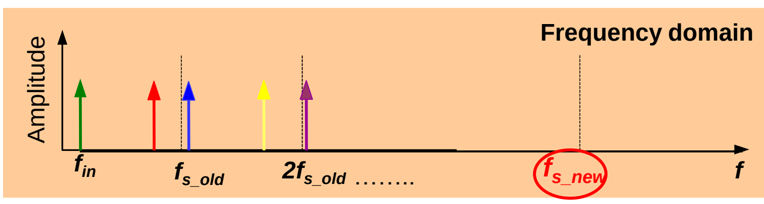
$$f_{max\text{-}signal} < f_s/2$$

*Note:

Minimum sampling rate of $f_s = 2xf_{max\text{-}Signal}$ is called <u>Nyquist rate</u>

- Two possibilities:
  1. Sample fast enough to cover all spectral components, including "parasitic" ones outside band of interest
  2. Limit f_{max_Signal} through filtering → attenuate out-of-band components prior to sampling

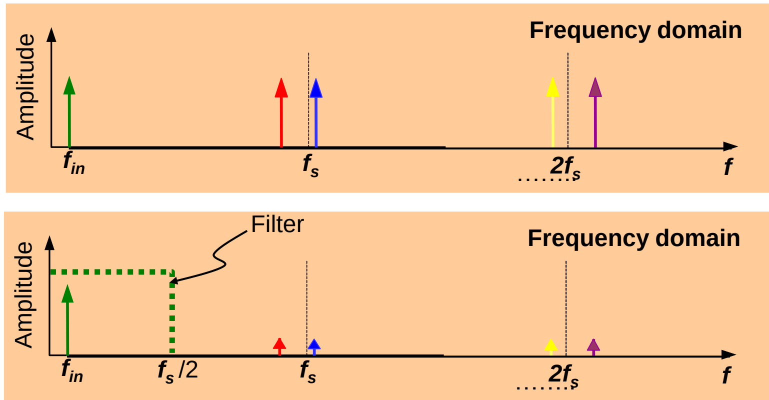
How to Avoid Aliasing?
1-Sample Fast



Push sampling frequency to *x2* of the highest frequency signal to cover all unwanted signals as well as wanted signals

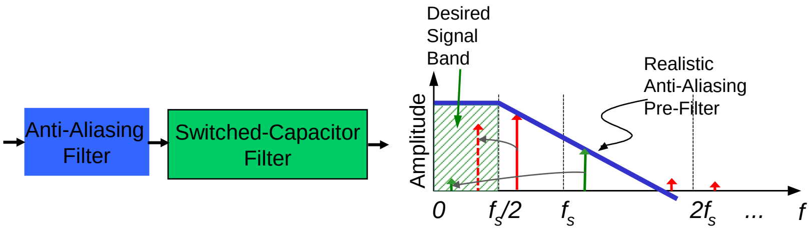
→ In vast majority of cases <u>not practical</u>

How to Avoid Aliasing?
2-Filter Out-of-Band Signal Prior to Sampling



Pre-filter signal to eliminate/attenuate signals above $f_s/2$- then sample

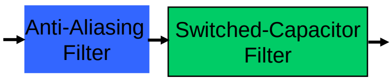
Anti-Aliasing Filter Considerations

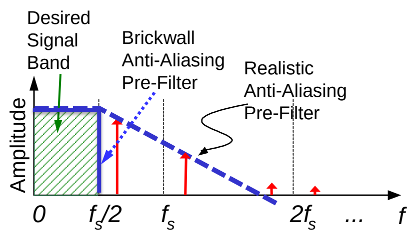


Case1- $B = f_{sig}^{max} = f_s/2$

- Practical anti-aliasing filter →Non-zero filter "transition band"
- Note out-of-band signal close to $f_s/2$ aliases down to the band of interest without much attenuation

Anti-Aliasing Filter Considerations



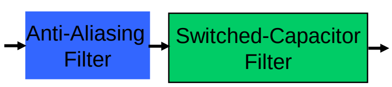


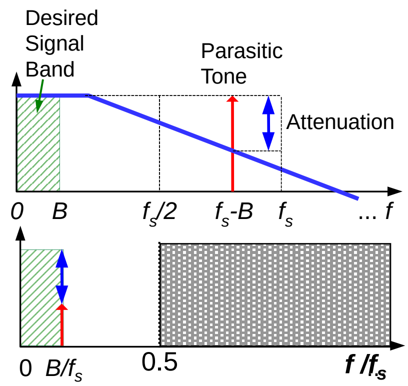
Case1- $B = f_{sig}^{max} = f_s/2$

- To achieve adequate out-of-band attentuation →extremely high order anti-aliasing filter (close to an ideal brickwall filter) is required
- Not practical anti-aliasing filter
- In order to make this work, we need to sample much faster than 2x the signal bandwidth

  →"Oversampling"

Practical Anti-Aliasing Filter





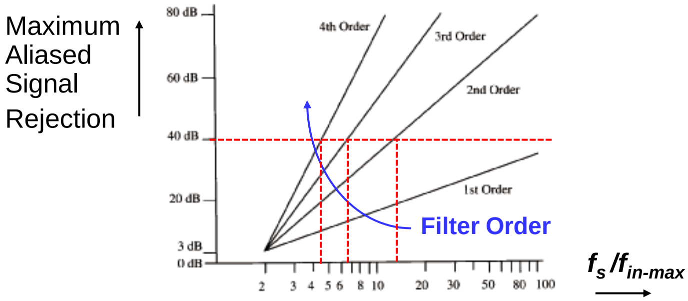
Case2 - $B = f_{max-Signal} << f_s/2$

- More practical anti-aliasing filter
- Preferable to have an anti-aliasing filter with:

  →The lowest order possible

  →No frequency tuning required (if frequency tuning is required then why use switched-capacitor filter, just use the prefilter!?)

Tradeoff
Oversampling Ratio versus Anti-Aliasing Filter Order



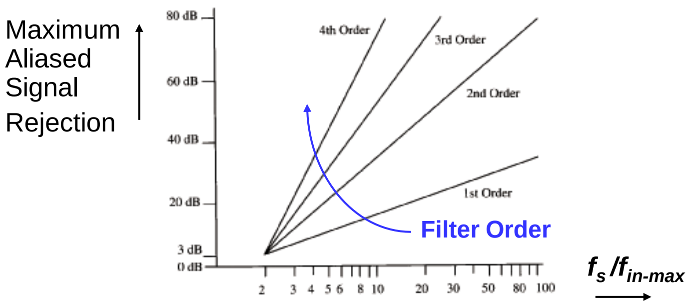
* *Assumption → anti-aliasing filter is Butterworth type (not a necessary requirement)*

Example:

Assume that a S.C. filter has 40dB in-band dynamic range and that out-of-band signals can have magnitude equal to in-band signals at the input → Find the minimum required anti-aliasing filter order versus oversampling rate

Ref: R. v. d. Plassche, *CMOS Integrated Analog-to-Digital and Digital-to-Analog Converters*, 2nd ed., Kluwer publishing, 2003, p.41

Tradeoff
Oversampling Ratio versus Anti-Aliasing Filter Order

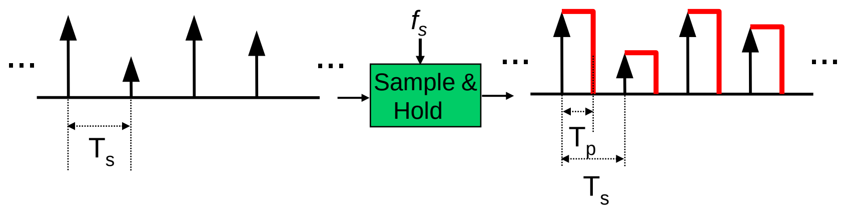


* *Assumption → anti-aliasing filter is Butterworth type (not a necessary requirement)*

→Tradeoff: Sampling frequency versus anti-aliasing filter order

Ref: R. v. d. Plassche, *CMOS Integrated Analog-to-Digital and Digital-to-Analog Converters*, 2nd ed., Kluwer publishing, 2003, p.41

Effect of Sample & Hold

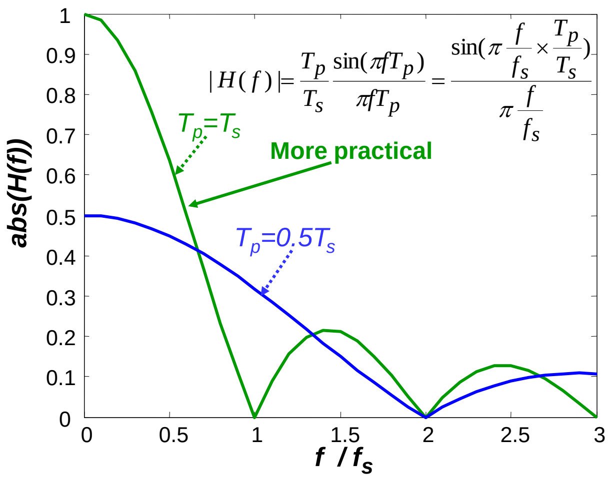


- Using the Fourier transform of a rectangular impulse:

$$|H(f)| = \frac{T_p}{T_s}\frac{\sin(\pi f T_p)}{\pi f T_p} \longrightarrow \frac{\sin x}{x} \text{ shape}$$

In literature also called Sinc function

Effect of Sample & Hold on Frequency Response



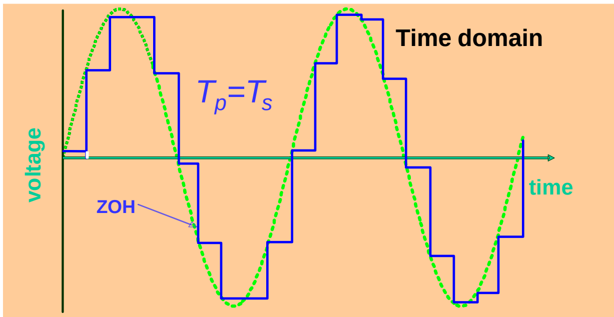
$$|H(f=0)| = \frac{T_p}{T_s}$$

$$|H(f = n f_s \frac{T_s}{T_p})| = 0$$

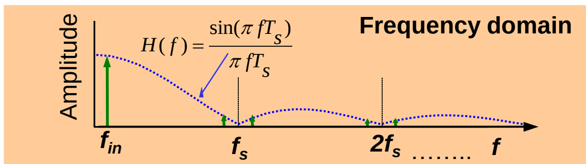
$n \rightarrow$ integer

Sample & Hold Effect
(Reconstruction of Analog Signals)

$T_p = T_s$



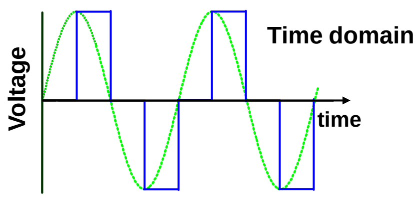
Magnitude droop due to *sinx/x* effect



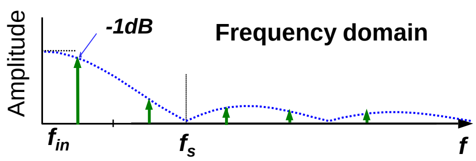
Sample & Hold Effect
(Reconstruction of Analog Signals)

Magnitude droop due to *sinx/x* effect:

Case 1) $f_{sig}=f_s/4$



Droop= *-1dB*



Sample & Hold Effect
(Reconstruction of Analog Signals)

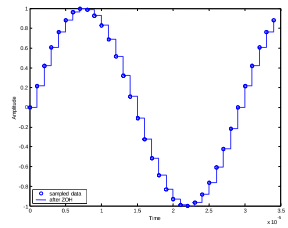
Magnitude droop due to ***sinx/x*** effect:

Case 2)
$f_{sig}=f_s/32$

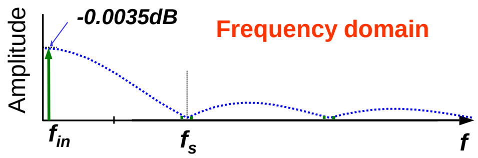
Droop= *-0.0035dB*

- ***Insignificant droop → High oversampling ratio desirable***

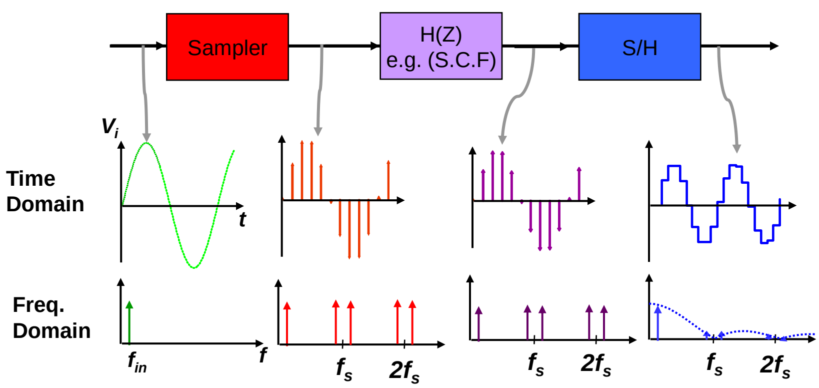
Time domain



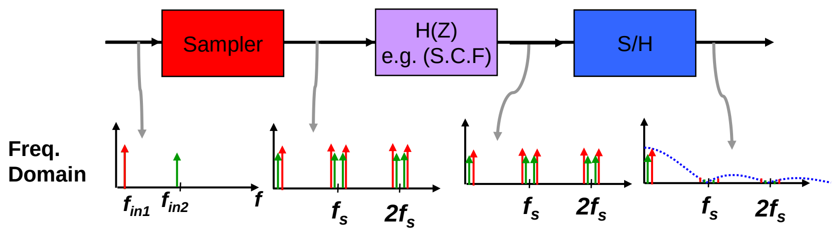
Frequency domain



Sampling Process Including S/H
Input: Single In-Band Sinusoid

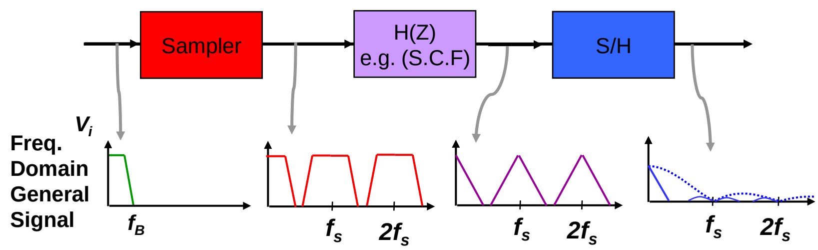


Sampling Process Including S/H
Input: In-Band Sinusoid + One Out-of-Band Sinusoid



Aliasing occurs prior to sinx/x shaping and hence sinx/x effect does not attenuate aliased components

Sampling Process Including S/H
Input: General In-Band Signal



Beware of sinx/x shaping (droop) of in-band signals

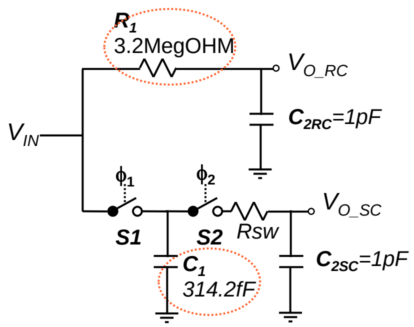
1^{st} Order Filter S.C. versus C.T. Transient Analysis

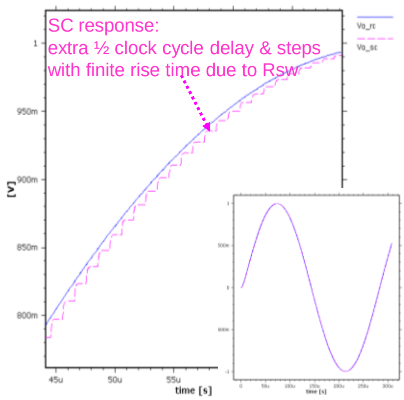
1^{st} Order RC versus SC LPF

$f_s = 1MHz$

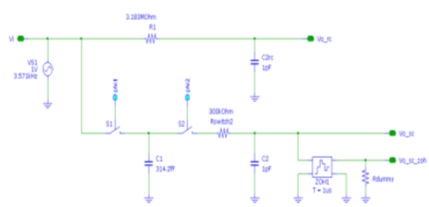
f_{-3dB}=50kHz

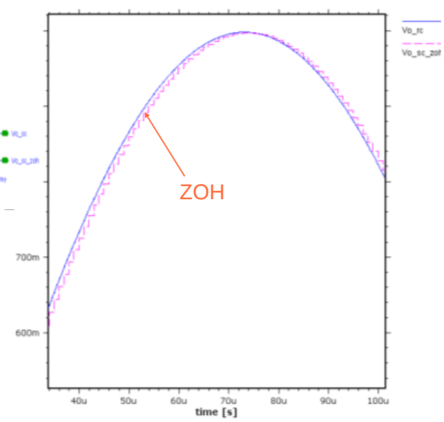
f_{in}= 3.6kHz





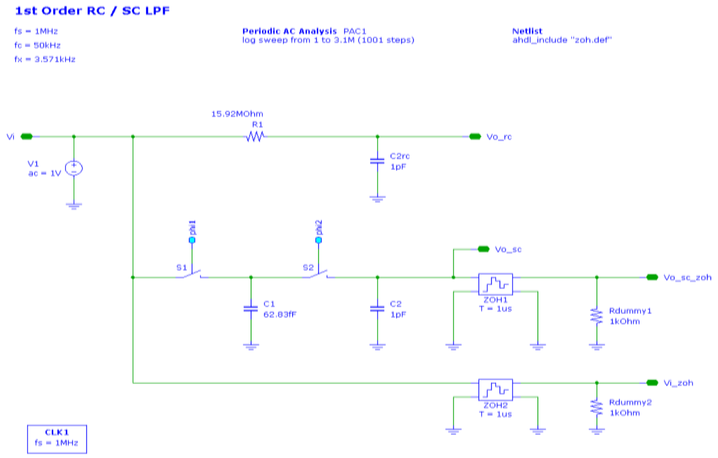
1^{st} Order Filter Transient Analysis



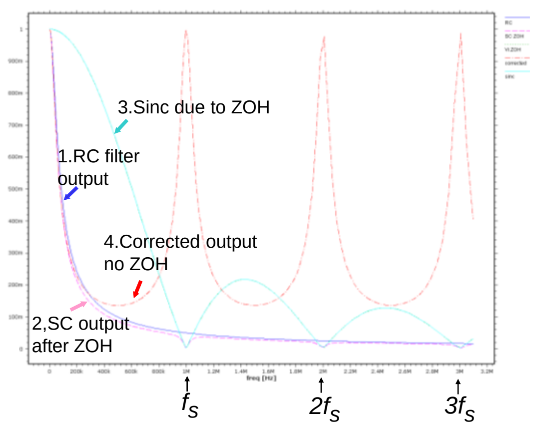


- ZOH: Emulates an ideal S/H→ pick signal after output is fully settled (usually at end of clock phase)
- Adds delay and sin(x)/x shaping

Periodic AC Analysis



1st Order Filter Magnitude Response



1. RC filter output
2. SC output after ZOH
3. Output after single ZOH
4. S.C. filter output w/o effect of ZOH
   - (2) over (3)
   - Repeats filter shape around nf_s
   - Identical to RC for $f << f_s/2$

Periodic AC Analysis

- SPICE frequency analysis
  - ac   linear, time-invariant circuits
  - pac   linear, time-variant circuits
- SpectreRF statements

```
V1 ( Vi 0 ) vsource type=dc dc=0 mag=1
 pacmag=1
PSS1 pss period=1u errpreset=conservative
PAC1 pac start=1 stop=1M lin=1001
```

- Output
  - Divide results by sinc(f/f_s) to correct for ZOH distortion

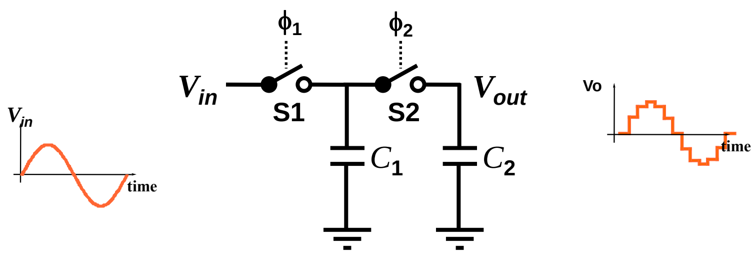
SpectreRF Circuit File

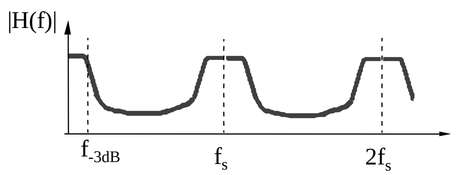
```
rc_pac
simulator lang=spectre
ahdl_include "zoh.def"

S1 ( Vi c1 phi1 0 ) relay ropen=100G rclosed=1 vt1=-500m vt2=500m
S2 ( c1 Vo_sc phi2 0 ) relay ropen=100G rclosed=1 vt1=-500m vt2=500m
C1 ( c1 0 ) capacitor c=314.159f
C2 ( Vo_sc 0 ) capacitor c=1p
R1 ( Vi Vo_rc ) resistor r=3.1831M
C2rc ( Vo_rc 0 ) capacitor c=1p
CLK1_Vphi1 ( phi1 0 ) vsource type=pulse val0=-1 val1=1 period=1u
                          width=450n delay=50n rise=10n fall=10n
CLK1_Vphi2 ( phi2 0 ) vsource type=pulse val0=-1 val1=1 period=1u
                          width=450n delay=550n rise=10n fall=10n
V1 ( Vi 0 ) vsource type=dc dc=0 mag=1 pacmag=1
PSS1 pss period=1u errpreset=conservative
PAC1 pac start=1 stop=3.1M log=1001
ZOH1 ( Vo_sc_zoh 0 Vo_sc 0 ) zoh period=1u delay=500n aperture=1n tc=10p
ZOH2 ( Vi_zoh 0 Vi 0 ) zoh period=1u delay=0 aperture=1n tc=10p
```

ZOH Circuit File

```
// Copy from the SpectreRF Primer

module zoh (Pout, Nout, Pin, Nin) (period,
     delay, aperture, tc)

node [V,I] Pin, Nin, Pout, Nout;
parameter real period=1 from (0:inf);
parameter real delay=0 from [0:inf);
parameter real aperture=1/100 from (0:inf);
parameter real tc=1/500 from (0:inf);
{
integer n; real start, stop;
node [V,I] hold;
  analog {
    // determine the point when aperture
     begins
    n = ($time() - delay + aperture) / period
     + 0.5;
    start = n*period + delay - aperture;
    $break_point(start);

    // determine the time when aperture ends
    n = ($time() - delay) / period + 0.5;
    stop = n*period + delay;
    $break_point(stop);

    // Implement switch with effective series
    // resistence of 1 Ohm
    if ( ($time() > start) && ($time() <=
     stop))
      I(hold) <- V(hold) - V(Pin, Nin);
    else
      I(hold) <- 1.0e-12 * (V(hold) - V(Pin,
     Nin));

    // Implement capacitor with an effective
    // capacitance of tc
    I(hold) <- tc * dot(V(hold));

    // Buffer output
    V(Pout, Nout) <- V(hold);

    // Control time step tightly during
    // aperture and loosely otherwise
    if (($time() >= start) && ($time() <=
     stop))
      $bound_step(tc);
    else
      $bound_step(period/5);
  }
}
```


First Order S.C. Filter

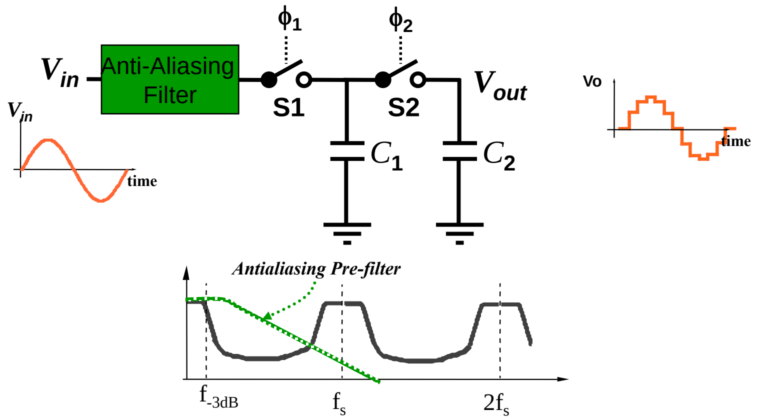




Output Frequency Spectrum prior to hold

Switched-Capacitor Filters → problem with aliasing

First Order S.C. Filter



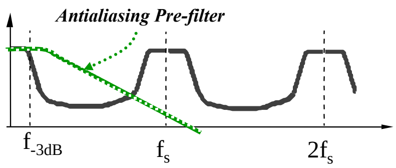
Output Frequency Spectrum prior to hold

Switched-Capacitor Filters → problem with aliasing

Sampled-Data Systems (Filters) Anti-aliasing Requirements

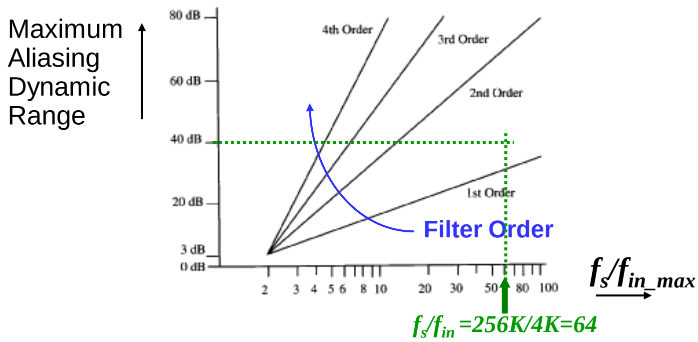
- Frequency response repeats at f_s , $2f_s$, $3f_s$.....
- High frequency signals close to f_s , $2f_s$,….folds back into passband (aliasing)
- Most cases must pre-filter input to sampled-data systems (filter) to attenuate signal at:
  $f > f_s/2$ (nyquist → $f_{max} < f_s/2$)
- Usually, anti-aliasing filter → included on-chip as continuous-time filter with relaxed specs. (no tuning)

Example : Anti-Aliasing Filter Requirements



- Voice-band CODEC S.C. filter high order low-pass with f_{-3dB} =4kHz & f_s =256kHz
- Anti-aliasing continuous-time pre-filter requirements:
  - Need at least 40dB attenuation of all out-of-band signals which can alias inband
  - Incur no phase-error from 0 to 4kHz
  - Gain error due to anti-aliasing filter → 0 to 4kHz < 0.06dB
  - Allow +-30% variation for anti-aliasing filter corner frequency (no tuning) **Need to find minimum required filter order**

Oversampling Ratio versus Anti-Aliasing Filter Order



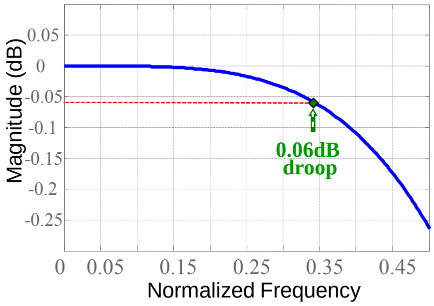
* *Assumption → anti-aliasing filter is Butterworth type*

→ 2^{nd} order Butterworth

→ Need to find minimum corner frequency for mag. droop < 0.06dB

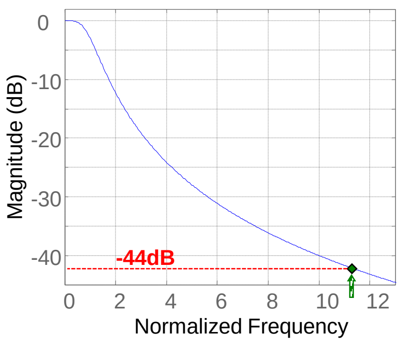
Example : Meeting the Anti-Aliasing Filter Specifications

- Note that since the anti-aliasing filter is not tuned have to make sure all specifications are met under worst-case conditions
- Worst case passband droop occurs at narrowest possible anti-aliasing filter bandwidth
- Find the AA filter bandwidth for with droop <0.06dB
- Normalized frequency for 0.06dB droop: need perform passband simulation→ normalized ω=0.34
- Narrowest AA filter bandwidth→ 4kHz/0.34=**12kHz**

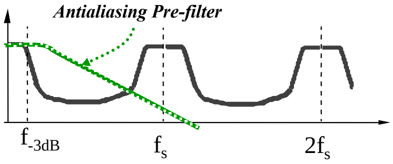


Example : Anti-Aliasing Filter Specifications

- Since +-30% variation of AA filter corner frequency is expected
- Set anti-aliasing filter corner frequency for minimum corner frequency **12kHz** → Find nominal corner frequency: 12kHz/0.7=**17.1kHz**
- Check if min. attenuation requirement is satisfied for widest filter bandwidth → 17.1x1.3=**22.28kHz**
- Find $(f_s\text{-}f_{sig})/f_{-3dB}^{\ max}$ →252/22.2=11.35→ make sure enough attenuation
- Check phase-error within 4kHz signal band for min. filter bandwidth via simulation



Example : Anti-Aliasing Filter



- Voice-band S.C. filter $f_{-3dB}=4kHz$ & $f_s=256kHz$
- Anti-aliasing filter requirements:
  - Need 40dB attenuation at clock freq.
  - Incur no phase-error from 0 to 4kHz
  - Gain error 0 to 4kHz < 0.06dB
  - Allow +-30% variation for anti-aliasing corner frequency (no tuning)

→2-pole Butterworth LPF with nominal corner freq. of 17kHz & no tuning (min.=12kHz & max.=22kHz corner frequency)

Summary

- Sampling theorem → $f_s > 2f_{max\_Signal}$
- Signals at frequencies $nf_S \pm f_{sig}$ fold back down to desired signal band, f_{sig}

  → This is called aliasing & usually mandates use of anti-aliasing pre-filters combined with oversampling
- Oversampling helps reduce required order for anti-aliasing filter
- S/H function shapes the frequency response with *sinx/x* shape

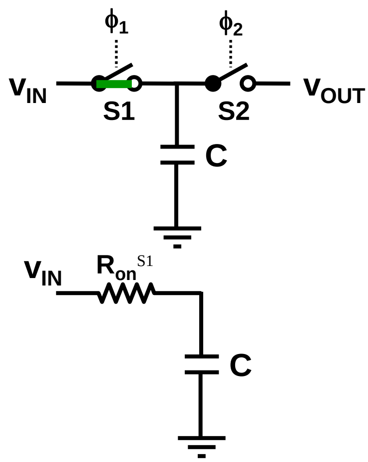
  → Need to pay attention to droop in passband due to *sinx/x*
- If the above requirements are not met, CT signals can NOT be recovered from sampled-data networks without loss of information

Switched-Capacitor Network Noise

- During ϕ_1 high: Resistance of switch S1 (R_{on}^{S1}) produces a noise voltage on C with variance kT/C (lecture 1- first order filter noise)
- The corresponding noise charge is:

$$Q^2 = C^2V^2 = C^2.\ kT/C = kTC$$

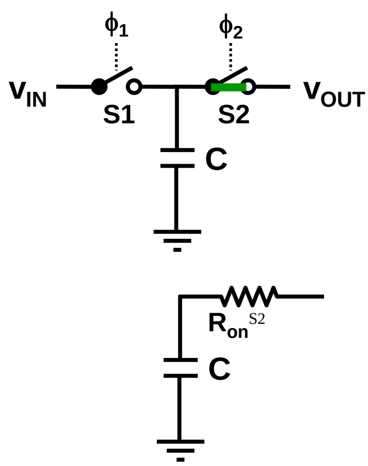
- ϕ_1 low: S1 open → This charge is sampled



Switched-Capacitor Noise

- During ϕ_2 high: Resistance of switch S2 contributes to an uncorrelated noise charge on C at the end of ϕ_2 : with variance kT/C
- Mean-squared noise charge transferred from v_{IN} to v_{OUT} per sample period is:

$$Q^2 = 2kTC$$



Switched-Capacitor Noise

- The mean-squared noise current due to S1 and S2's *kT/C* noise is :

$$Since \;\; i = Q/t \quad then \rightarrow \overline{i^2} = (Qf_s)^2 = 2k_BTCf_s^2$$

- This noise is approximately white and distributed between 0 and $f_s/2$ (noise spectra → single sided by convention)
  The spectral density of the noise is found:

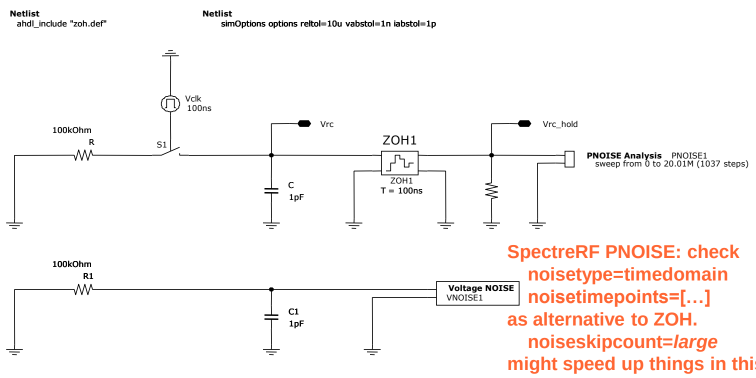
$$\frac{\overline{i^2}}{\Delta f} = \frac{2k_BTCf_s^2}{f_s/2} = 4k_BTCf_s$$

$$Since \;\; R_{EQ} = \frac{1}{f_sC} \quad then: \quad \boxed{\frac{\overline{i^2}}{\Delta f} = \frac{4k_BT}{R_{EQ}}}$$

→S.C. resistor noise = a physical resistor noise with same value!

Periodic Noise Analysis SpectreRF

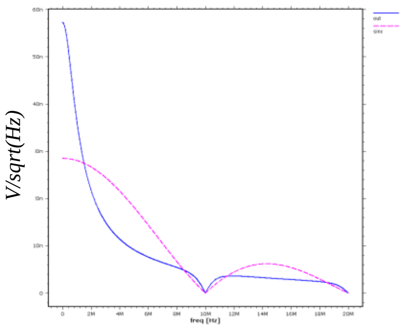
Sampling Noise from SC S/H



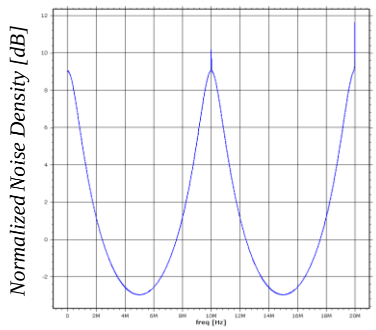
SpectreRF PNOISE: check
noisetype=timedomain
noisetimepoints=[…]
as alternative to ZOH.
noiseskipcount=*large*
might speed up things in this case.

```
PSS pss period=100n maxacfreq=1.5G errpreset=conservative
PNOISE ( Vrc_hold 0 ) pnoise start=0 stop=20M lin=500 maxsideband=10
```

Sampled Noise Spectrum

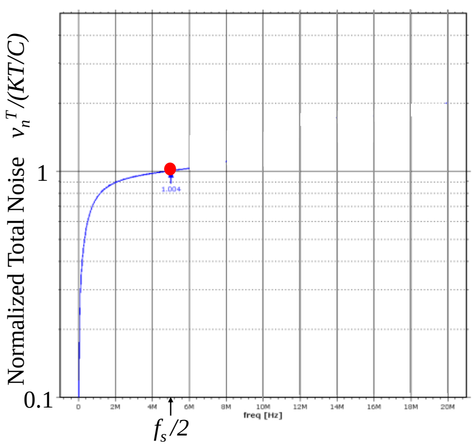


Spectral density of sampled noise including sinx/x effect



Noise spectral density with sinx/x effect taken out

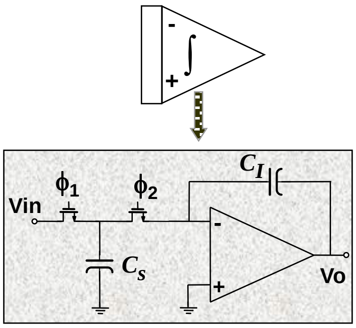
Total Noise

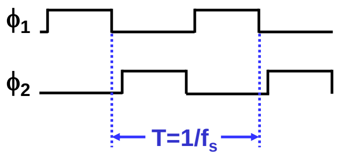


Sampled simulated noise in 0 … $f_s/2$: 62.2μV rms

(expect 64μV for 1pF)

Switched-Capacitor Integrator





for $f_{signal} << f_{sampling}$

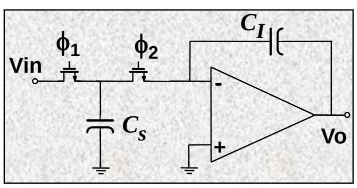
$$\rightarrow V_0 = \frac{f_s \times C_s}{C_I} \int V_{in}\, dt$$

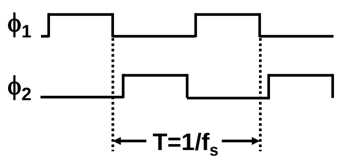
$$\omega_0 = f_s \times \frac{C_s}{C_I}$$

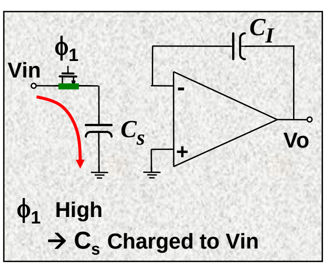
Main advantage: No tuning needed

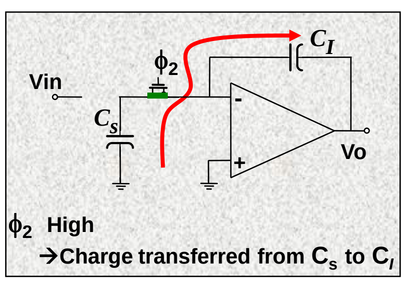
→ Critical frequency function of ratio of capacitors & clock freq.

Switched-Capacitor Integrator









Continuous-Time versus Discrete-Time Analysis Approach

Continuous-Time

- Write differential equation
- Laplace transform ($F(s)$)
- Let $s=j\omega$ → $F(j\omega)$
- Plot $|F(j\omega)|$, $phase(F(j\omega))$

Discrete-Time

- Write difference equation → relates output sequence to input sequence

$$V_o(nT_s) = V_i[(n-1)T_s] - \ldots\ldots\ldots$$

- Use delay operator z^{-1} to transform the recursive realization to algebraic equation in z domain

$$V_o(z) = z^{-1} V_i(z) \ldots\ldots$$

- Set $z = e^{j\omega T}$
- Plot mag./phase versus frequency

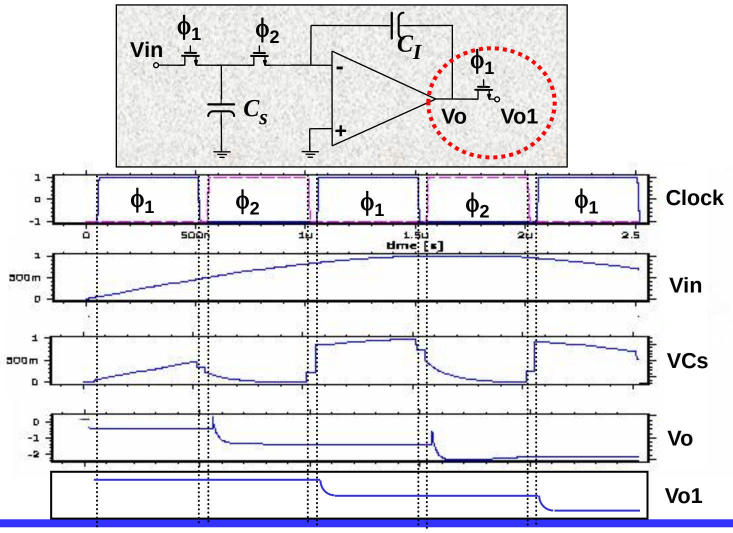
Discrete Time Design Flow

- Transforming the recursive realization to algebraic equation in z domain:
  - Use delay operator z :

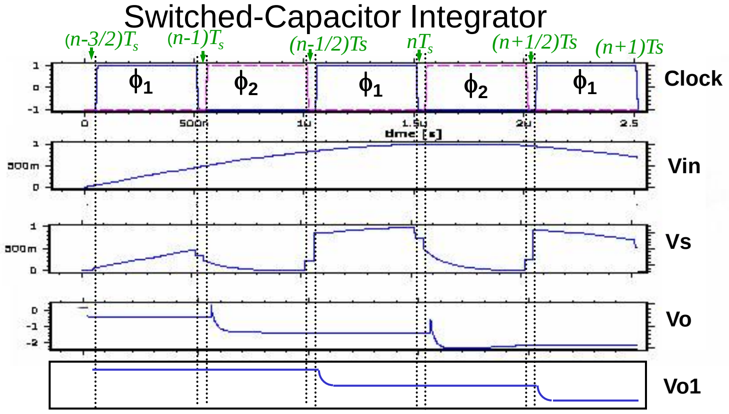
$$\begin{aligned}
nT_S & \ldots\ldots\ldots\ldots \rightarrow 1 \\
\left[(n-1)T_S\right] & \ldots\ldots\ldots\ldots \rightarrow z^{-1} \\
\left[(n-1/2)T_S\right] & \ldots\ldots\ldots \rightarrow z^{-1/2} \\
\left[(n+1)T_S\right] & \ldots\ldots\ldots\ldots \rightarrow z^{+1} \\
\left[(n+1/2)T_S\right] & \ldots\ldots\ldots \rightarrow z^{+1/2}
\end{aligned}$$

* Note: $z = e^{j\omega T_s} = \cos(\omega T_s) + j\sin(\omega T_s)$

Switched-Capacitor Integrator Output Sampled on $\phi 1$



Switched-Capacitor Integrator



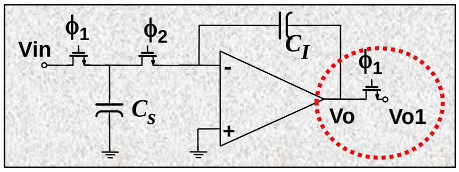
$\Phi_1 \rightarrow \quad Q_s[(n-1)T_s] = C_s V_i[(n-1)T_s]\,, \quad Q_I[(n-1)T_s] = Q_I[(n-3/2)T_s]$

$\Phi_2 \rightarrow \quad Q_s[(n-1/2)T_s] = 0 \,, \quad Q_I[(n-1/2)T_s] = Q_I[(n-1)T_s] + Q_s[(n-1)T_s]$

$\Phi_1 \rightarrow \quad Q_s[nT_s] = C_s V_i[nT_s]\,, \quad \boxed{Q_I[nT_s] = Q_I[(n-1)T_s] + Q_s[(n-1)T_s]}$

Since $V_{o1} = -Q_I/C_I$ & $V_i = Q_s/C_s \rightarrow C_I V_{o1}(nT_s) = C_I V_{o1}[(n-1)T_s] - C_s V_i[(n-1)T_s]$

Switched-Capacitor Integrator
Output Sampled on ϕ_1



$$C_I\ V_o(nT_s) = C_I\ V_o\left[(n-1)T_s\right] - C_s\ V_{in}\left[(n-1)T_s\right]$$

$$V_o(nT_s) = V_o\left[(n-1)T_s\right] - \frac{C_s}{C_I} V_{in}\left[(n-1)T_s\right]$$

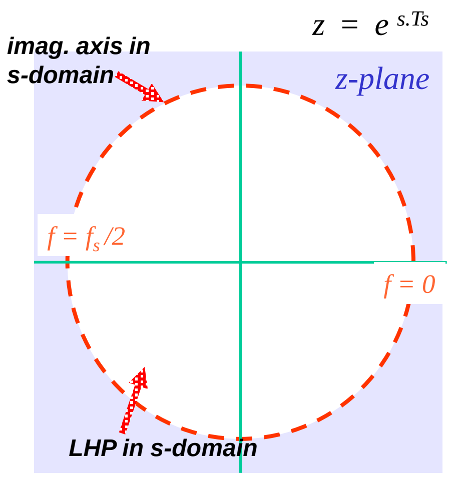
$$V_o(Z) = Z^{-1} V_o(Z) - Z^{-1}\frac{C_s}{C_I} V_{in}(Z)$$

$$\frac{V_o}{V_{in}}(Z) = -\frac{C_s}{C_I} \times \frac{Z^{-1}}{1-Z^{-1}}$$

DDI (Direct-Transform Discrete Integrator)

z-Domain Frequency Response

- Sampled-data systems→ z plane singularities analyzed via z-plane
- The s-plane jω axis maps onto the unit-circle
- LHP singularities in s-plane map into inside of unit-circle in z-domain
- RHP singularities in s-plane map into outside of unit-circle in z-domain
- Particular values:
  - $f = 0 \rightarrow z = 1$
  - $f = f_s/2 \rightarrow z = -1$

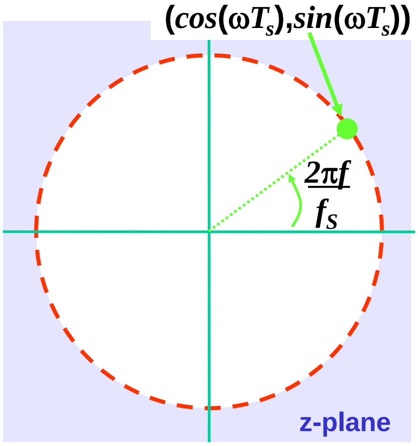


z-Domain Frequency Response

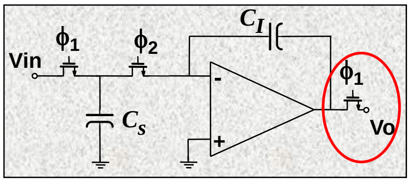
- The frequency response is obtained by evaluating H(z) on the unit circle at:

  $z = e^{j\omega T} = \cos(\omega T_s) + j\sin(\omega T_s)$

- Once $z=-1$ ($f_s/2$) is reached, the frequency response repeats, as expected
- The angle to the pole is equal to 360° (or 2π radians) times the ratio of the pole frequency to the sampling frequency



Switched-Capacitor Direct-Transform Discrete Integrator



$$\frac{V_o}{V_{in}}(z) = -\frac{C_S}{C_I} \times \frac{z^{-1}}{1-z^{-1}}$$

$$= -\frac{C_S}{C_I} \times \frac{1}{z-1}$$

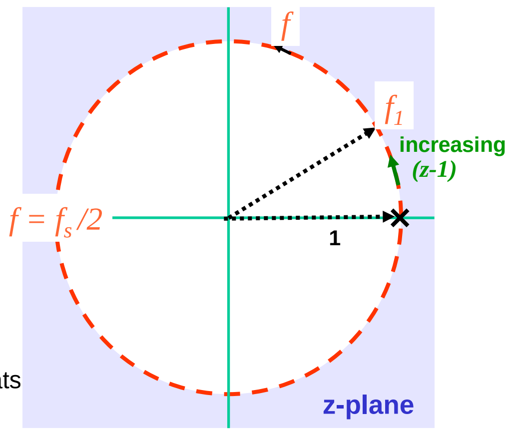
DDI Integrator
Pole-Zero Map in z-Plane

$z-1=0 \rightarrow z=1$
on unit circle

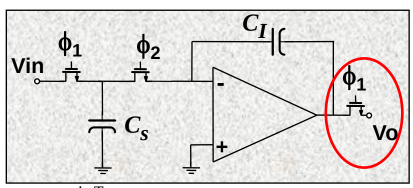
Pole from $f \rightarrow 0$
in s-plane mapped to $z=+1$

As frequency increases z domain point moves on unit circle (CCW)

Once frequency gets to:
$z=-1$ $(f=f_s/2)$
→ frequency response repeats



DDI Switched-Capacitor Integrator



$$\frac{V_o}{V_{in}}(z) = -\frac{C_s}{C_I} \times \frac{z^{-1}}{1-z^{-1}} = \frac{C_s}{C_I} \times \frac{1}{1-z} \quad , \quad z = e^{j\omega T}$$

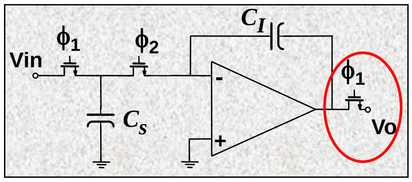
$$= \frac{C_s}{C_I} \times \frac{1}{1-e^{j\omega T}} = \frac{C_s}{C_I} \times \frac{e^{-j\omega T/2}}{e^{-j\omega T/2} - e^{j\omega T/2}} \qquad since: \quad \sin\alpha = \frac{e^{j\alpha} - e^{-j\alpha}}{2j}$$

$$= -j\frac{C_s}{C_I} \times e^{-j\omega T/2} \times \frac{1}{2\sin(\omega T/2)}$$

$$= -\frac{C_s}{C_I}\frac{1}{j\omega T} \times \frac{\omega T/2}{\sin(\omega T/2)} \times e^{-j\omega T/2}$$

Ideal Integrator Magnitude Error Phase Error

DDI Switched-Capacitor Integrator



$$\frac{V_O}{V_{in}}(z) = = -\frac{C_s}{C_I}\frac{1}{j\omega T} \times \frac{\omega T/2}{\sin(\omega T/2)} \times e^{-j\omega T/2}$$

Magnitude Error | Phase Error

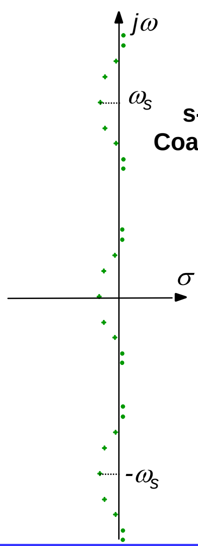
Example: Mag. & phase error for:

1- f/f_s=1/12 → *Mag. error = 1% or 0.1dB*
Phase error=15 degree
Q_{intg} = -3.8

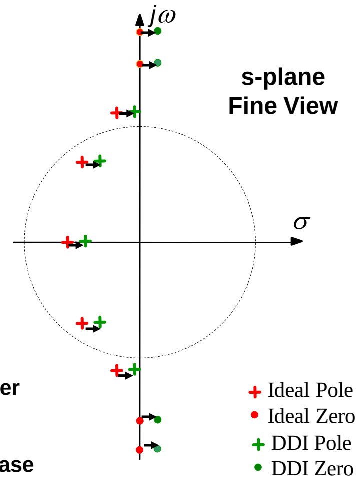
2- f/f_s=1/32 → *Mag. error=0.16% or 0.014dB*
Phase error=5.6 degree
Q_{intg} = -10.2

DDI Integrator:
→ magnitude error no problem
phase error major problem

5th Order Low-Pass Switched Capacitor Filter Built with DDI Integrators

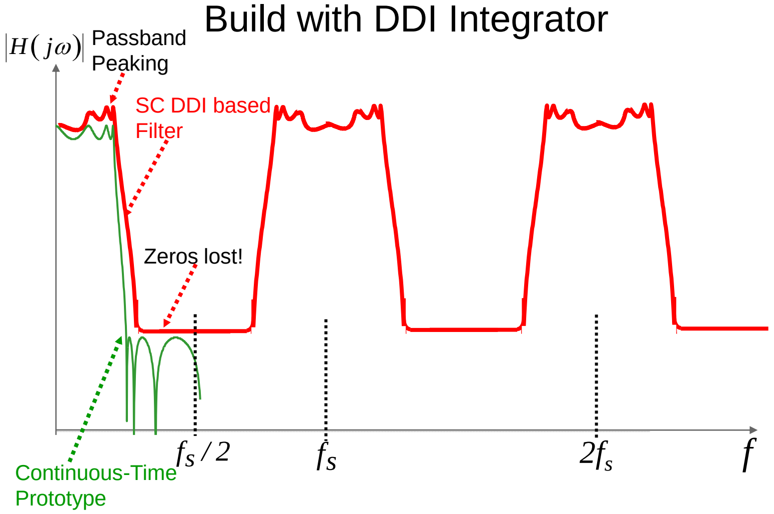


s-plane Coarse View

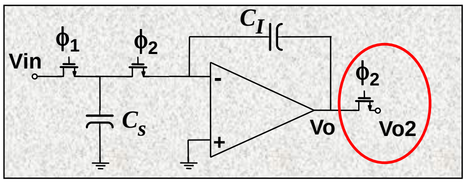


Example:
5th Order Elliptic Filter
Singularities pushed towards RHP due to integrator excess phase

Switched Capacitor Filter Build with DDI Integrator



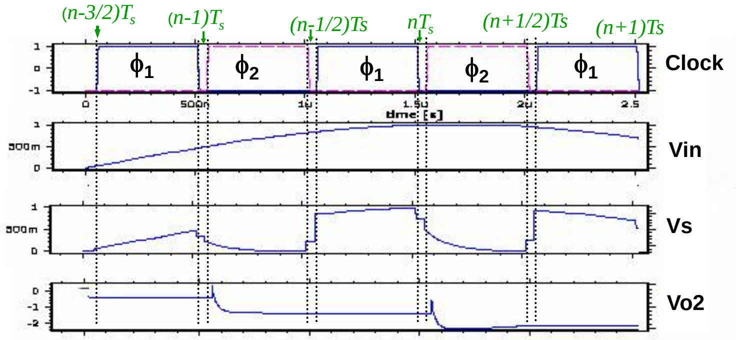
Switched-Capacitor Integrator Output Sampled on ϕ2



Sample output ½ clock cycle earlier

→ Sample output on ϕ_2

Switched-Capacitor Integrator
Output Sampled on $\phi 2$



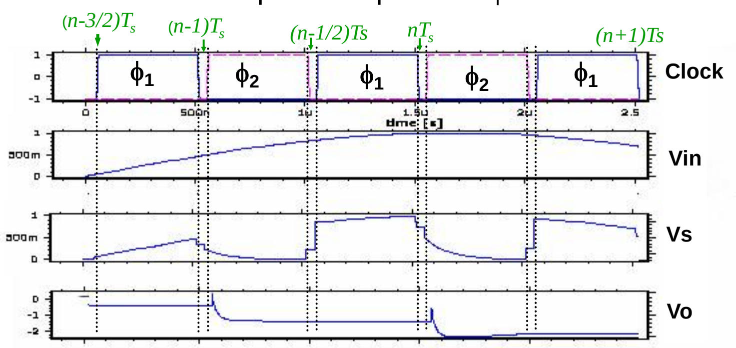
$\Phi_1 \rightarrow \quad Q_s[(n-1)T_s] = C_s V_i[(n-1)T_s]\,, \quad Q_I[(n-1)T_s] = Q_I[(n-3/2)T_s]$

$\Phi_2 \rightarrow \quad Q_s[(n-1/2)\,T_s] = 0 \quad , \quad Q_I[(n-1/2)\,T_s] = Q_I[(n-3/2)\,T_s] + Q_s[(n-1)\,T_s]$

$\Phi_1 \rightarrow \quad Q_s[nT_s] = C_s V_i[nT_s]\,, \quad Q_I[nT_s] = Q_I[(n-1)\,T_s] + Q_s[(n-1)\,T_s]$

$\Phi_2 \rightarrow \quad Q_s[(n+1/2)\,T_s] = 0 \quad , \quad Q_I[(n+1/2)\,T_s] = Q_I[(n-1/2)\,T_s] + Q_s[n\,T_s]$

Switched-Capacitor Integrator
Output Sampled on $\phi 2$



$Q_I[(n+1/2)\,T_s] = Q_I[(n-1/2)\,T_s] + Q_s[n\,T_s]$

$V_{o2} = -Q_I/C_I \;\&\; V_i = Q_s/C_s \rightarrow C_I V_{o2}[(n+1/2)\,T_s] = C_I V_{o2}[(n-1/2)\,T_s] - C_s V_i[n\,T_s]$

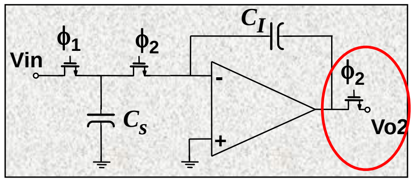
Using the z operator rules:

$\rightarrow C_I V_{o2} z^{1/2} = C_I V_{o2} z^{-1/2} - C_s V_i$

$$\frac{V_{o2}}{V_{in}}(z) = -\frac{C_s}{C_I} \times \frac{z^{-1/2}}{1-z^{-1}}$$

LDI Switched-Capacitor Integrator

LDI (Lossless Discrete Integrator) → same as DDI but output is sampled ½ clock cycle earlier



LDI

$$\frac{V_{o2}}{V_{in}}(z) = -\frac{C_s}{C_I} \times \boxed{\frac{z^{-1/2}}{1-z^{-1}}} \quad , \quad z = e^{j\omega T}$$

$$= -\frac{C_s}{C_I} \times \frac{e^{-j\omega T/2}}{1-e^{-j\omega T}} = \frac{C_s}{C_I} \times \frac{1}{e^{-j\omega T/2} - e^{+j\omega T/2}}$$

$$= -j\frac{C_s}{C_I} \times \frac{1}{2\sin(\omega T/2)}$$

$$= -\frac{C_s}{C_I}\frac{1}{j\omega T} \times \frac{\omega T/2}{\sin(\omega T/2)}$$

Ideal Integrator — Magnitude Error

No Phase Error!
For signals at frequencies << sampling freq.
→ Magnitude error negligible

Switched-Capacitor Filter Built with LDI Integrators