EECS 40, Fall 2006
Prof. Chang-Hasnain
Final Exam

December 12, 2006
Total Time Allotted: 180 minutes
Total Points: 350 pts

1. This is a closed book exam. However, you are allowed to bring one page (8.5” x 11”), single-sided notes PLUS your 1-page notes from midterm 1, 2 and 3.
2. No electronic devices, i.e. calculators, cell phones, computers, etc.
3. Slide rules are allowed.
4. SHOW all the steps on the exam. **Answers without steps will be given only a small percentage of credits.** Partial credits will be given if you have proper steps but no final answers.
5. **Remember to put down units.** Points will be taken off for answers without units.

Last (Family) Name:____

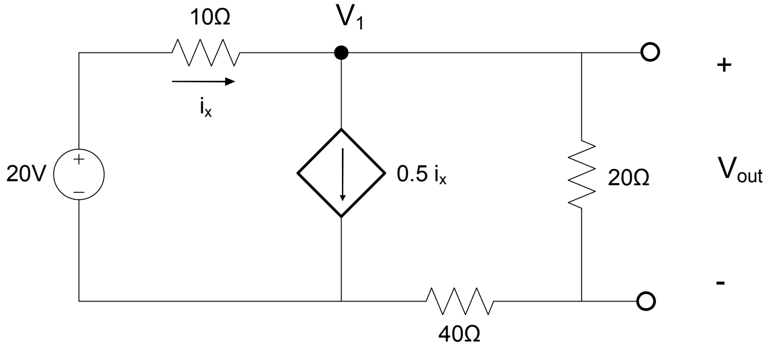
First Name: ____

Student ID: ___________________________Discussion Session: _______________

Signature: ____

	Score
1. [30 pts] Thevenin Equivalent Circuit	
2. [30 pts] Equivalent Impedances	
3. [50 pts] Semiconductor Device Physics	
4. [40 pts] Diodes/Op-Amps	
5. [60 pts] Second Order Circuits	
6. [140 pts] MOSFET	
TOTAL	

1. [30pts] Thevenin Equivalent Circuit



a) What is the current i_x through the 10Ω resistor?

$$KVL: 20V - (i_x \cdot 10\Omega) - (0.5i_x \cdot 20\Omega) - (0.5i_x \cdot 40\Omega) = 0$$
$$20V = 10\Omega \cdot i_x + 30\Omega \cdot i_x$$
$$i_x = \left(\frac{20V}{40\Omega}\right) = 0.5A$$

b) Find V_1 when there is no load connected to the ports a and b (as depicted by the figure above).

Voltage drop across the 2 resistors:
$$V_1 = (0.5i_x)(20\Omega + 40\Omega)$$
$$= (0.25A)(60\Omega)$$
$$= 15V$$

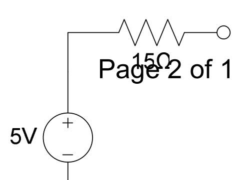
c) What is the open circuit voltage across a-b, V_{oc}?

Voltage across the 20Ω resistor:
$$V_{oc} = (0.5i_x)(20\Omega)$$
$$= (0.25A)(20\Omega)$$
$$= 5V$$

d) What is the short circuit current across a-b, I_{sc}?

$0.5i_x$ flows directly from V_1 to 40Ω resistor, KVL now reads:
$$20V - (i_x \cdot 10\Omega) - (0.5i_x \cdot 40\Omega) = 0$$
$$20V = 10\Omega \cdot i_x + 20\Omega \cdot i_x$$
$$i_x = \left(\frac{20V}{30\Omega}\right) = \frac{2}{3}A$$
$$i_{sc} = 0.5i_x = \frac{1}{3}A$$

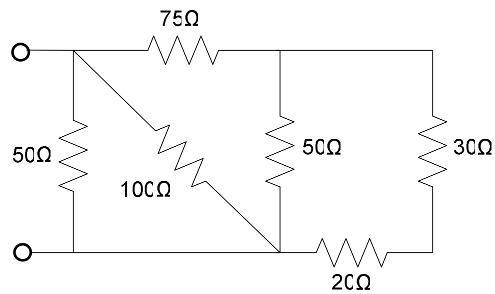
e) What is R_{th}? Draw the Thevenin equivalent circuit.



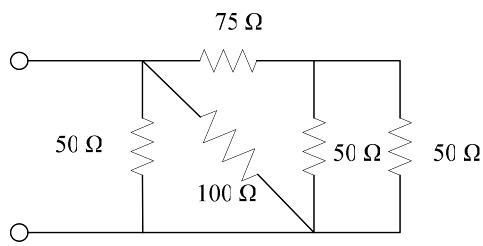
$$R_{Th} = \frac{V_{oc}}{i_{sc}}$$
$$= \frac{5V}{1/3A}$$
$$= 15\Omega$$

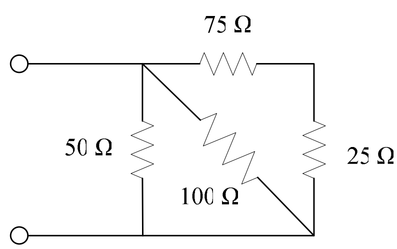
2. [30pts] Equivalent Impedances

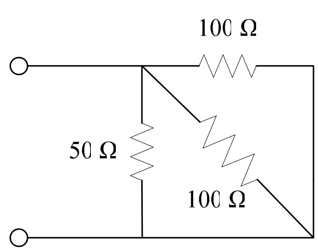
a) What is Req across the indicated nodes?

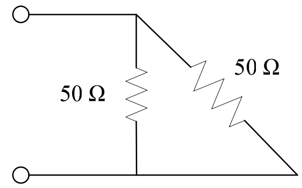


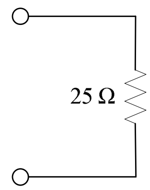
Break down 2 resistors at a time:





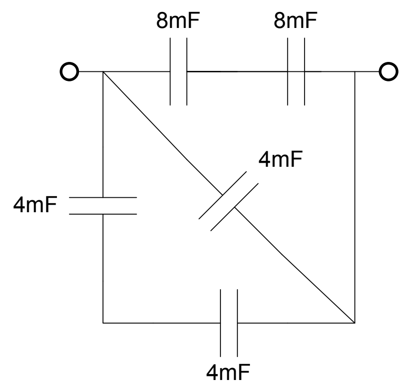




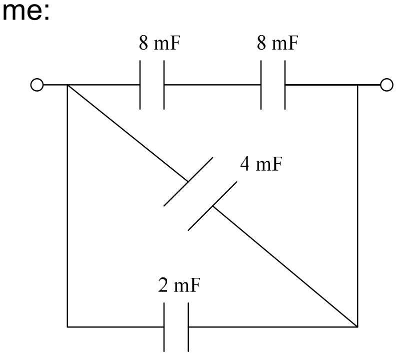


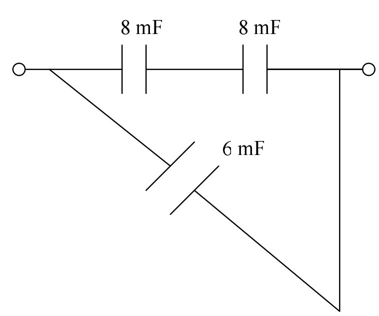
$R_{eq} = 25\,\Omega$

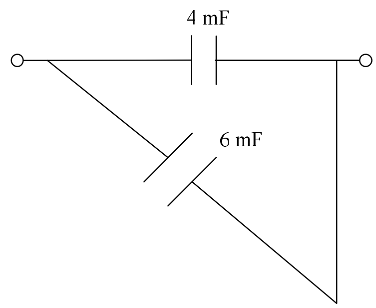
b) What is Zeq across the indicated nodes?

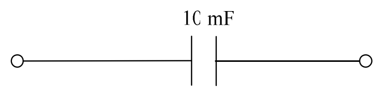


Break down 2 capacitors at a time:







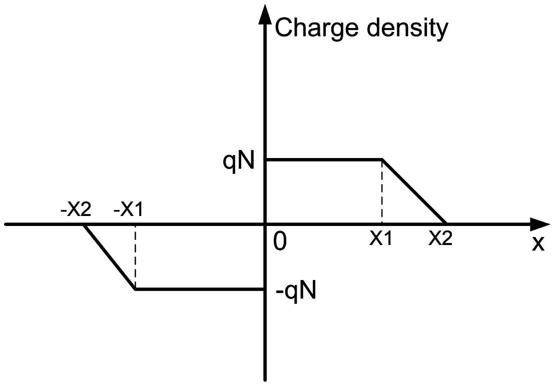


$$C_{eq} = 10mF$$

$$Z_{eq} = \frac{1}{j\omega C_{eq}} = \frac{100}{j\omega}\Omega \ \ or \ \ \frac{1}{10j\omega}k\Omega$$

3. [50 pts] Semiconductor Device Physics

Suppose we have the charge density distribution given by the figure. The permittivity of the material is ε.



a) (5 pts) Which side is p side and which side n side?

Left side (x<0): p
Right side (x>0): n

b) (5 pts) What is the direction of the built-in electric field?

The built-in electric field is in –x direction

c) (10 pts) What is the value for the electric field E(x) for $x \le -x_2$ and $x_2 \le x$?

E(x)=0 for both regions.

d) (10 pts) What is E(x) for $-x_2 \le x \le -x_1$?

From the figure, we can write down the expression for charge density in this region

$$\rho(x) = \frac{qN}{x_1 - x_2}(x + x_2) \ .$$

According to Gauss's law

$$\frac{d}{dx}E(x) = \frac{\rho(x)}{\varepsilon},$$

we can arrive at the following result by carrying out the integration

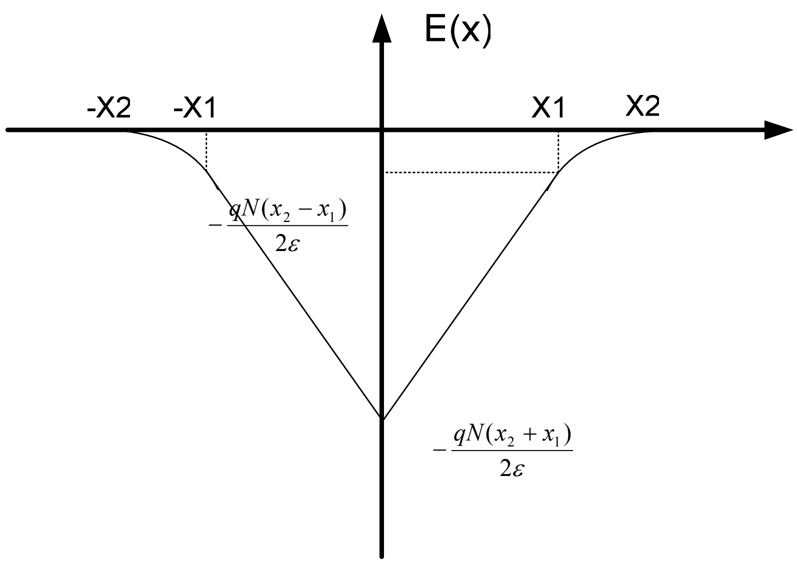
$$E(x) = \frac{qN}{\varepsilon(x_1 - x_2)}\frac{(x + x_2)^2}{2}$$

e) (10 pts) What is E_{max}? Here E_{max} represents the maximum absolute value.

The electric field reaches its maximum value at the x=0. The magnitude can be calculated by evaluating the area enveloped by the curve on the left hand side, and divide the area by ε. Therefore,

$$E_{\max} = \frac{qN(x_2 - x_1)}{2\varepsilon} + \frac{qNx_1}{\varepsilon}$$
$$= \frac{qN(x_2 + x_1)}{2\varepsilon}$$

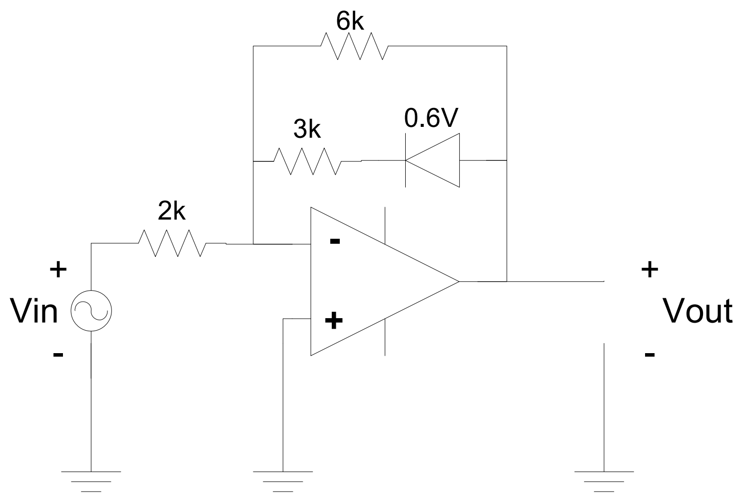
f) (10 pts) Sketch E(x) vs. X. Label your plot. Also describe the curves.



E(x) is symmetrical with respect to x. It is a quadratic function when $x \in [-x_2, -x_1] \cup [x_1, x_2]$, a linear function when $x \in [-x_1, 0] \cup [0, x_1]$, and zero anywhere else.

4. [40 pts] Diodes/Op-Amps

Assume the op-amp is ideal.
Use the large-signal model for the diode with turn-on voltage of 0.6V.



(a) (10 pts) When the diode is on, what is V_{out} as a function of V_{in}?

When the diode is on, it acts like a 0.6V source.
By the summing-point constraint, V- = V+ = 0V.
By applying KCL at V-, we get Vin/2k + Vout/6k + (Vout-0.6V)/3k = 0V.
Solving yields Vout = 0.4 - Vin.

(b) (10 pts) When the diode is off, what is V_{out} as a function of V_{in}?

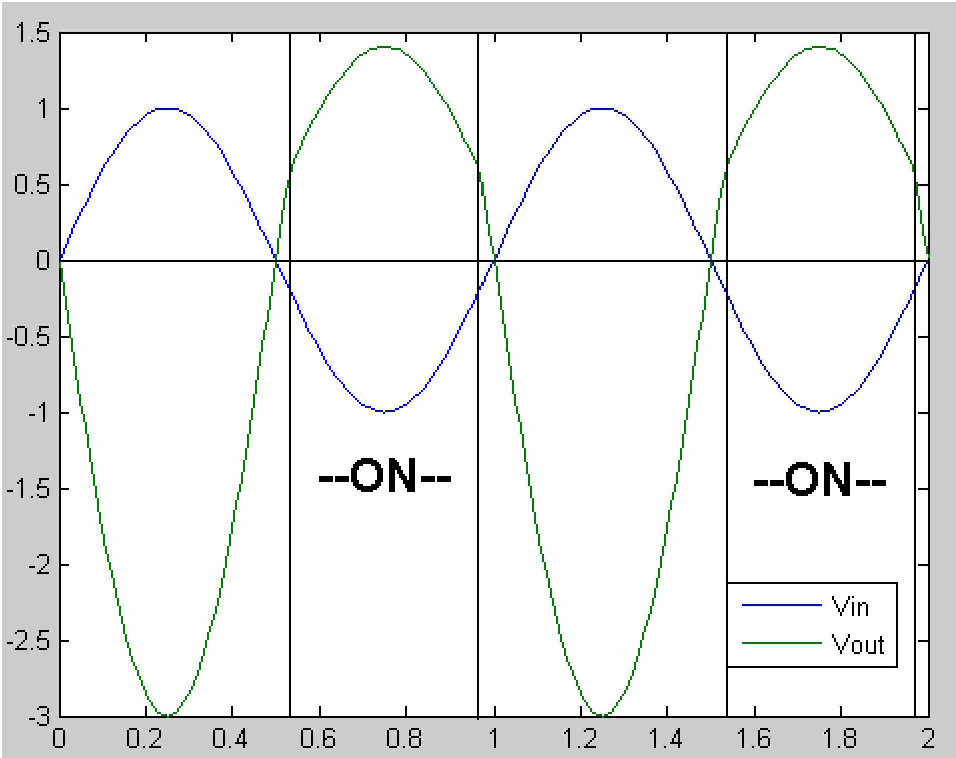
When the diode is off, it acts as a open circuit.
Now KCL at V- gives Vin/2k + Vout/6k = 0V.
Solving yields Vout = -3Vin.
Also notice that this is just a regular inverting amplifier.

(c) (5 pts) For what range of V_{in} will the diode be on?

For the diode to be on, there must be positive current through the diode or, alternatively, through the 3k resistor (since they are in series).
This means there must be a voltage drop across the resistor must be positive.
So that gives us Vout - 0.6V > 0V.
Substituting in our expression from (a), we get 0.4V - Vin - 0.6V > 0V.
Solving yields Vin < -0.2V.

Alternatively, we can see that for the diode to be off, we need the diode voltage to be less than the threshold of 0.6V, which gives us Vout -0 < 0.6V.
Plugging in our expression from (b) gives us -3Vin < 0.6V.
Solving yields Vin > -0.2V.
This means the diode will be on for Vin < -0.2V

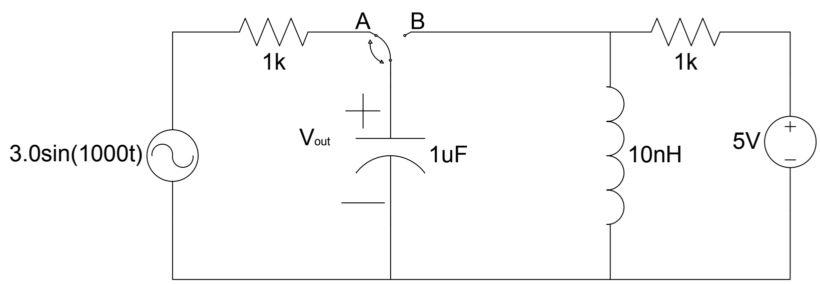
(d) (15 pts) Complete the following plot for Vout. Indicate which regions the diode is on. Make sure to label your axes.



The key points are that the diode switches when Vin = -0.2V and Vout = 0.6V.
The negative peak is -3V, and the positive peak is 1.4V.
Also notice that When Vin = 0V, Vout = 0V.

5. [60 pts] Second Order Circuits

Suppose the following circuit. Assume that the switch has been in position A for an infinitely long time before t = 0. $v_{in} = 3.0\sin(1000t)$



a) (10 pts) Write the input voltage in the Phasor format, $\mathbf{V_{in}}$?

Noting the input is a sin, we apply the identity:

$\cos(\omega t - \pi/2) = \sin(\omega t)$

Thus, $V_{in} = 3.0\angle -\frac{\pi}{2}$

This angle is equivalent to -90 degrees.

b) (10 pts) Assuming an AC steady state condition and the switch is in the A position. What is the output voltage in Phasor format, $\mathbf{V_{out}}$?

$$V_{out} = \frac{Z_c}{Z_c + R}V_{in} = \frac{1}{1 + j\omega RC}V_{in} = \frac{1}{1+j}3.0\angle -\frac{\pi}{2} = \frac{3.0\angle -\frac{\pi}{2}}{\sqrt{2}\angle\frac{\pi}{4}} = \frac{3}{\sqrt{2}}\angle -\frac{3\pi}{4}$$

c) (10 pts) What is the output voltage in time domain, $v_{out}(t)$?

By inspection, $V_{out} = \frac{3}{\sqrt{2}}\cos\left(1000t - \frac{3\pi}{4}\right)$

This angle is equivalent to -135 degrees or +225 degrees.

d) (20 pts) At time t = π ms, the switch flips from position A to B, disconnecting the AC source from the output. Provide a differential equation describing v_{out} with respect to time. The solution should be a second-order differential equation of V_{out}.

$$\frac{V_{out} - 5V}{R} + C\frac{dV_{out}}{dt} + \frac{1}{L}\int V_{out}dt = 0$$

Taking the derivative,

$$\frac{d^2V_{out}}{dt^2} + \frac{1}{RC}\frac{dV_{out}}{dt} + \frac{1}{LC}V_{out} = 0$$

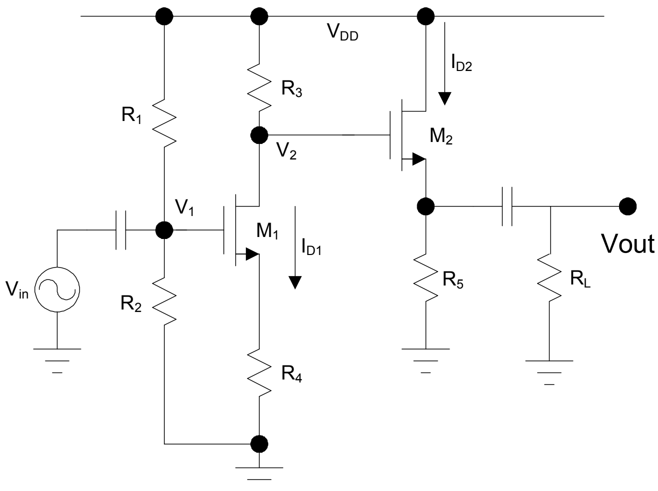
Plugging in values,

$$\frac{d^2V_{out}}{dt^2} + 10^3\frac{dV_{out}}{dt} + 10^{14}V_{out} = 0$$

e) (10 pts) What is the V_{out} at very large t?
In steady state, the inductor becomes a short circuit (while it is in parallel with the capacitor).
So, $V_{out} \rightarrow 0V$.

6. [140 pts] MOSFET

In the circuit below, V_{DD}=5V, and both transistors are identical with V_{to}=1V, K=0.1 mA/V^2.



(A) (50 pts) DC analysis

a) (5 pts) What is V_1? Given $R_1 = R_2 = 5k\Omega$

$$V_1 = Vdd\frac{R2}{R1+R2} = 5\frac{5k\Omega}{5k\Omega+5k\Omega} = 2.5V$$

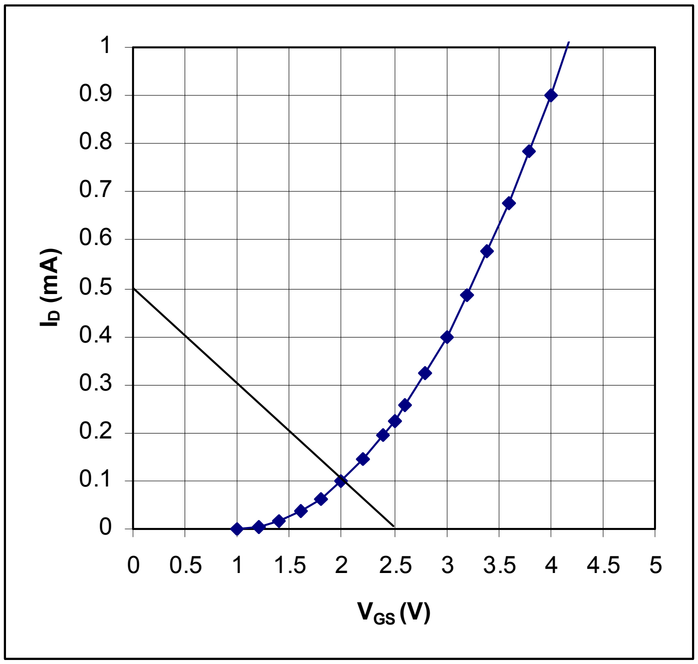
b) (10 pts) What is I_{D1}? Given $R_3 = R_4 = 5k\Omega$. Write down the necessary equation(s) and draw load line in the following graph.

$$V_1 - I_D R_4 = V_{GS}$$
$$2.5 - I_D * 5k = V_{GS}$$
$$I_D = \frac{2.5 - V_{GS}}{5k} = \frac{2.5 - V_{GS}}{5} mA$$

From the Load Line

I_D = 0.1 mA

Vgs = 2V



c) (5 pts) Which mode is transistor M1 in?

Saturation

d) (5 pts) What is V_{DS1}?

$$V_{DS1} = Vdd - I_{D1}R3 - I_{D1}R4 = 5 - .1m * 5k - .1m * 5k = 5 - 0.5 - 0.5 = 4V$$

e) (5 pts) What is V_2?

$$V_{DS1} = Vdd - I_{D1}R3 = 5 - .1m * 5k = 4.5V$$

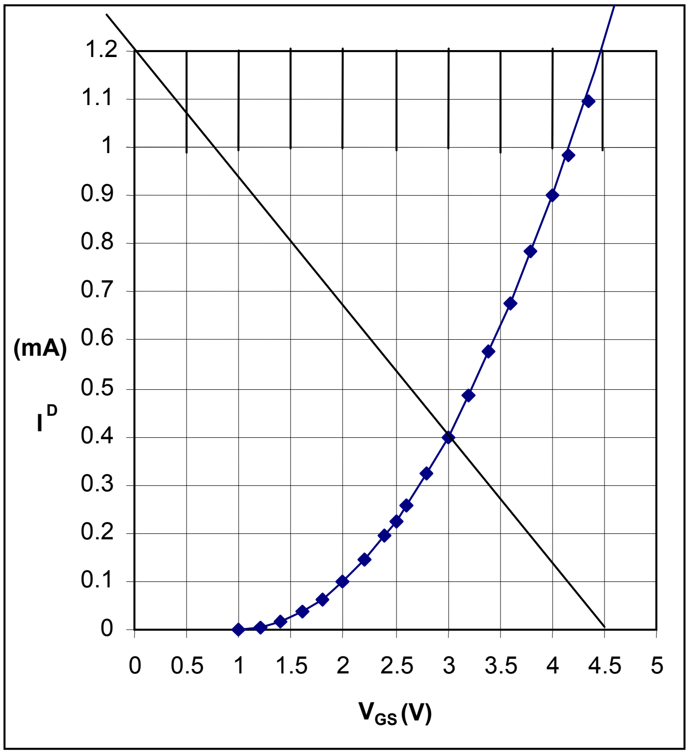
f) (5 pts) For transistor M2, write down the equations for V_{DS2} and V_{GS2}. What mode is this transistor in?

$$V_{DS1} = Vdd - I_{D2}R5$$
$$V_{GS2} = 4.5 - I_{D2}R5$$

The transistor is also in saturation.

g) (10 pts) What is I_{D2}? Given R_5 = 3.75kΩ. Write down the necessary equation(s) and draw load line in the following graph.

$$V_{GS2} = V_2 - I_{D2}R5$$
$$I_{D2} = \frac{4.5 - V_{GS2}}{3.75k} = \frac{4.5 - V_{GS2}}{3.75} mA$$
$$I_{D2} = 0.4mA @ V_{GS} = 3V$$

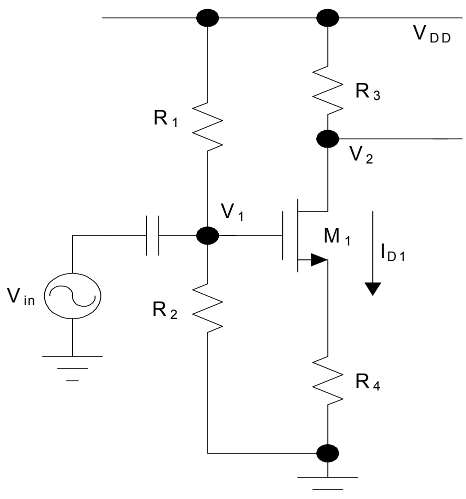


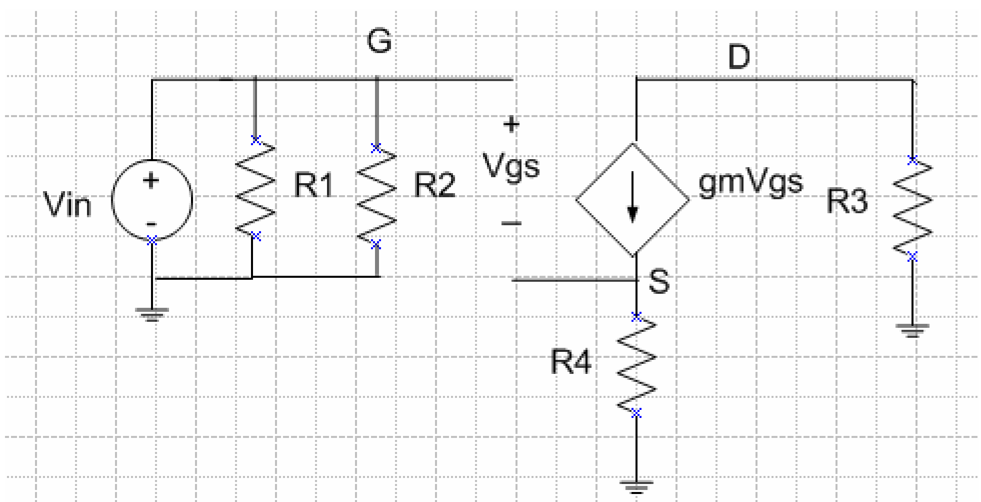
h) (5 pts) What is V_{DS2}?

$$V_{DS1} = Vdd - I_{D1}R5 = 5 - .4m * 3.75k = 3.5V$$

(B) (90 pts) AC small-signal analysis:

a) (20 pts) For transistor M1, the small signal gain $A_{V1} = V_2/V_{in}$. Draw the small signal model for M1. Consider $r_d \to \infty$ for both transistors.





b) (15 pts) What is A_{V1}?

To find Av, we first find GM and then find Rout.
To find GM, we short the output (V2) and apply a test voltage at the input.
Applying KCL at V2 and VS (source of M1):

$$\frac{Vs}{R4} - gm(Vt - Vs) = 0$$

$$Vs = \frac{gmR4 * Vt}{1 + gmR4}$$

$$Iout + gm(Vt - Vs) = 0$$

$$Iout = -gm(Vt - \frac{gmR4 * Vt}{1 + gmR4}) = -gmVt(1 - \frac{gmR4 * Vt}{1 + gmR4}) = -gmVt(\frac{1 + gmR4}{1 + gmR4} - \frac{gmR4}{1 + gmR4})$$

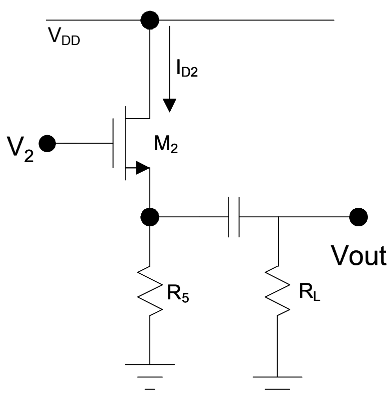
$$Iout = \frac{-gmVt}{1 + gmR4}$$

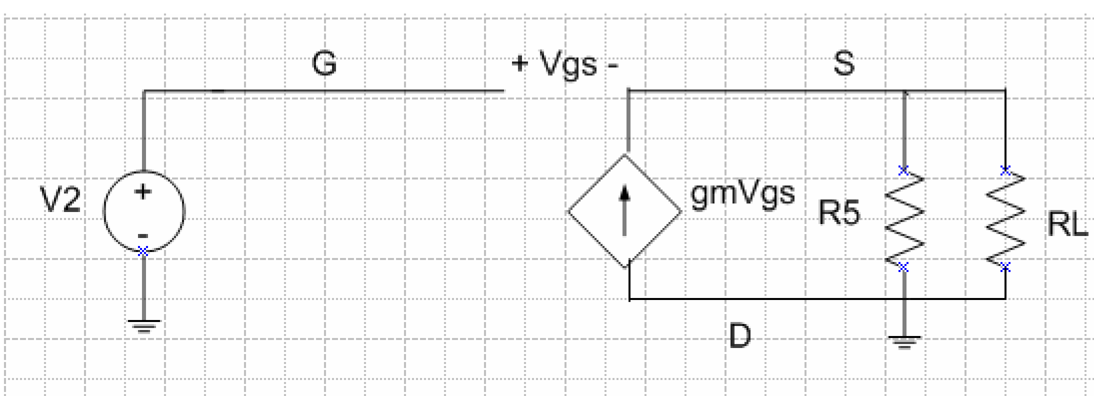
$$GM = \frac{Iout}{Vt} = \frac{-gm}{1 + gmR4}$$

Ro is simply R3, because rd is infinity.
Therefore the gain Av1:

Av1 = GM*Ro = $-\frac{gmR3}{1 + gmR4}$

c) (20 pts) For transistor M2, the small signal gain $A_{V2} = V_{out}/V_2$. Draw the small signal model for M2.





d) (15 pts) What is A_{V2}?

To find the gain, we find GM and Ro.
If we short the output VS (the source of M2), and apply a test voltage at the gate (V2), we see that GM = Iout / Vt = gm.

$$Iout - gm(Vgs) = Iout - gm(V2 - 0)$$

$$Iout = gmV2$$

$$GM2 = \frac{Iout}{V2} = gm$$

To find Ro, we short the input (V2) and apply a test voltage at the output VS. Then we measure the current flowing into the output node.
First, combine R5 and RL together to form RL'.

Using KCL:

$$-Iout + \frac{Vt}{RL'} - gm(0 - Vt) = 0$$

$$Iout = Vt(\frac{1}{RL'} + gm)$$

$$RO2 = \frac{Vt}{Iout} = \frac{1}{(\frac{1}{RL'} + gm)} = \frac{RL'}{1 + gmRL'}$$

Av2=GM2 * RO2

Av2= $\frac{gmRL'}{1 + gmRL'}$

e) (5 pts) What is the overall gain A_{V2} = V_{out}/V_{in}?

Av = Av1 * Av2 = $-\frac{gmR3}{1 + gmR4} \frac{gmRL'}{1 + gmRL'}$

f) (5 pts) What is the input impedance of the overall circuit?

The input impedance is simply R1 || R2 because the input impedance of a NMOS is infinity.
Rin = R1 || R2 = 2.5kΩ

g) (10 pts) What is the output impedance of the overall circuit?

As found above in part d), the output impedance of the overall circuit is the same as the output impedance of the second stage (consisting of M2).

Zo = RL'|| (1/gm) = $\frac{RL'}{1 + gmRL'}$