Quiz #1, EECS 40, Fall 2006
Total: 100 pts and 20 pts Bonus

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1. (14 pts) Consider a transfer function, $H(f) = \dfrac{-jQ\left(\dfrac{f_0}{f}\right)}{1 + jQ\left(\dfrac{f}{f_0} - \dfrac{f_0}{f}\right)}$

(a) What is $|H(f)|^2$?

$$|H(f)|^2 = \frac{\left(Q\dfrac{f_0}{f}\right)^2}{1^2 + Q^2\left(\dfrac{f}{f_0} - \dfrac{f_0}{f}\right)^2}$$

(b) What is the phase $\angle H(f)$?

$$\angle\left(-jQ\frac{f_0}{f}\right) = -\frac{\pi}{2}$$

$$\angle\left(1 + jQ\left(\frac{f}{f_0} - \frac{f_0}{f}\right)\right) = \tan^{-1}\left(Q\left(\frac{f}{f_0} - \frac{f_0}{f}\right)\right)$$

$$\angle H(f) = -\frac{\pi}{2} - \tan^{-1}\left(Q\left(\frac{f}{f_0} - \frac{f_0}{f}\right)\right)$$

2. (50 pts) Bode Magnitude Plot: Use the same transfer function in 1, $y = 10\log|H(f)|^2$

(a) What is the break frequency (or resonance frequency in the text book)?

$$f_B = f_0$$

(b) What is y at the break frequency?

$$y = 10\log|H(f)|^2 = 10\log\left(\frac{\left(Q\frac{f_0}{f_0}\right)^2}{1^2 + Q^2\left(\frac{f_0}{f_0} - \frac{f_0}{f_0}\right)^2}\right)$$
$$= 10\log\left(\frac{Q^2}{1+Q^2(0)}\right) = 10\log\left(Q^2\right)$$
$$= 20\log Q$$

(c) What is y for very small f?

The f term in the numerator is by itself, cannot be ignored. The f^{-1} term in the denominator dominates at very small f.

$$y = 10\log\frac{\left(Q\frac{f_0}{f}\right)^2}{Q^2\left(\frac{f_0}{f}\right)^2} = 10\log 1 = 0\ dB$$

(d) What is y for very large f? What is the slope of this portion?

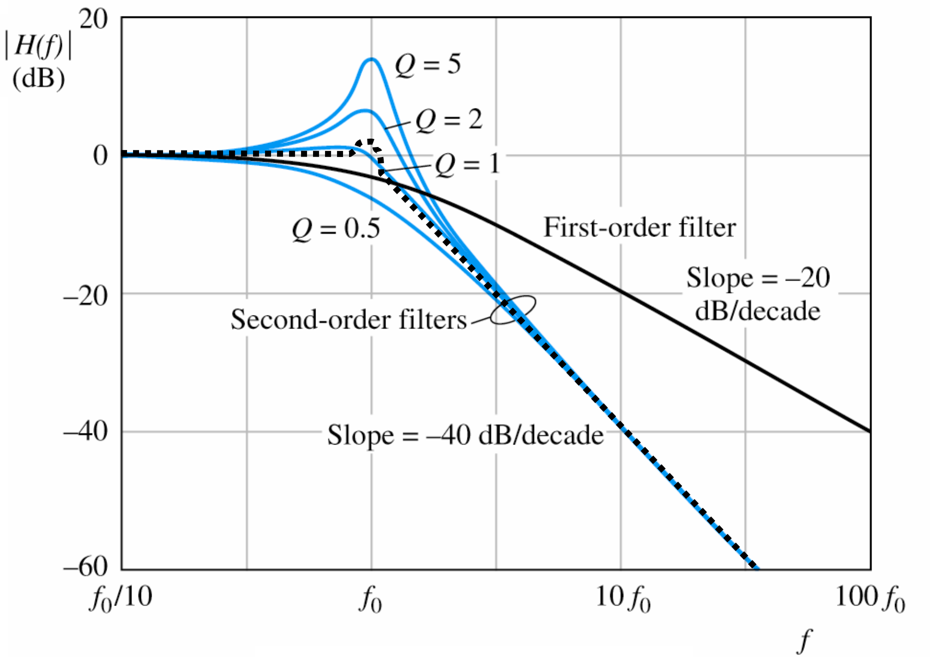
Again, we cannot ignore the lone f term in the numerator. At large f, the f^{-1} term in the denominator becomes 0. The f term dominates.

$$y = 10\log\frac{\left(Q\frac{f_0}{f}\right)^2}{Q^2\left(\frac{f}{f_0}\right)^2} = 10\log\left(\frac{f_0}{f}\right)^4$$
$$= 40\log f_0 - 40\log f$$

Thus, the slope is -40 dB/decade.

(e) Sketch the Bode magnitude plot.

The figure below shows the Bode magnitude plot (blue lines) for different values of Q. The humps at f_0 have the value 20 log Q. The thick dashed line represents the approximate values, emphasizing the linear portions of the Bode plot.



3. (36 pts) Bode Magnitude Plot: Use the same transfer function in 1, let $y = 10\log|H(f)|^2$

(a) At the break frequency, $\angle H(f) = ?$

$$\angle H(f) = -\frac{\pi}{2} - \tan^{-1}\left(Q\left(\frac{f_0}{f_0} - \frac{f_0}{f_0}\right)\right)$$
$$= -\frac{\pi}{2} - \tan^{-1}(0) = -\frac{\pi}{2}$$

(b) In radians, what does $\angle H(f)$ approach as $f \to \infty$?

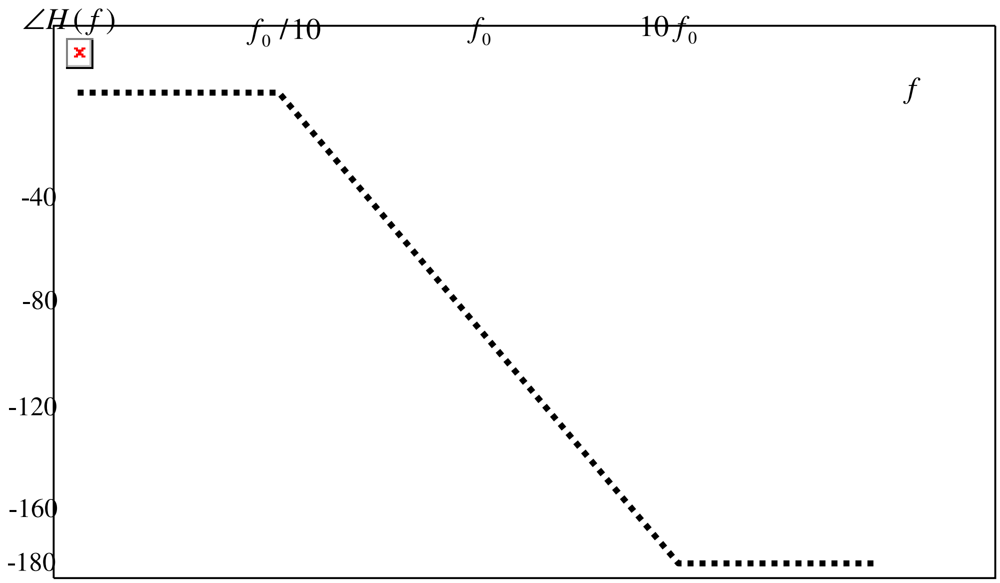
$$\angle H(f) = -\frac{\pi}{2} - \tan^{-1}(\infty)$$
$$= -\frac{\pi}{2} - \frac{\pi}{2} = -\pi$$

(c) In radians, what does $\angle H(f)$ approach as $f \to 0$?

$$\angle H(f) = -\frac{\pi}{2} - \tan^{-1}(-\infty)$$
$$= -\frac{\pi}{2} - \left(-\frac{\pi}{2}\right) = 0$$

(d) Sketch the Bode Phase plot.

The curve shows the actual values, the dotted line is the Bode approximation.



3. (20 pts) Bonus: Given the transfer function $H(f) = \dfrac{jQ(\frac{f}{f_0} - \frac{f_0}{f})}{1 + jQ(\frac{f}{f_0} - \frac{f_0}{f})}$

What kind of filter is this? Sketch the Bode Magnitude Plot.

This is a notch filter, with V_{out} taken across the L and C in a series RLC circuit.

The slope of the response does not take a constant asymptotic slope around the resonant frequency, but does approach $-\infty$ at the resonant frequency. The width of the stop band decreases with higher Q.