Lecture #13

ANNOUNCEMENTS

- Graded HW can be picked up in 278 Cory

OUTLINE

- Mutual inductance
- First-order circuits
- Natural response of an RL circuit

Reading

Chapter 6.4, Chapter 7.1

Mutual Inductance

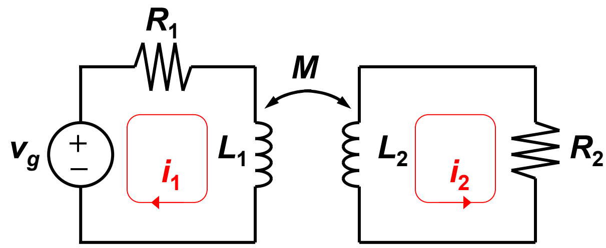
- Mutual inductance occurs when two circuits are arranged so that the change in current in one causes a voltage drop to be induced in the other.

  Example: Consider inductor L_1 in the circuit below

  – self-induced voltage is $L_1(di_1/dt)$

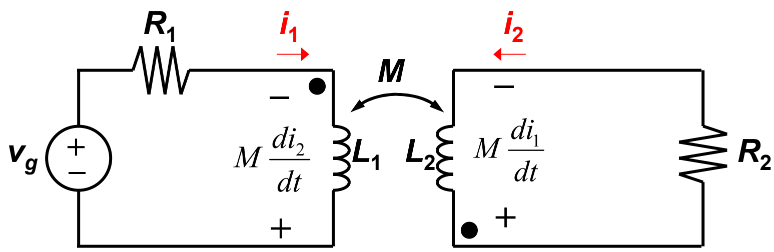
  – mutually induced voltage is $M(di_2/dt)$

  …but what is the polarity of this voltage?



The "Dot Convention"

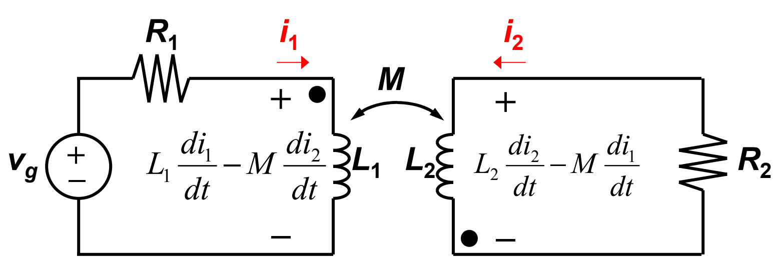
- If a current "enters" the dotted terminal of a coil, the reference polarity of the voltage induced in the other coil is positive at its dotted terminal.
- If a current "leaves" the dotted terminal of a coil, the reference polarity of the voltage induced in the other coil is negative at its dotted terminal.



Induced Voltage Drop

- The total induced voltage drop across an inductor is equal to the sum of the self-induced voltage and the mutually induced voltage

Example (cont'd): Apply KVL to loops



Relationship between *M* and L_1, L_2

- The value of mutual inductance is a function of the self-inductances:

$$M = k\sqrt{L_1 L_2}$$

(This equation is valid only if ***L*** is a constant function of ***i***)

where ***k*** is the ***coefficient of coupling***

$$0 \le k \le 1$$

Total Energy Stored in Inductors

- The total energy stored in inductors L_1 and L_2 is

$$w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 + M i_1 i_2$$

if each coil current enters the dotted terminal
(or if each coil current leaves the dotted terminal)

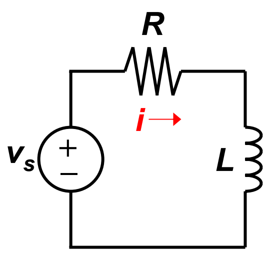
- The total energy stored in inductors L_1 and L_2 is

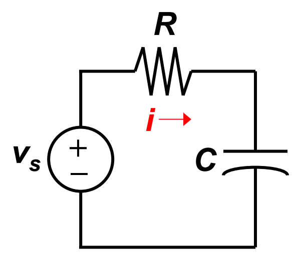
$$w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 - M i_1 i_2$$

if one coil current enters the dotted terminal and the other coil current leaves the dotted terminal

First-Order Circuits

- A circuit which contains only sources, resistors and an inductor is called an ***RL circuit***.
- A circuit which contains only sources, resistors and a capacitor is called an ***RC circuit***.
- RL and RC circuits are called first-order circuits because their voltages and currents are described by first-order differential equations.





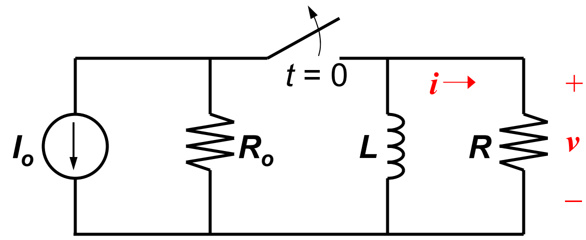
Natural Response & Step Response

- The ***natural response*** of an RL or RC circuit is its behavior (*i.e.* current and voltage) when stored energy in the inductor or capacitor is suddenly released to the resistive network.

- The ***step response*** of an RL or RC circuit is its behavior when voltage or current is suddenly applied (by a source) to the inductor or capacitor.

Natural Response of an RL Circuit

- Consider the following circuit, for which the switch is closed for $t < 0$, and then opened at $t = 0$:



Notation:

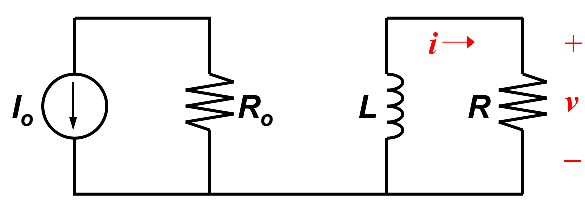
0^- is used to denote the time just prior to switching

0^+ is used to denote the time immediately after switching

- The current flowing in the inductor at $t = 0^-$ is I_o

Solving for the Current ($t \geq 0$)

- For t > 0, the circuit reduces to

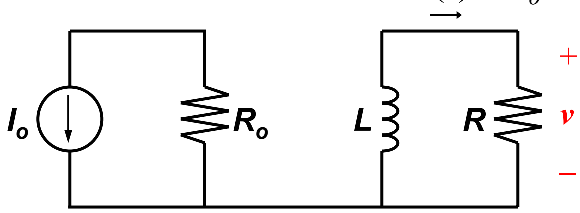


- Applying KVL to the LR circuit:

- Solution: $i(t) = i(0)e^{-(R/L)t}$

Solving for the Voltage ($t > 0$)

$$i(t) = I_o e^{-(R/L)t}$$



- Note that the voltage changes abruptly:

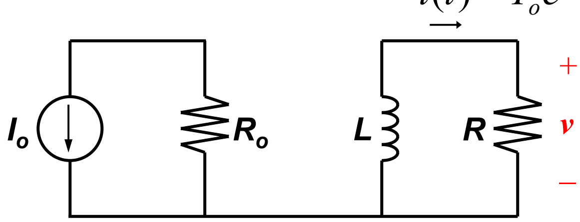
$$v(0^-) = 0$$

$$\text{for } t \geq 0,\ \ v(t) = iR = I_o R e^{-(R/L)t}$$

$$\Rightarrow\ v(0^+) = I_o R$$

Solving for Power and Energy Delivered ($t > 0$)

$$i(t) = I_o e^{-(R/L)t}$$



$$p = i^2 R = I_0^2 R e^{-2(R/L)t}$$

$$w = \int_0^t p(x)dx = \int_0^t I_0^2 R e^{-2(R/L)x} dx$$

$$= \frac{1}{2} L I_0^2 \left(1 - e^{-2(R/L)t}\right)$$

Time Constant τ

- In the example, we found that

$$i(t) = I_o e^{-(R/L)t} \quad \text{and} \quad v(t) = I_o R e^{-(R/L)t}$$

- Define the ***time constant*** $\tau = \frac{L}{R}$
  - At $t = \tau$, the current has reduced to $1/e$ (~0.37) of its initial value.
  - At $t = 5\tau$, the current has reduced to less than 1% of its initial value.

Transient *vs.* Steady-State Response

- The momentary behavior of a circuit (in response to a change in voltage or current) is referred to as its ***transient response***.

- The behavior of a circuit a long time (many time constants) after the change in voltage or current is called the ***steady-state response***.