Network Theorems & Devices

Science is emphatically an important part of culture today, as scientific knowledge and its applications continue to transform the world, and condition every aspect of the relations between men and nations.

- Vannevar Bush

Contents:

Educational Objectives

In this lab, you will test the predictions of two important network theorems as well as experimentally verify:

Thevenin's Theorem

The Maximum Power Transfer Theorem

Background Information

1. Thevenin's Theorem

Any linear active one-port network can be replaced by a single voltage source, equal to the open-circuit voltage of the one-port, in series with the network in which all independent sources are set to zero.

Proof:

Consider the linear circuit represented in the top panel of Figure 1 (LAN: linear active network) which contains assorted sources and resistors, and has a single "port" by which it can be connected to the outside world. We attach a load resistor R_L at the port as shown, and observe that R_L draws load current i_1 and voltage v_1 from the network.

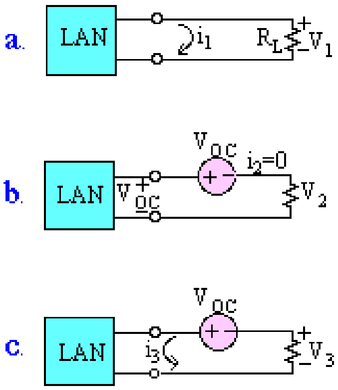


Figure 1: Development of Thevenin's Theorem

In Figure 1b, a voltage source has been added in series with the load so as to oppose the current flow. We have increased the voltage of this source until the current i_2 is brought to zero. When zero current is flowing at a port, we say the port is "open-circuited". Therefore, in Figure 1b, the port voltage (not V_2) is the open-circuit voltage, V_{oc} of the LAN. By Kirchoff's Voltage Law, this is equal and opposite to the voltage of the external source we have applied.

In Figure 1c, all independent sources inside the LAN have been turned to zero, creating a linear passive network or LPN. This means that voltage sources have been replaced by short circuits (zero voltage) and current sources have been replaced by open-circuits (zero current). The only source now active is the external source of the center panel, V_{oc}. Since this is the only active source, we now expect the current to flow in the direction shown as i_3 in the bottom panel.

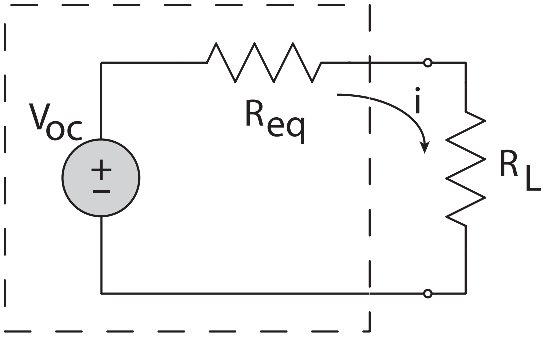
According to Thevenin's theorem, the load should receive the same current in the bottom panel 1c as it did in the top panel 1a. To prove that this is the case, apply the superposition principle. In the top panel, with the LAN sources active and the V_{oc} source zero, current i_1 flows in the load. In the bottom panel, with the LAN sources all zero and the V_{oc} source active, the load current is i_3. Now, according to the superposition principle, the current flow when all sources are active should be the sum of the currents produced by each acting alone. But this sum is zero according to the center panel. Therefore, i_1 and i_3 must be equal in magnitude, but opposite in direction. If we reverse the polarity of the V_{oc} source in the third panel, the load receives exactly the current and voltage it did from the original network.

2. Maximum Power Transfer Theorem

To draw the maximum power from a resistive LAN, the load resistance should be set equal to the Thevenin equivalent resistance of the LAN.

Proof:

Consider a LAN with Thevenin equivalent resistance R_{eq} to which a load resistor, R_L is connected, as shown in Figure 2. The power to R_L is $P = i^2 R_L$



$P_{R_L} = i^2 R_L$

Figure 2: R_L drawing power from the Thevenin equivalent representation of a LAN.

We want to find the value of R_L which will draw maximum power from the network. To determine this, we will first obtain an expression for the power P in terms of R_L by substituting for the current *i* in the above expression the following fractional form:

$$i = \frac{V}{R_{eq} + R_L}$$

Thus the square of the current can be written:

$$i^2 = \frac{V^2}{(R_{eq} + R_L)^2}$$

Using this form the power *P* can be written:

$$P = i^2 R_L = \frac{V^2 R_L}{(R_{eq} + R_L)^2}$$

To find the value of the load resistance R_L for which the power *P* is a maximum, we differentiate *P* with respect to R_L and set the result equal to zero. This is just an application of Fermat's Theorem in Calculus which says that the derivative must be zero at the maxima and minima of a differentiable function. After differentiating, we obtain the result that:

$$\frac{dP}{dR_L} = \frac{V^2 (R_{eq} - R_L)}{(R_{eq} + R_L)^3} = 0$$

This equation shows that $R_{eq} = R_L$, thus proving the maximum power transfer theorem.

This theorem has many practical applications. One which you may be familiar with is the "matching" of components in an audio system. For example, if the output impedance of an amplifier is 8 Ohms, it should be used with an 8 Ohm loudspeaker to achieve maximum power output.

Maximum Power Transfer Without Calculus

The maximum power transfer theorem can also be derived without calculus by completing the square. Recall:

$$P = \frac{V^2 R_L}{(R_{eq} + R_L)^2}$$

This expression is a rational function in the variable R_L, when V and R_{eq} are considered fixed.

We will work instead with the variable $i = \frac{V}{R_{eq} + R_L}$ which implies that $R_L = \frac{V}{i} - R_{eq}$. Substituting these expressions into the equation for power gives:

$$P = -R_{eq}\left(i^2 - \frac{Vi}{R_{eq}} \right)$$

Notice that the expression for power is a simple quadratic function of the current i and no longer a rational function. The maximum of a quadratic function can be found by completing the square. Applying this technique to the expression for power we obtain:

$$P = -R_{eq}\left(\left(i - \frac{V}{2R_{eq}} \right)^2 - \frac{V^2}{4R_{eq}{}^2} \right)$$

The parabola opens downwards and achieves a maximum of $P_{\max} = \frac{V^2}{4R_{eq}}$ when $i = \frac{V}{2R_{eq}}$.

Since $i = \frac{V}{R_{eq} + R_L} = \frac{V}{2R_{eq}}$, we see by comparing the two denominators, that the maximum power occurs when $R_L = R_{eq}$.

Problem Formulation

There are three parts to this experiment. In part 1 you will first build a LAN and then determine experimentally the value of the load resistance which will draw the maximum power. In part 2, you will determine through measurements, the values of the Thevenin open-circuit voltage and equivalent resistance of your LAN, and then build a Thevenin equivalent circuit. In part 3, you will verify the maximum power theorem and Thevenin's theorem using the results of parts 1 and 2.

Procedures

The circuit you will use as the LAN is shown in Figure 3. You will be given 5 resistors, each of which can be connected in any of the positions (except one resistor with longer leads which must be used to reach between non-adjacent terminal posts). The 10 volt source is the lab power supply (set to +10V). The load resistor R_L is a resistor box. You may want to use the color code on the resistors to try to calculate the expected Thevenin values. However, the required calculations are a little difficult for this configuration since no two resistors are in series or in parallel.

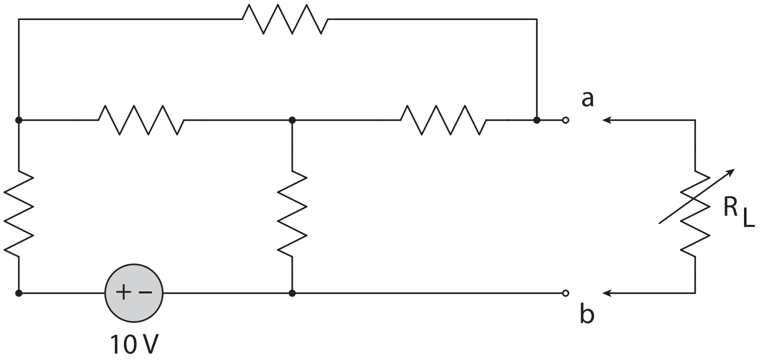


Figure 3: Linear active circuit with port at terminals a-b.

Part 1 - Maximum Power Transfer Measurement

1.1 Construct the circuit of Figure 3 using the lab terminal board as shown in Figure 4.

1.2 Vary the load resistor from 15.5 kilo-ohms to 0.5 kilo-ohms in 1 kilo-ohm steps, and record the voltage across R_L. Power consumed by R_L in each case can be calculated using: $P = V^2/R_L$.

1.3 Create a plot of power consumed against load resistance R_L. Determine where a maximum occurs and select 5 data points around the peak (two to the left and two to the right) and take measurements with 500 Ohm increments in the interval defined by this range. Locate the peak under this resolution.

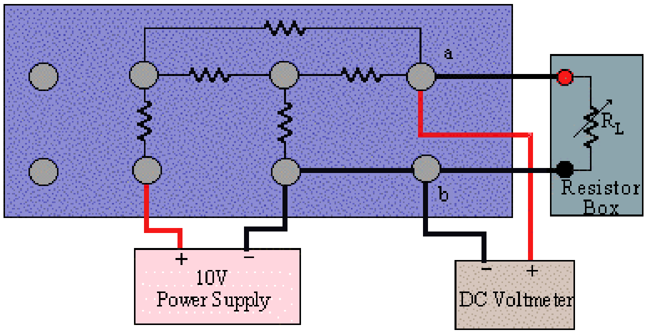


Figure 4: Lab implementation of the circuit of Figure 3.

Part 2 - Thevenin Equivalent Measurements

2.1 Disconnect the load resistor box and measure and record the LAN's open-circuit voltage across terminals a-b.

2.2 Disconnect the supply voltage and replace it by a short circuit as shown in Figure 5.

Warning: Be sure to disconnect the supply before adding the short!

Use the lab ohmmeter to measure the resistance looking back into terminals a-b with the source replaced by a short circuit.

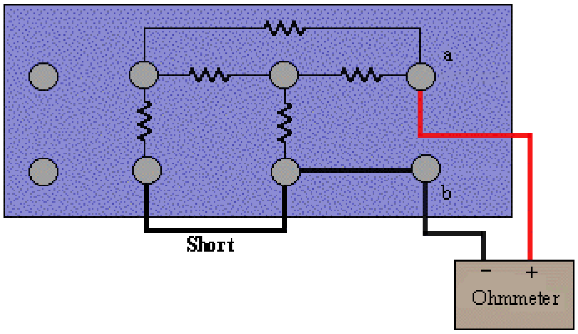


Figure 5: Measurement of the Thevenin equivalent resistance.

Part 3 - Verification of the Theorems

3.1 Compare the R_L value for the peak you located in Part 1.3 to the Thevenin resistance measured in Part 2.

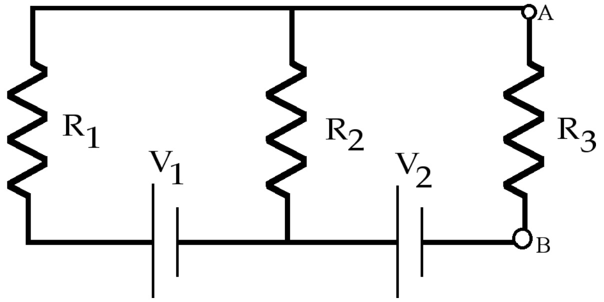
3.2 Using the open-circuit voltage and equivalent resistance values measured in Part 2, construct a Thevenin Equivalent circuit. That is, set the Lab power supply to provide the open-circuit voltage value, and set a resistor box to R_{eq}. Connect these two in series and apply this equivalent circuit to a second resistor box representing a load. Set the load resistance to a few of the values used in Part 1 and compare the voltage provided by the equivalent circuit to that read in part 1 for the same load.

Instrumentation Lab 4 Quiz A

Name **ID #** **Section** **Grade:**

1. For the circuit below, calculate the Thevenin equivalent voltage V_{th} across R_3.

$R_1 = 5\ \text{k}\Omega$ $R_2 = 2\ \text{k}\Omega$ $V_1 = 20\ \text{V}$ $V_2 = 10\ \text{V}$



2. For the circuit above, calculate the Thevenin equivalent resistance R_{th} across R_3.

3. How will the power delivered to the resistor R_3 change, if <u>all</u> the resistors R_1, R_2 and R_3 are doubled?

a. the same **b.** one half **c.** doubles **d.** quadruples

4. Plot power versus load resistance using your data from Part One.

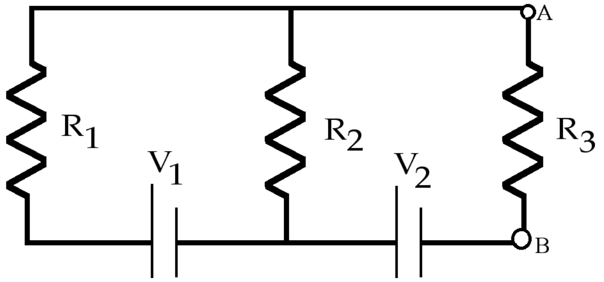
5. The workstation was cleaned and all the parts for the lab were returned to the workstation drawer. (TA must answer this.) True False

Instrumentation Lab 4 Quiz B

Name	**ID #**	**Section**	**Grade:**

1. For the circuit below, calculate the Thevenin equivalent voltage V_{th} acrosss R_3.

$R_1 = 5\ \text{k}\Omega$ $R_2 = 2\ \text{k}\Omega$ $V_1 = 40\ \text{V}$ $V_2 = 20\ \text{V}$



2. For the circuit above, calculate the Thevenin equivalent resistance R_{th} across R_3.

3. How will the power delivered to the resistor R_3 change, if the polarity of each voltage source is reversed?

a. the same **b.** one half **c.** doubles **d.** quadruples

4. Sketch the plot of your data from Part One of the procedures in the space below.

5. The workstation was cleaned and all the parts for the lab were returned to the workstation drawer. (TA must answer this.) True False