Lecture 2
Cast of Characters

- Basic quantities
  - Charge
  - Current
  - Voltage
  - Power
- Basic elements
  - Resistor
  - Voltage Source
  - Current Source
  - Capacitor
  - Inductor

CHARGE

Most matter is macroscopically electrically neutral most of the time.

Exceptions: clouds in thunderstorm, people on carpets in dry weather, plates of a charged capacitor…

Microscopically, of course, matter is full of charges.

The application of an electric field causes charges to drift, or move. Electrons will naturally move from lower electric potential to higher potential.

The rate at which the charges move depends on the magnitude of the potential difference and the properties of the matter.

CURRENT – VOLTAGE RELATIONSHIP

MOVING CHARGE

Measuring Charge

Charge is measured in Coulombs.

An electron has charge -1.6×10^{-19} C.

Charge flow ➠ Current

Charge storage ➠ Energy

Definition of Current

1 Ampere = flow of 1 Coulomb per second

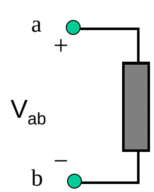
$$i\,(A) = \frac{dq}{dt}\left(\frac{C}{S}\right)$$

where q is the charge in Coulombs and t is the time in seconds

Current is defined as flow of positive charge!

VOLTAGE

Voltage is the difference in electric potential between two points.

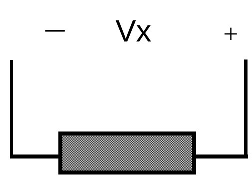


"V_{ab}" means the potential at a minus the potential at b.

Potential is always defined by two points.

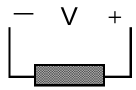
We can use the subscript convention above to define a voltage between two labeled points (e. g. a and b above),

or draw a + and – indicating polarity.



REFERENCE DIRECTIONS

A question like “Find the current” or “Find the voltage” is always accompanied by a definition of the direction:

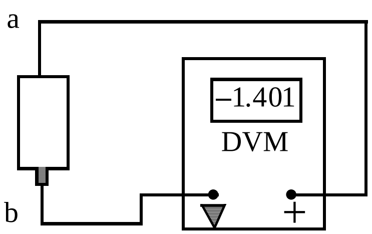


In this example if the current turned out to be 1mA, but flowing to the left we would merely say I = -1mA.

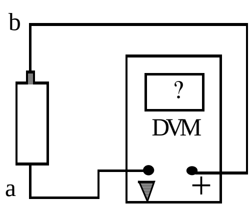
To solve circuits, you may need to specify reference directions for currents. But there is no need to guess the reference direction so that the answer comes out positive….Your guess won’t affect what the charge carriers are doing!

SIGN CONVENTIONS

Suppose you have an unlabelled battery and you measure its voltage with a digital voltmeter. It will tell you magnitude **and sign** of the voltage.



With this circuit, you are measuring $V_a - V_b$ (or V_{ab}).
DVM indicates –1.401, so $V_a < V_b$ by 1.401 V.
Which is the positive battery terminal?

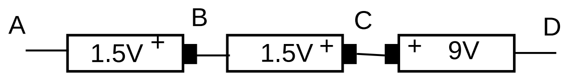


Now you make a change. What would this circuit measure?

Note that we have used “ground” symbol () for reference node on DVM. Often it is labeled “C” or “common.”

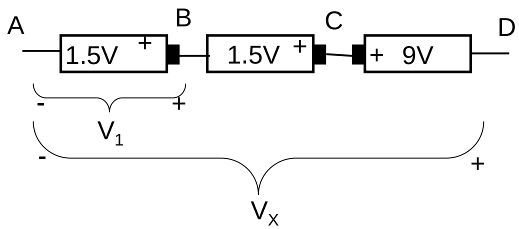
SIGN CONVENTIONS

Example 1



What is V_{AD} ?

Example 2 Find V_1 and V_x.



POWER IN ELECTRIC CIRCUITS

Power: Transfer of energy per unit time (Joules per second = Watts)

In falling through a potential drop V>0, a positive charge q gains energy

Potential energy change = qV for each charge q

Power = P = V (dq/dt) = VI

P = V × I Volt × Amps = Volts × Coulombs/sec = Joules/sec = Watts

Circuit elements can *absorb* power from or *release* power to the circuit.

How to keep the signs straight for absorbing and releasing power?

Memorize our convention:

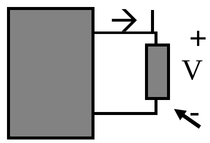
+ Power ≡ absorbed into element
– Power ≡ delivered from element

"ASSOCIATED REFERENCE DIRECTIONS"

If an element is absorbing power, positive charge will flow from higher potential to lower potential—over a voltage drop.

P = VI > 0 corresponds to the element absorbing power if the definitions of I and V are "associated".

This box represents the rest of the circuit



Here, I and V are "associated"

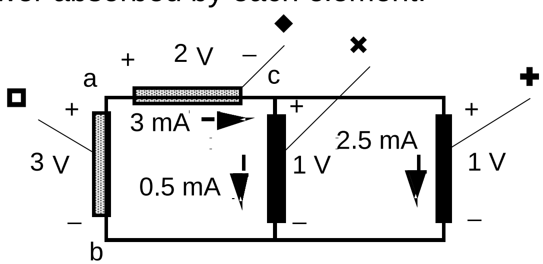
Circuit element

How can a circuit element absorb power?

By converting electrical energy into heat (resistors in toasters); light (light bulbs); acoustic energy (speakers); by storing energy (charging a battery).

EXAMPLES OF CALCULATING POWER

Find the power absorbed by each element.



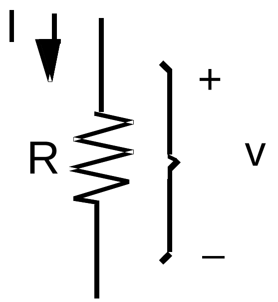
Element ◘ :

Element ◆ :

Element ✖ :

Element ✚ :

RESISTOR



With the “associated” current and voltage relationship shown, we get Ohm’s law:

V = I R

where R is the resistance in Ohms.

In reality, R is never negative => resistor always absorbs power

If you know resistor current, you know resistor voltage and vice-versa.

WIRE AND AIR

In class, we will mostly assume that wire is a perfect conductor.
In reality, wire does have a very small resistance.

Wire: No voltage drop, all points on wire at same potential
Current does flow, defined by other circuit elements

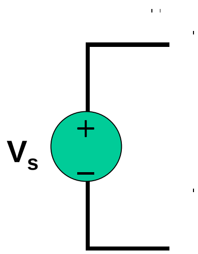
We will also assume that air is a perfect insulator (no “arcing”).

Air: No current flow
May carry voltage, defined by other circuit elements

There can be a nonzero voltage over a “hole” in a circuit!

IDEAL VOLTAGE SOURCE

Symbol



The ideal voltage source explicitly defines the voltage between its terminals.

Constant (DC) voltage source: $V_s = 5$ V

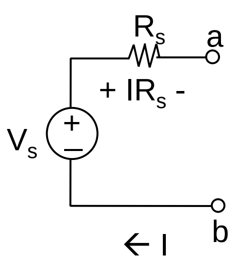
Time-Varying voltage source: $V_s = 10 \sin(t)$ V

The ideal voltage source has known voltage, but unknown current!

The current through the voltage source is defined by the rest of the circuit to which the source is attached.

You cannot assume that the current is zero!

REALISTIC VOLTAGE SOURCE



In reality, the voltage across a voltage source decreases slightly as more current is drawn.

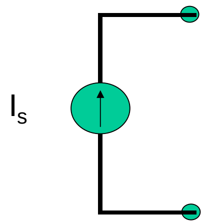
Real voltage sources can be thought of as an ideal voltage source with a small resistance.

When you are using a voltage source such as a battery or function generator to provide power to a circuit, positive current will flow towards the rise in source voltage.

The small resistance will carry a small voltage, causing the output voltage V_{ab} to be slightly less than V_s.

IDEAL CURRENT SOURCE

Symbol



The ideal current source sets the value of the current running through it.

Constant (DC) current source: $I_s = 2$ A

Time-Varying current source: $I_s = -3 \sin(t)$ A

The ideal current source has known current, but unknown voltage!

The voltage over the current source is defined by the rest of the circuit to which the source is attached.

You cannot assume that the voltage is zero!

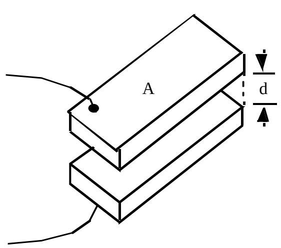
CAPACITOR

Any two conductors a and b separated by an insulator with a difference in voltage V_{ab} will have an equal and opposite charge on their surfaces whose value is given by

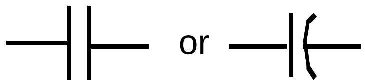
$Q = CV_{ab}$,

where C is the **capacitance** of the structure, and the + charge is on the more positive electrode.

We learned about the *parallel-plate capacitor* in physics. If the area of the plate is A, the separation d, and the *dielectric constant* of the insulator is ε, the capacitance equals $C = A\,\varepsilon/d$.



CAPACITOR

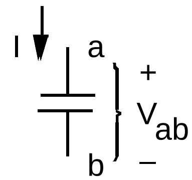
Symbol  or

Constitutive relationship: $Q = C\,V_{ab}$.

(Q is positive on plate a if $V_a > V_b$)

But $I = \dfrac{dQ}{dt}$ so $I = C\dfrac{dV}{dt}$

where we use the *associated reference directions*.



Charges do not go directly through dielectric:
current is actually process of collecting/removing charge from plates

ENERGY STORED IN A CAPACITOR

You might think the energy (in Joules) is QV, which has the dimension of joules. But during charging the average voltage was only half the final value of V.

Thus, energy is $\dfrac{1}{2}QV = \boxed{\dfrac{1}{2}CV^2}$

INDUCTORS

Inductors are the **dual** of capacitors – they store energy in magnetic fields that are proportional to current.

Capacitor	**Inductor**
$I = C\frac{dV}{dt}$	$V = L\frac{dI}{dt}$
$E = \frac{1}{2}CV^2$	$E = \frac{1}{2}LI^2$