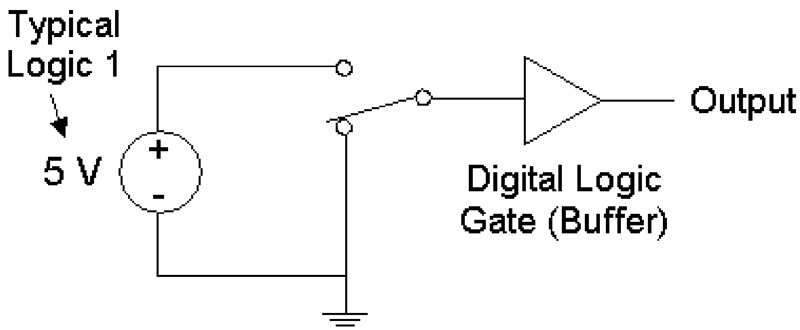
Sheila Ross and W. G. Oldham

Lecture 4

When we perform a sequence of computations using a digital circuit, we switch the input voltages between logic 0 and logic 1.



The output of the digital circuit fluctuates between logic 0 and logic 1 as computations are performed.

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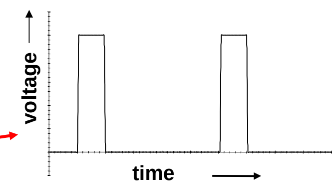
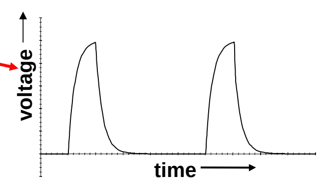
RC Circuits

We compute with pulses

We send beautiful pulses in



But we receive lousy-looking pulses at the output



Capacitor charging effects are responsible!

Every node in a circuit has capacitance to ground, and it’s the charging of these capacitances that limits real circuit performance (speed)

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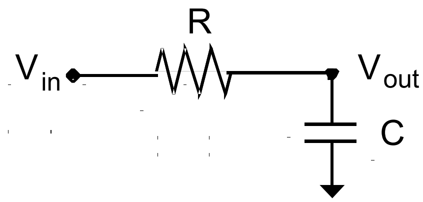
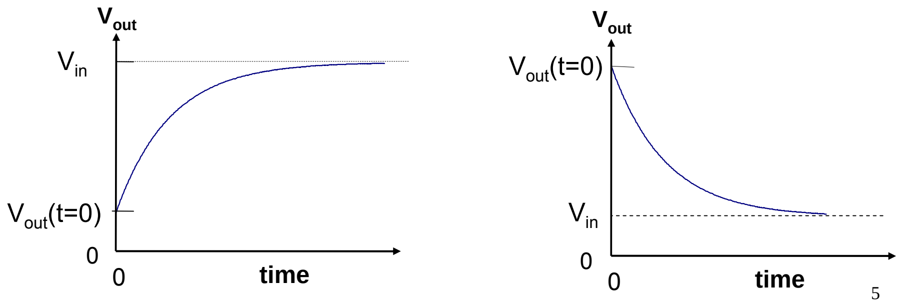
RC Response

Internal Model

of Logic Gate



Behavior of V_{out} after change in V_{in}

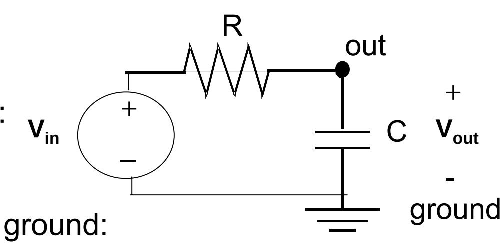


RC Response Derivation

KCL at node “out”:

Current into “out” from the left:

$(V_{in} - V_{out}) / R$



Current out of “out” down to ground:

$C\, dV_{out} / dt$

KCL: $(V_{in} – V_{out}) / R = C\, dV_{out} / dt$

$$\rightarrow \quad \frac{dV_{out}}{dt} = \frac{1}{RC}(V_{in} - V_{out})$$

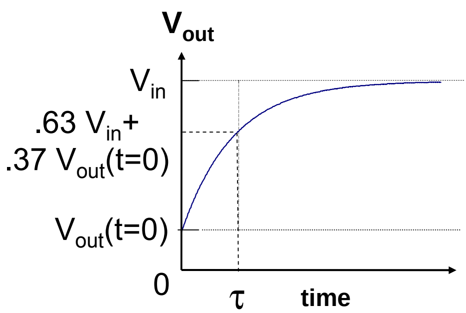
“Time Constant”
$\tau = RC$

Solution: $\boxed{V_{out}(t) = V_{in} + [\, V_{out}(t=0) – V_{in} \,]\, e^{-t/(RC)}}$

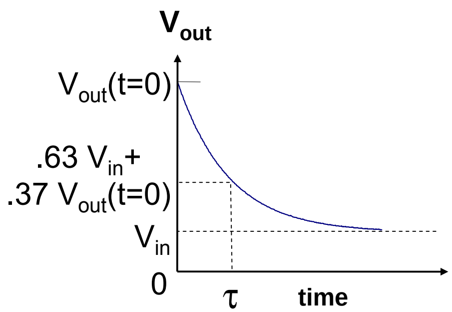
Charging and Discharging

$$\boxed{V_{out}(t) = V_{in}(1-e^{-t/\tau}) + V_{out}(t=0)e^{-t/\tau}}$$

Charging: $V_{in} > V_{out}(t=0)$



Discharging: $V_{in} < V_{out}(t=0)$



63% of transition complete after 1 τ

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Common Cases

Transition from logic 0 to logic 1

(charging)

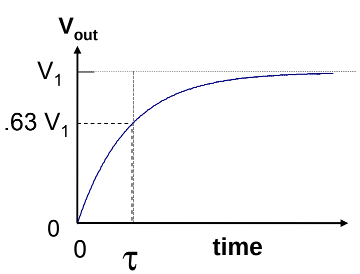
$V_{out}(t) = V_1(1-e^{-t/\tau})$

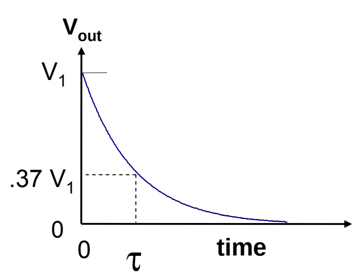
Transition from logic 1 to logic 0

(discharging)

$V_{out}(t) = V_1 e^{-t/\tau}$

(V_1 is logic 1 voltage)





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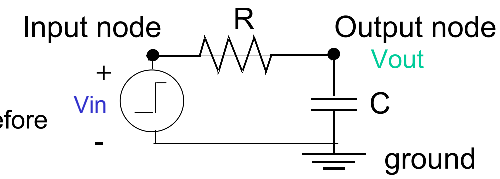
Charging and discharging in RC Circuits (The official EE40 Easy Method)

Method of solving for any node voltage in a single capacitor circuit.

1) Simplify the circuit so it looks like one resistor, a source, and a capacitor (it will take another two weeks to learn all the tricks to do this.) But then the circuit looks like this:

2) The time constant is τ = RC.

3) Solve for the capacitor voltage before the transient, $V_{out}(t=0)$.



4) Solve the for asymptotic value of capacitor voltage. Hint: Capacitor eventually conducts no current (dV/dt dies out asymptotically).

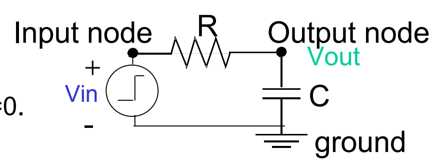
5) Sketch the transient. It is 63% complete after one time constant.

6) Write the equation by inspection.

Example

R = 1kΩ, C = 1pF.

Assume V_{in} has been zero for a long time, then steps from zero to 10 V at t=0.



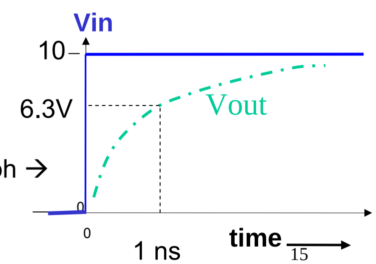
At t=0, since V_{in} has been constant for a long time, the circuit is in “steady-state”. Capacitor current is zero (since dV/dt = 0), so by KVL, Vout(t=0) = 0.

Asymptotically, the capacitor will have no current, so the capacitor voltage will be equal to V_{in}, 10 V (resistor will have 0 V).

The time constant τ = RC = 1 ns.

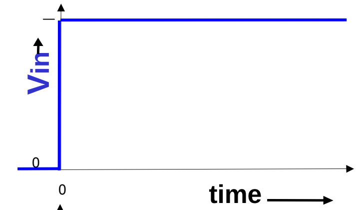
We can plug into the equation and draw the graph →

$$V_{out}(t) = 10 - 10e^{-t/1\ ns}$$

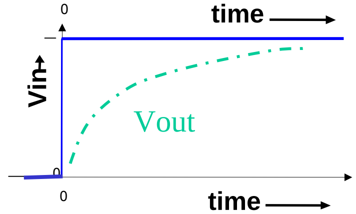


PULSE DISTORTION

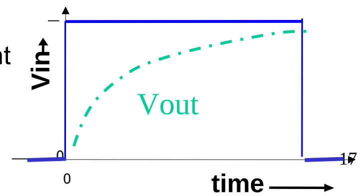
What if I want to step up the input,



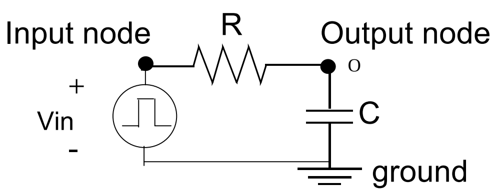
wait for the output to respond,



then bring the input back down for a different response?



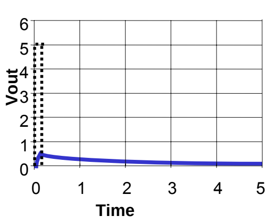
PULSE DISTORTION



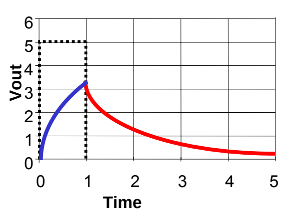
The pulse width must be long enough, or we get severe pulse distortion.

We need to reach a recognizable logic level.

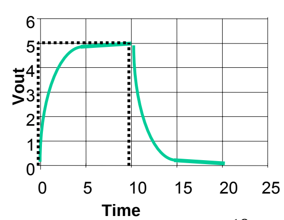
PW = 0.1RC



PW = RC

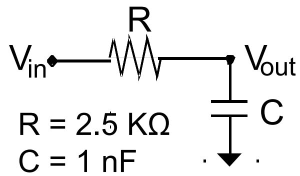


PW = 10RC



EXAMPLE

Suppose a voltage pulse of width 5 μs and height 4 V is applied to the input of the circuit at the right.

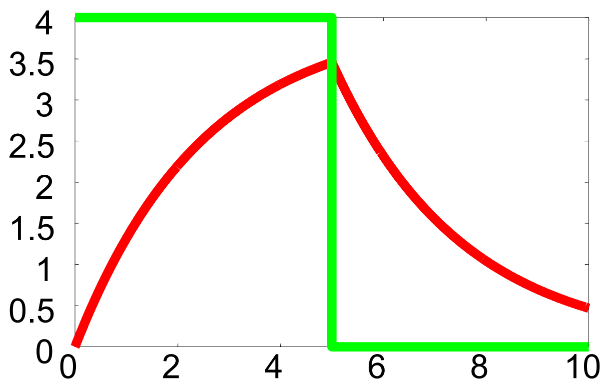


Sketch the output voltage.

First, the output voltage will increase to approach the 4 V input, following the exponential form. When the input goes back down, the output voltage will decrease back to zero, again following exponential form.

How far will it increase? Time constant = RC = 2.5 μs
The output increases for 5μs or 2 time constants.
It reaches $1-e^{-2}$ or 86% of the final value.
0.86 x 4 V = 3.44 V is the peak value.

EXAMPLE



The equation for the output is:

$$V_{out}(t) = \begin{cases} 4-4e^{-t/2.5\mu s} & \text{for } 0 \le t \le 5\ \mu s \\ 3.44e^{-(t-5\mu s)/2.5\mu s} & \text{for } t > 5\ \mu s \end{cases}$$

APPLICATIONS

- Now we can find "propagation delay" t_p; the time between the input reaching 50% of its final value and the output to reaching 50% of final value.
- For instantaneous input transitions between 0 V and logic 1,

  $0.5 = e^{-tp}$

  $t_p = -\ln 0.5 = 0.69$

  It takes 0.69 time constants, or 0.69 RC.

- We can find the time it takes for the output to reach other desired levels. For example, we can find the time required for the output to go from 0 V to the minimum voltage level recognizable as logic 1 (known as V_{IH}).

- Knowing these delays helps us design clocked circuits.