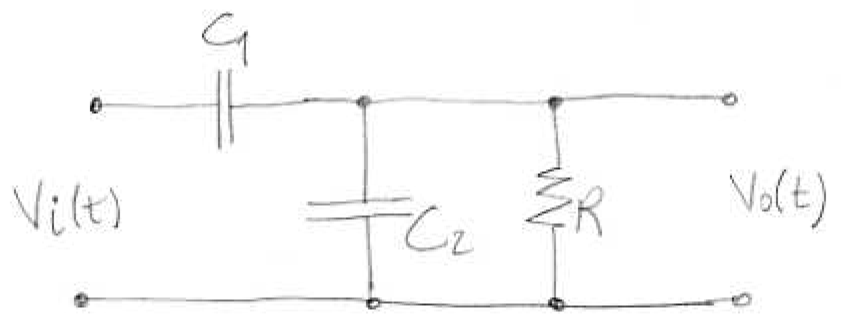
EE40 SUMMER '04 MIDTERM #2

A) Given the following circuit



A.1) Compute the transfer function

$$F(s) = \frac{V_o(s)}{V_i(s)}$$

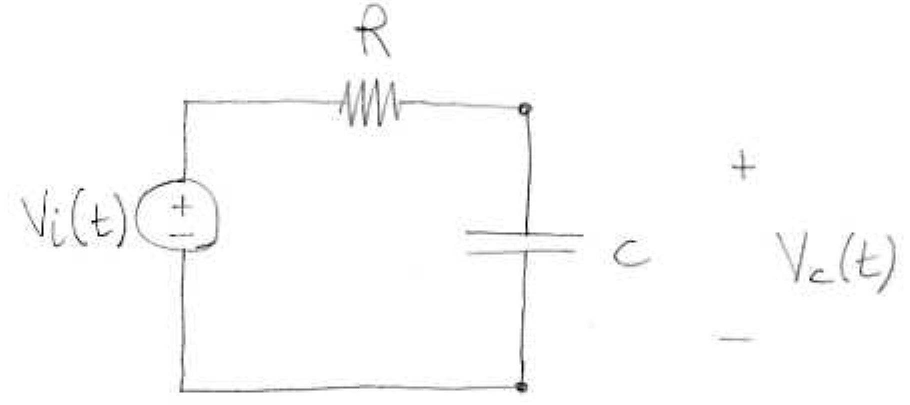
A.2) Compute the frequency response

$$F(j\omega)$$

A.3) Compute $|F(j\omega)|$

A.4 Extra question) Plot $|F(j\omega)|$ and $\measuredangle F(j\omega)$ as a function of ω

B) Given the following circuit:



B.1) Considering the input $V_i(t)$ as in figure B.f1, plot the output $V_c(t)$. (You have to do the work, don't just plot $V_c(t)$ but compute $V_c(t)$ as a function of time).

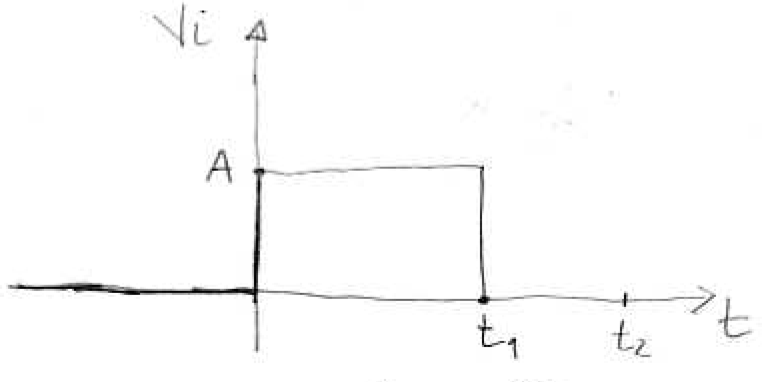
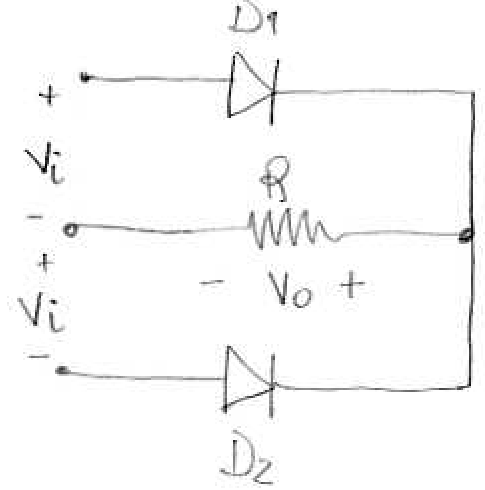


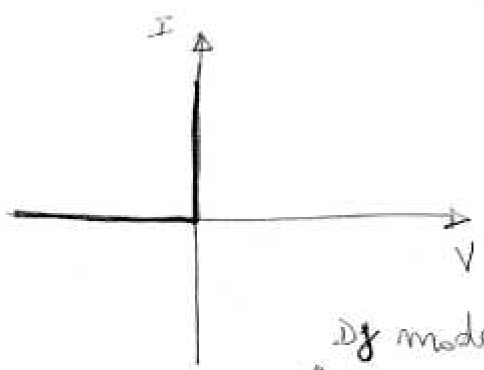
fig. B.f1

B.2) Compute $V_c(t_2)$

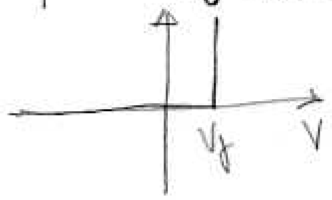
C) Consider the following circuit



Ideal Diode model

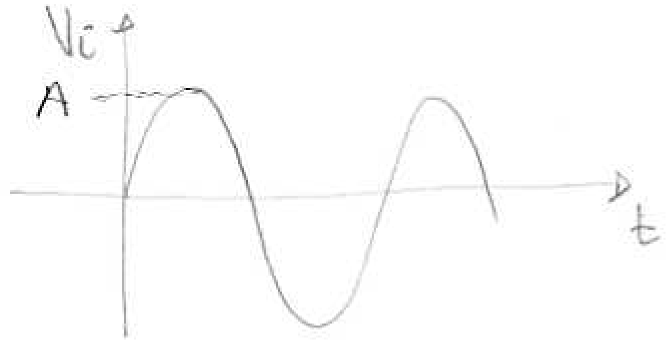


Dγ model



C.1) Using the ideal model for the diode (open-circuit, short-circuit) plot the transfer characteristic (Vo vs. Vi)

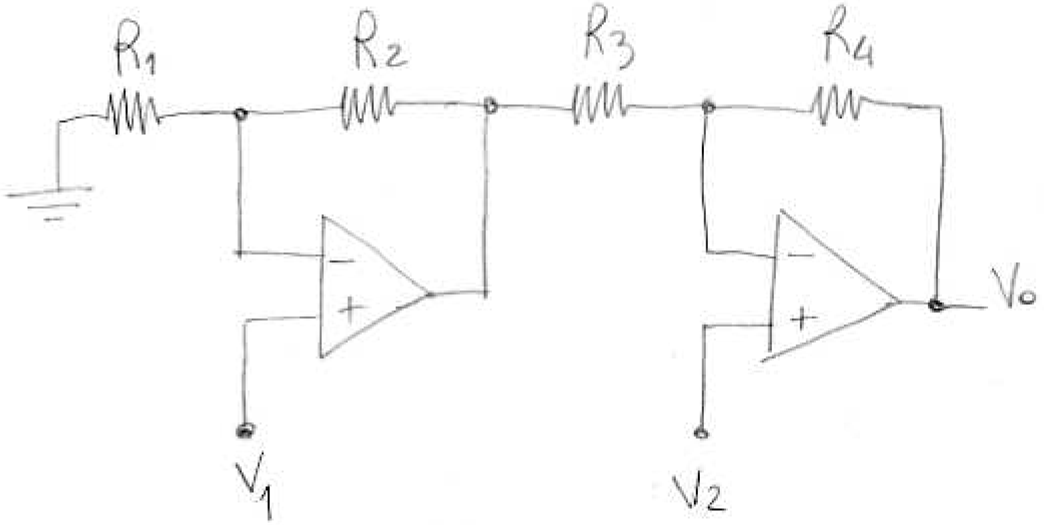
C.2) If the input is a sinusoid:



Plot the output

C.3 Extra question) Plot the transfer characteristic using the Dγ model

D) Given the circuit:



with $R_1 = R_4$, $R_2 = R_3$

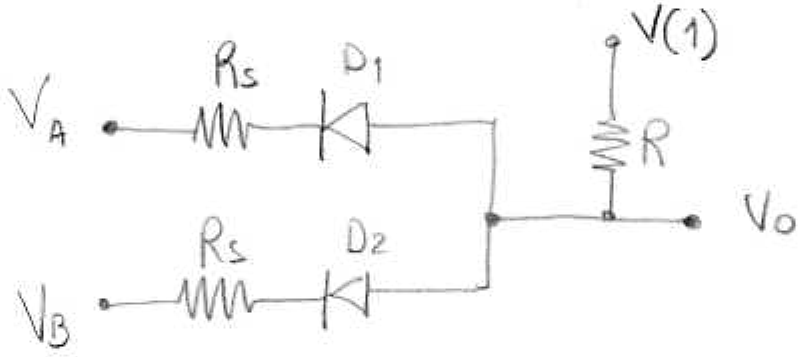
D.1) Compute V_o as a function of V_1, V_2

D.2) What is the input resistance for V_1 and V_2 ?

D.4 Extra question) If a noise signal $n(t)$ is injected in both V_1 and V_2, what is the noise at the output?

E Extra of the Extras) Consider a positive logic where the two digits 0 or 1 are encoded by two voltages V(0), V(1) where $V(1) > V(0)$.

Given the following logic device:

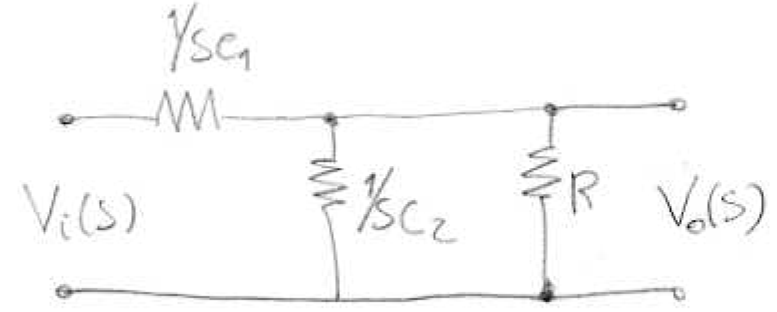


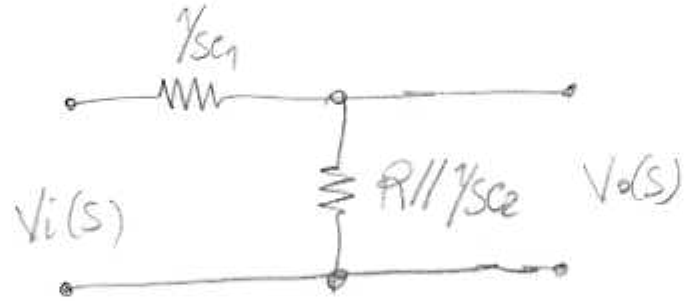
E.1) Write the thruth table of this device:

A	B	O

(Considering the diode ideal and $R \gg R_S$.)

A.1) Using Laplace transform:





$R // 1/sC_2 = Z_2 =$

$$= \frac{R}{sC_2R+1}$$

$$V_o(s) = \frac{Z_2}{1/sC_1 + Z_2} V_i(s)$$

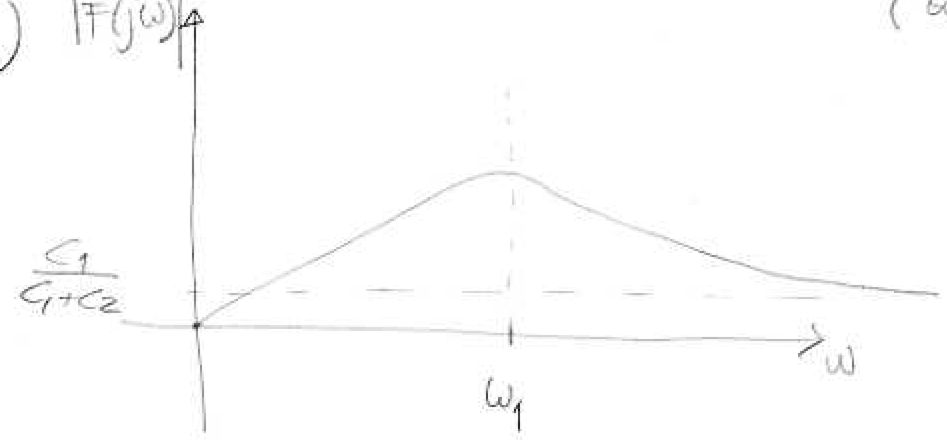
$$F(s) = \frac{R}{(sC_2R+1)\frac{1}{sC_1} + (sC_2R+1)\frac{R}{sC_2R+1}} =$$

$$= \frac{sC_1R}{sC_2R+1+sC_1R} = \frac{sC_1R}{sR(C_1+C_2)+1}$$

A.2) $$F(j\omega) = \frac{j\omega C_1 R}{j\omega R(C_1+C_2)+1}$$

A 3) $|F(j\omega)| = \dfrac{\omega C_1 R}{\sqrt{1+\omega^2 R^2 (C_1+C_2)^2}}$

A.4)



(assuming you don't know anything about Bode diagram)

$$\frac{d|F(j\omega)|}{d\omega} = \frac{\omega R C_1 \dfrac{2\omega R^2 (C_1+C_2)^2}{\sqrt{1+\omega^2R^2(C_1+C_2)^2}} - C_1 R\sqrt{1+\omega^2 R^2 (C_1+C_2)^2}}{1+\omega^2 R^2 (C_1+C_2)^2}$$

$$\frac{dF(j\omega)}{d\omega} = 0 \Rightarrow \frac{2\omega^2 R^3 C_1 (C_1+C_2)^2}{\sqrt{1+\omega^2R^2(C_1+C_2)}} = C_1 R\sqrt{1+\omega^2R^2(C_1+C_2)^2}$$

$$\Rightarrow 2\omega^2 R^3 C_1 (C_1+C_2)^2 - \cancel{C_1}\cancel{R}\left(1+\omega^2R^2(C_1+C_2)^2\right)$$

$$\Rightarrow 2R^2(C_1+C_2)^2\omega^2 = 1+\omega^2R^2(C_1+C_2)^2$$

$$\Rightarrow R^2(C_1+C_2)^2\omega^2 = 1 \Rightarrow \omega_1 = \sqrt{\frac{1}{R^2(C_1+C_2)^2}} = \frac{1}{R(C_1+C_2)}$$

this is a max

for $\omega \to 0$ $|F(j\omega)| \to 0$ also for $\omega \to \infty$

$$\lim_{\omega\to\infty} \frac{\omega C_1 R}{\sqrt{1+\omega^2R^2(C_1+C_2)^2}} = \lim_{\omega\to\infty} \frac{C_1 R}{\sqrt{1/\omega^2 + R^2(C_1+C_2)^2}} = \frac{C_1}{C_1+C_2}$$

$$\measuredangle F(j\omega) = \pi/2 - tg^{-1}\left(\frac{1}{\omega R(C_1+C_2)}\right)$$

B.1) Using N.V.A.

$$\frac{V_i(t) - V_c(t)}{R} = i_c(t) = C\,\frac{d\,V_c(t)}{dt}$$

$$\frac{dV_c(t)}{dt} + \frac{1}{RC}V_c(t) = \frac{V_i(t)}{RC}$$

If in the example I did using the inductor, you replace $I(t)$ with $V_c(t)$ you obtain the same solution.

$$V_c(t) = K_1 e^{\alpha t} + K_2$$

$\boxed{0 \le t\ t_1}$ $\quad V_i(t) = A$

$K_2 = A$ because if $V_c(t) = K_2$ then

$$\cancel{\frac{dK_2}{dt}} + \frac{K_2}{RC} = \frac{A}{RC}$$

$$\frac{d\,K_1 e^{\alpha t}}{dt} + \frac{1}{RC}K_1 e^{\alpha t} = 0 \Rightarrow \cancel{K_1}\alpha \cancel{e^{\alpha t}} + \frac{\cancel{K_1}}{RC}\cancel{e^{\alpha t}} = 0$$

$$\alpha = -1/RC$$

$V_c(t) = K_1 e^{-t/RC} + A$ but $V_c(0) = 0 \Rightarrow K_1 = -A$

$V_c(t) = A\left(1 - e^{-t/\tau}\right)$

In $t_1 \Rightarrow \quad V_c(t_1) = A\left(1 - e^{-t_1/\tau}\right)$

now we shift the time to t_1

$\boxed{t \geq t_1} \quad V_i(t) = 0$

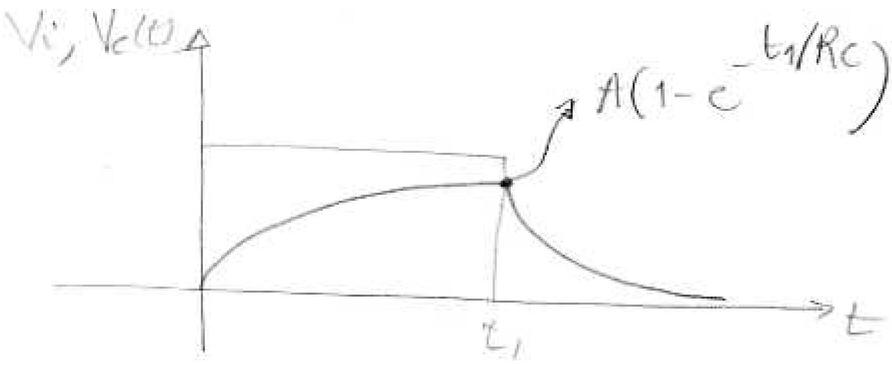
$$\frac{dV_c(t)}{dt} + \frac{1}{RC} V_c(t) = 0$$

$$V_c(t_1) = A\left(1 - e^{-t_1/\tau}\right)$$

$$V_c(t) = K_1 e^{-t/RC} \Rightarrow V_c(t_1) = K_1 e^{-t_1/RC} = A\left(1 - e^{-t_1/RC}\right)$$

$$\Rightarrow K_1 = \frac{A\left(1 - e^{-t_1/RC}\right)}{e^{-t_1/RC}} = A e^{t_1/RC} - A$$

$$V_c(t) = \left(A e^{t_1/RC} - A\right) e^{-t/RC} = A e^{(t_1 - t)/RC} - A e^{-t/RC}$$



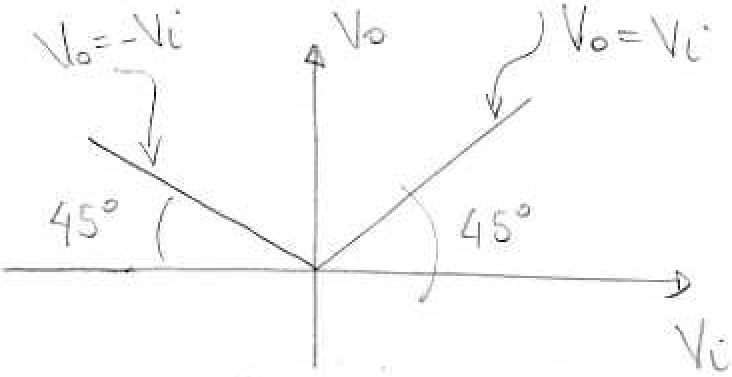
B.2) $V_c(t_2) = \left(Ae^{t_1/RC} - A\right) e^{-t_2/RC} = Ae^{t_1-t_2/RC} - Ae^{-t_2/RC}$ MIDZ 11

C.1) If $V_i > 0 \Rightarrow D_1$ is on and D_2 is OFF

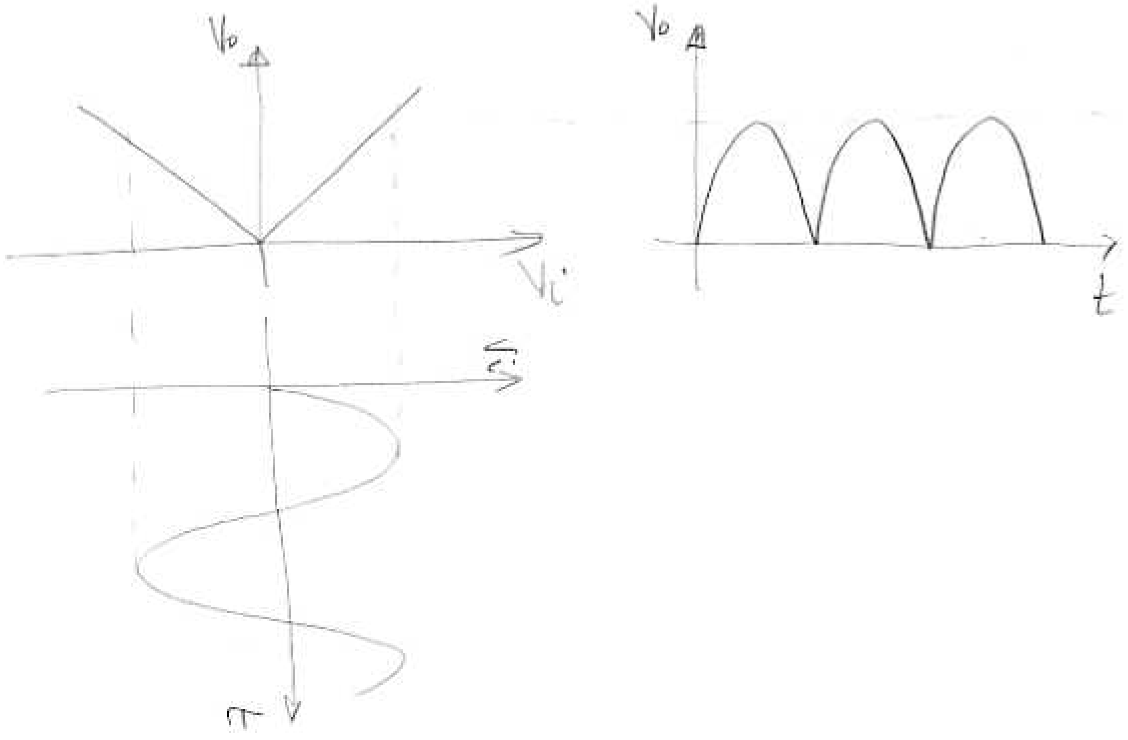
$V_o = V_i$

If $V_i < 0 \Rightarrow D_2$ is ON and D_1 is OFF

$V_o = -V_i$



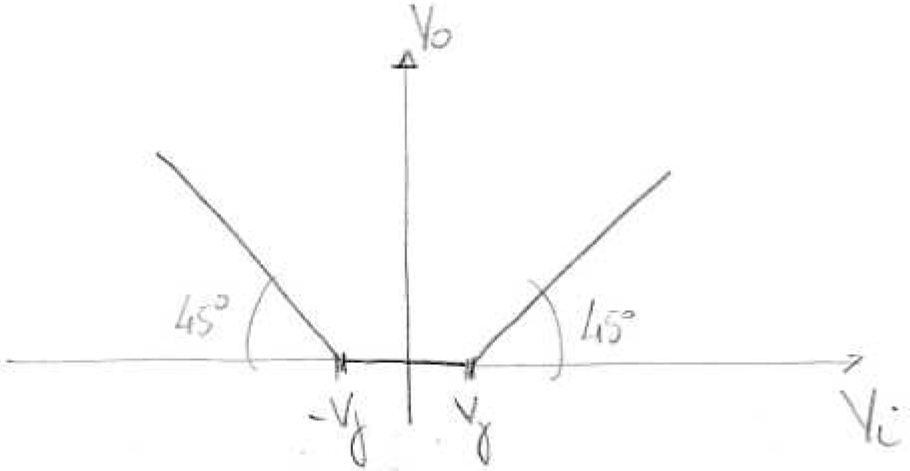
C.2)



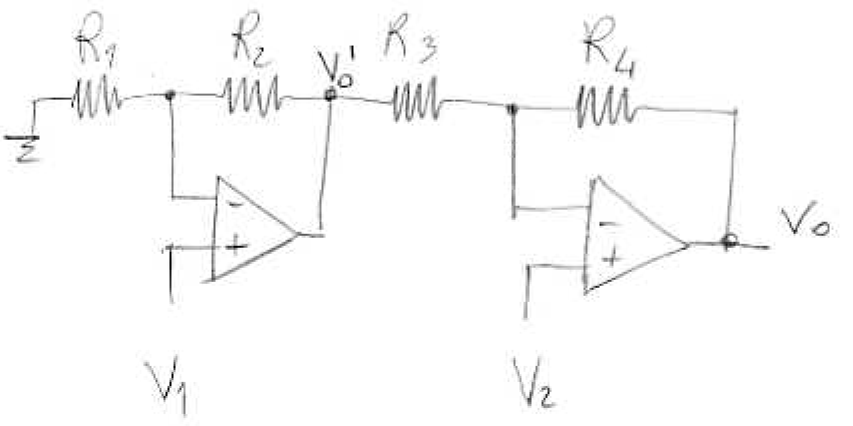
C.3) If we use D_γ model then V_i has to be greater than V_γ in order for D_1 to be on, and less than $-V_\gamma$ in order for D_2 to be on.

If for instance D_1 is on then $V_0 = V_i - V_\gamma$

The trans characteristic is like the following



D.1)



By superposition:

$V_1 = 0 \qquad V_2 \neq 0$

$V_o' = 0 \quad , \quad V_{o_2} = V_2\left(1 + \frac{R_4}{R_3}\right)$ (non inverting ampl.)

$V_2 = 0 \qquad V_1 \neq 0$

$V_o' = V_1\left(1 + \frac{R_2}{R_1}\right)$ non inverting ampl.

$V_{o_1} = -V_o'\left(\frac{R_4}{R_3}\right)$ inverting ampl.

$$V_o = V_2\left(1 + \frac{R_4}{R_3}\right) - V_1\left(1 + \frac{R_2}{R_1}\right)\frac{R_4}{R_3} =$$

$$= V_2\left(1 + \frac{R_4}{R_3}\right) - V_1\left(\frac{R_4}{R_3} + 1\right) = (V_2 - V_1)\left(1 + \frac{R_1}{R_2}\right)$$

D.2) Since the op amp is ideal, I_+ is zero.

So

$$R_{in_1} = \infty$$

$$R_{in_2} = \infty$$

D. 4)

$$V_o = (V_2' - V_1')\left(1 + \frac{R_1}{R_2}\right) = \left(V_2 + n(t) - V_1 - n(t)\right)\left(1 + \frac{R_1}{R_2}\right)$$

$$= (V_2 - V_1)\left(1 + \frac{R_1}{R_2}\right)$$

So the output noise is zero.

E.1) (0,0) If $V_A = V_B = V(0)$ then D_1 and D_2 are on so

$$V_o = V(0) + \left(V(1) - V(0)\right) \frac{R_S // R_S}{R + R_S // R_S} =$$

$$= V(0) + \left(V(1) - V(0)\right) \frac{R_S}{2R + R_S} \underset{R \gg R_S}{\simeq} V(0)$$

(0,1) If $V_A = V(0)$ and $V_B = V(1)$ then D_1 is on and D_2 is off

$$V_o = V(0) + \left(V(1) - V(0)\right) \frac{R_S}{R + R_S} \simeq V(0)$$

(1,0) is symmetric to the previous case, so D_2 is on and D_1 is on.

(1,1) D_1 and D_2 are both off $\Rightarrow V_o = V(1)$

A	B	O
0	0	0
0	1	0
1	0	0
1	1	1

it is an AND gate