DIODES: APPROXIMATIONS

The current / voltage relation for a diode is:

$$I = I_0 \left(e^{V/\eta V_T} - 1 \right)$$

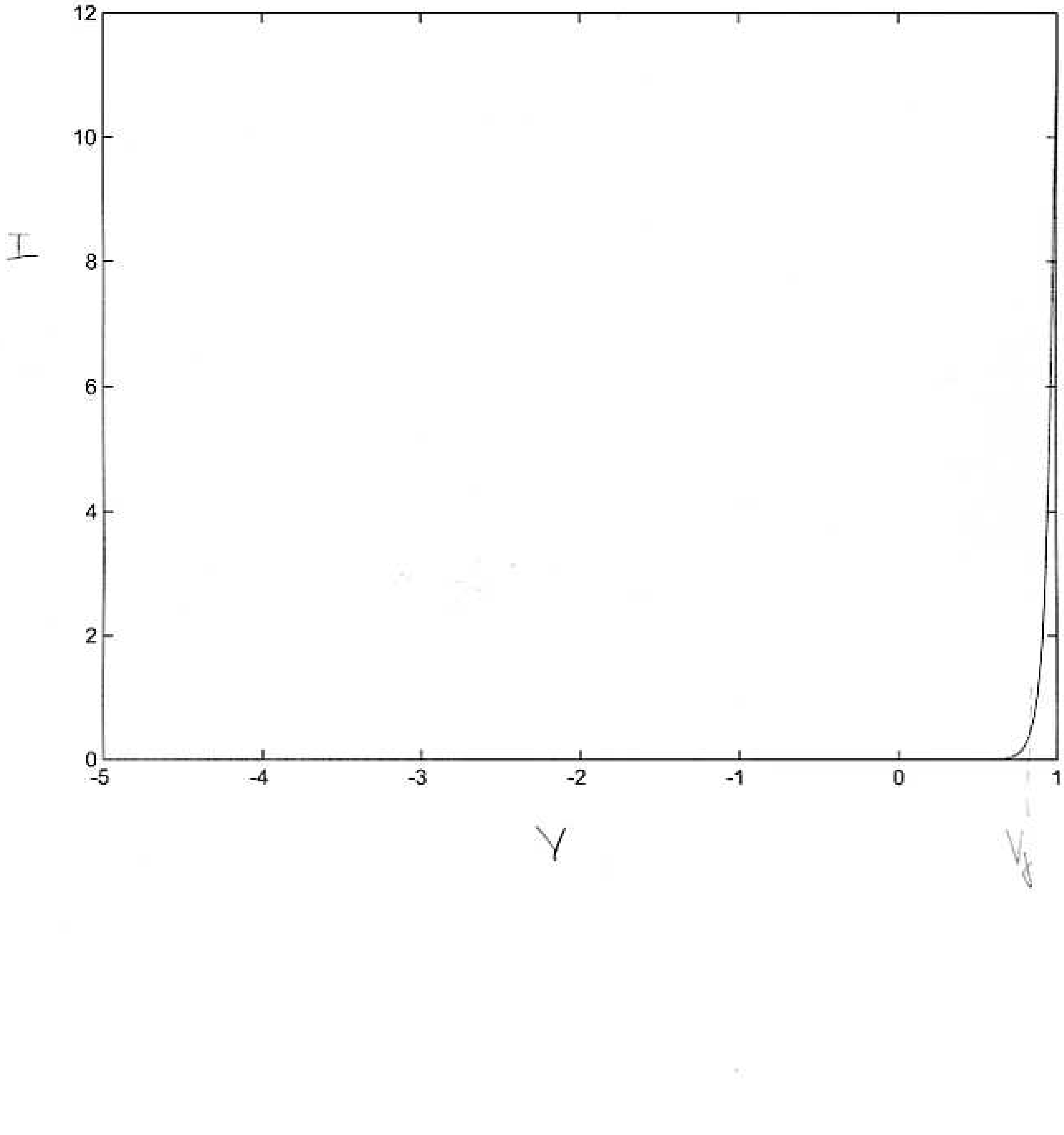
where $V_T \simeq 26\,mV$ @ $300\,K$ (remember that V_T depends on the temperature and its value is actually $V_T = \frac{KT}{q}$ where K is the Boltzman constant, q is the charge of an electron and T is the Temperature in Kelvin degrees).

η is a constant that depend on the semiconductore: $\eta = 1$ for Germanium (Ge) and 2 for silicon (Si).

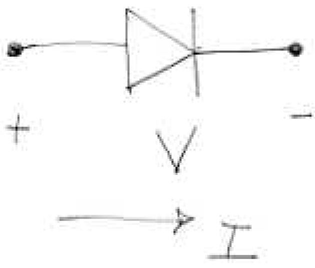
Finally I_0 is the reverse saturation current. For instance for the 1N4002, $I_0 \simeq 0.05\,\mu A$ @ $25°C$. Figure 1 on the next page shows the I/V plot obtained using these numbers.

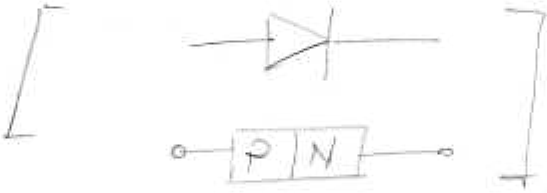
$V_T = 25$ mV

$I_0 = 0.05\ \mu A$



The circuit symbol that we use for diodes is like the following:





For $V < V_\gamma$, I is is very small and if $V<0$ then $I \simeq -I_0$ which is a very small reverse current.

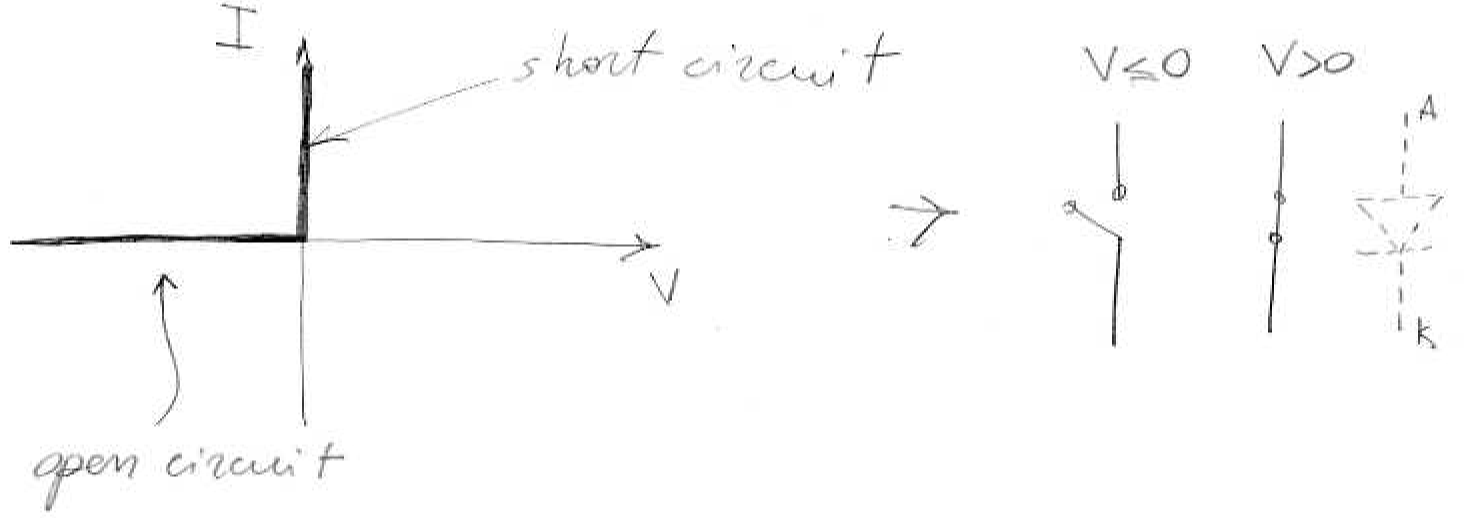
If $V > V_\gamma$ then $\frac{\partial I}{\partial V}$ is very high meaning that G (the conductance) is high and hence $R = \frac{1}{G}$ is very small. The diode behaves like a short circuit.

We want to abstract this behaviour. The first rough approximation is the following:

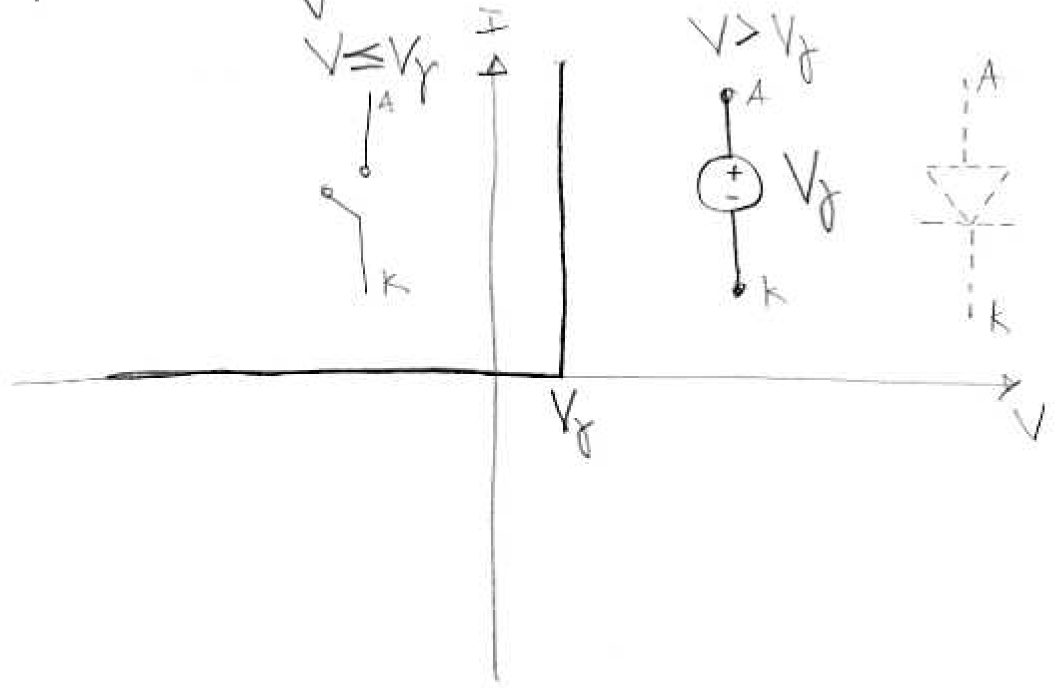
$V \leq 0$ open circuit

$V \geq 0$ short circuit

This behaviour can be graphically represented as follows:



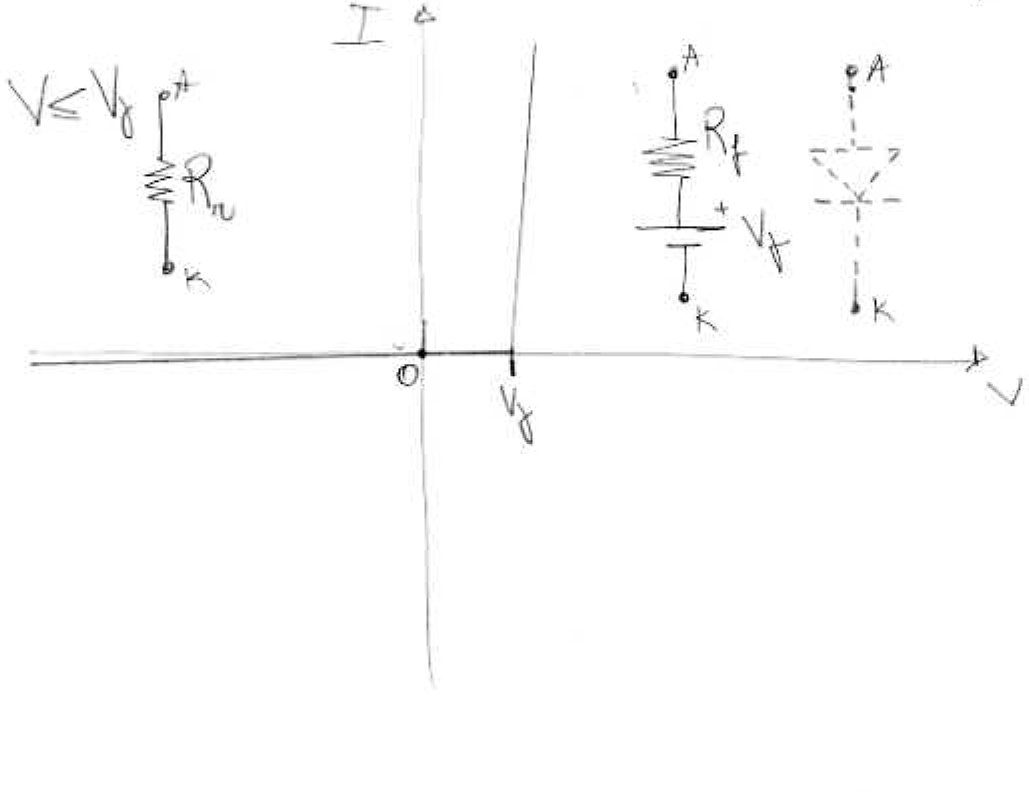
This model is an approximation because it doesn't consider I_0, in reverse polarization, and V_γ in forward polarization. We could take into account V_γ using the following model:



This model considers the diode an open circuit for $V \leq V_\gamma$, and a voltage source for $V > V_\gamma$.

Actually this is not the correct description of this model. We should say: $I=0$ for $V \leq V_\gamma$ and $V = V_\gamma$ otherwise, which means $V=V_\gamma$ for forward polarization. The reason is that in this model V across the diode cannot be greater than V_γ. When the diode is in forward polarization then not only the voltage il V_γ but also the current can assume any value greater than 0. This is basically a voltage source.

a more refined approximation is the following

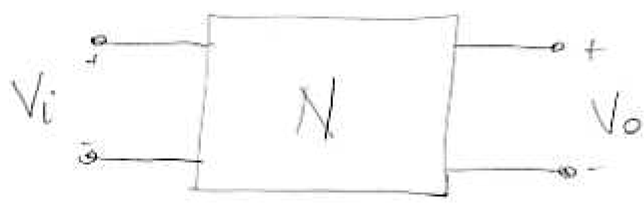


In reverse polarization, the diode can be considered as a resistor with a very high resistance. This situation can be graphically representet by a line that passes through the origin of the I-V plane.
The angular coefficient of this line is:

$$\frac{dI}{dV} = \frac{1}{R}$$

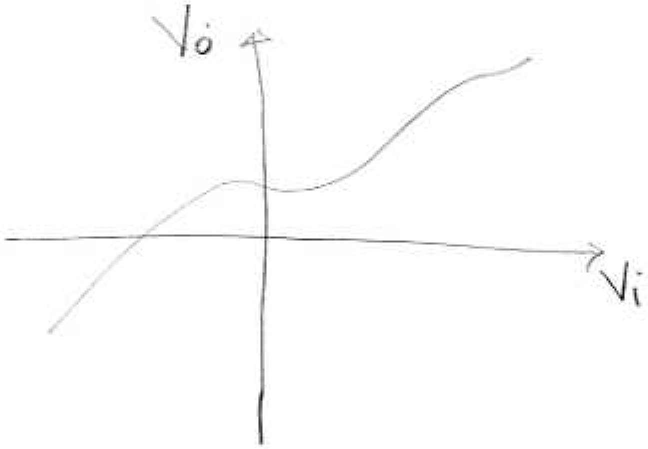
In forward polarization, the diode can be considered as a very small resistor R_r but we have to use a voltage source V_γ to consider the fact that the line doesn't pass through the origin.

- TRANS CHARACTERISTIC
Consider a circuit with an input and an output:



The question we want to answer is: what is the value of the output

if the value of the input is x Volts?
Of course we want to characterize the output ∀ values x.
We can buil a graph then:



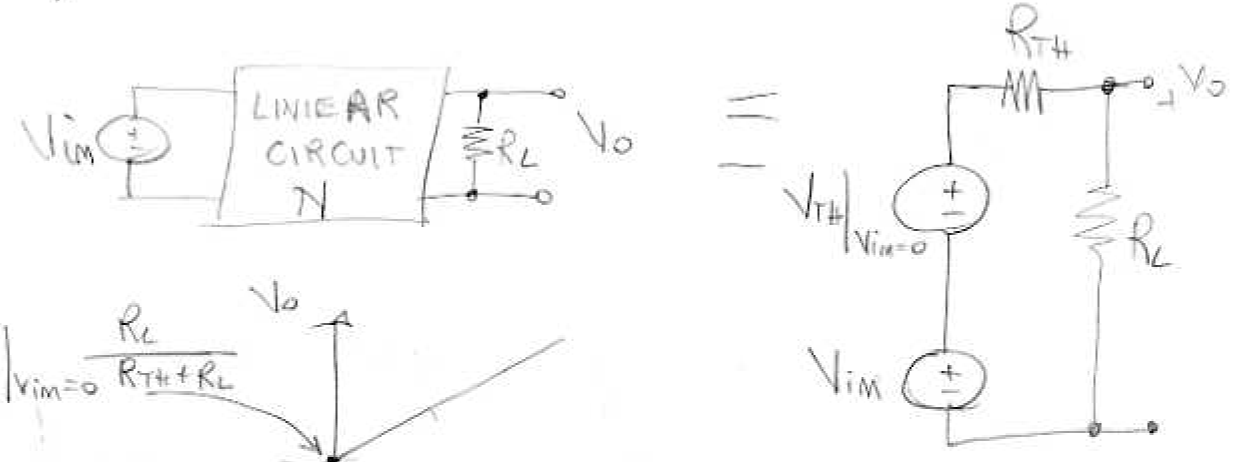
for each value of Vi we can look at the graph and find the value of the output Vo.

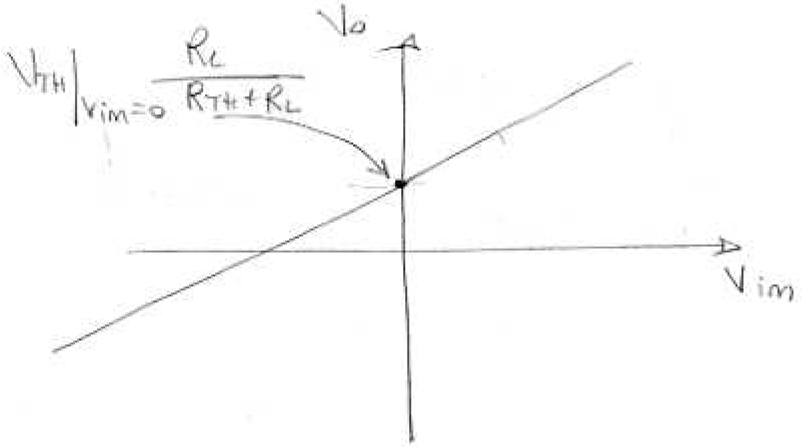
For linear circuits that graph will always be something like:

$$V_o = aV_i + b$$

Unfortunately, diodes are non linear circuits element. The relation betwee current and voltage is not linear, it is indeed exponential.

If we are given a circuit and we are asked to find the transch. then if the circuit is linear we can just find the thevenin equivalent circuit :

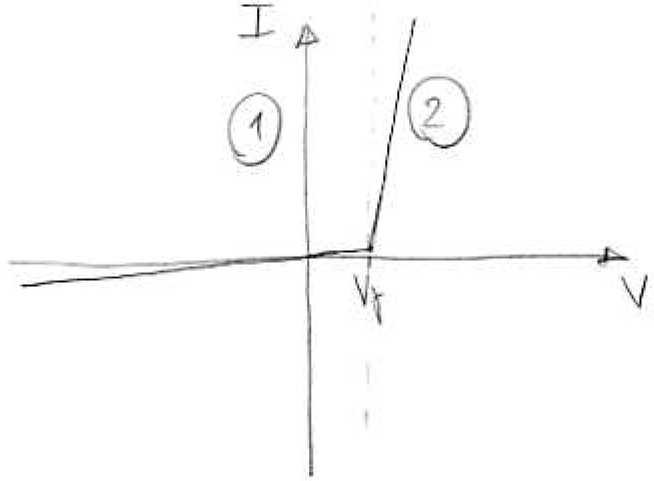




If the circuit is non linear then we cannot apply the methods that we know directly.

– STATES METHOD

Even if diodes are non linear devices, we have built approximations that are piecewise linear. We define a state of the device the portion of its I-V characteristit which is linear:



1 and 2 are the two states. They are defined by:

1 $\left\{ V \leq V_\gamma \; ; \; I = \dfrac{V}{R_r} \right\}$

2 $\left\{ V > V_\gamma \; ; \; I = \dfrac{V - V_\gamma}{R_f} \right\}$

If a circuit has D dioeds, then its state is a D-tuple S where S[i] (the i-th element of the tuple) represent the state of the i-th diode.

We can use the following method for analyzing a circuits contaning diodes:

1 - Pick an initial state S

2 - While a solution is not found

2.1 - $\forall$ diodes D_i

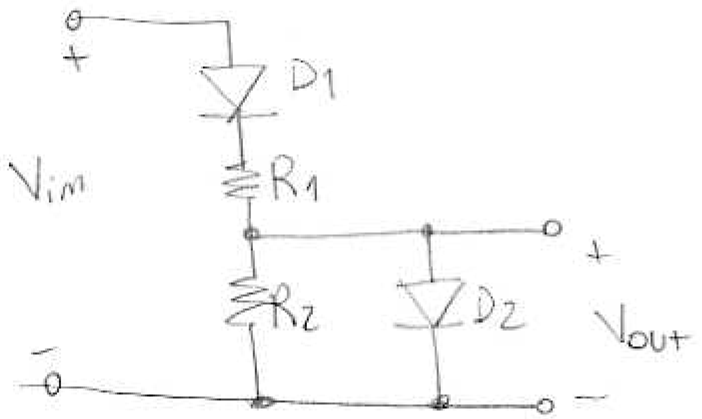
2.1.1 - substitute D_i with its equivalent circuit based on state $S[i]$

2.2 - Analyze the resulting linear circuit

2.3 - If all voltages across the diodes are compliant with the state S then this is the solution

2.4 - otherwise pick a state S that has not been visited already

EXAMPLE:



$R_1 = 100\,\Omega$

$R_2 = 50\,\Omega$

$R_r = 100\,M\Omega$

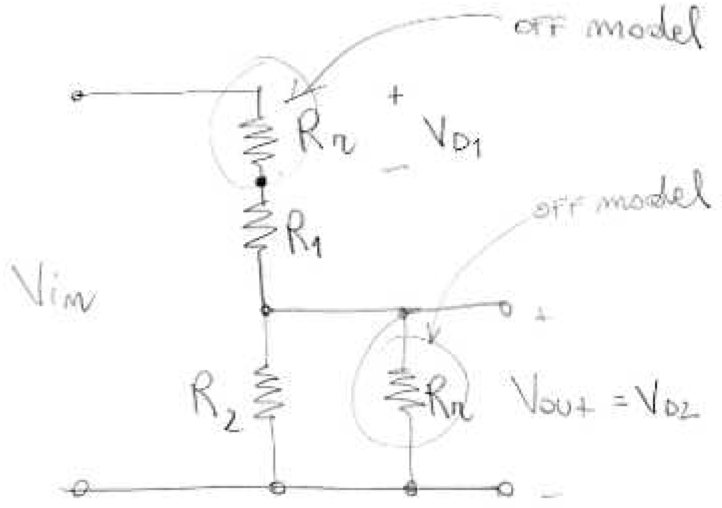
$R_f = 5\,\Omega$

$V_\gamma = 0.7$

Since there are two diodes then we have to explore at most 4 states:

We start with the initial state {OFF, OFF}

D1 D2



$$V_{out} = V_{D2} = V_{in} \frac{R_2 /\!/ R_r}{R_2 /\!/ R_L + R_1 + R_r}$$

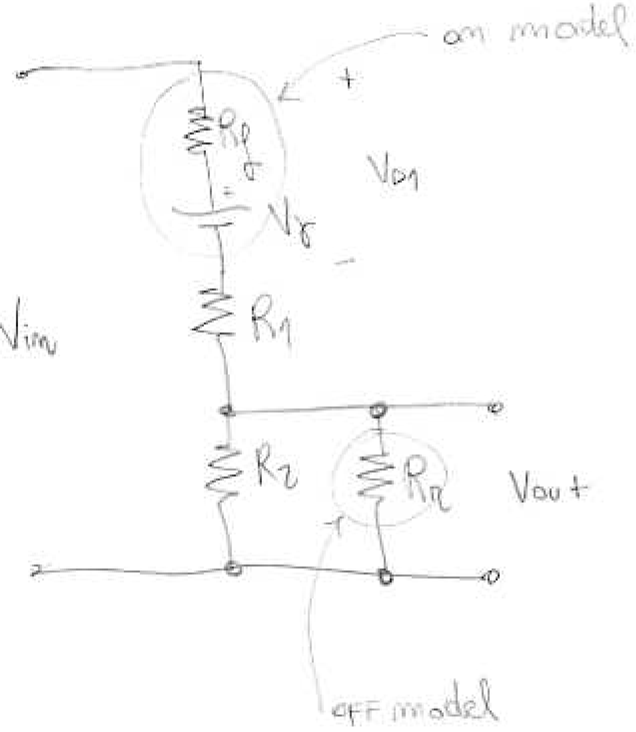
$$V_{D1} = V_{in} \frac{R_r}{R_r + R_1 + R_2 /\!/ R_r}$$

$$V_{D2} = V_{in} \frac{R_2}{R_r} \simeq 0$$

$$V_{D1} \simeq V_{in}$$

If $V_{in} \leq V_\gamma$ then this state is valid because both V_{D1} and V_{D2} are less than V_γ and the two diodes are both off which matches our original assumption.

When $V_{in} > V_\gamma$, then D1 switches to ON and we can try then with the new state {ON, OFF}



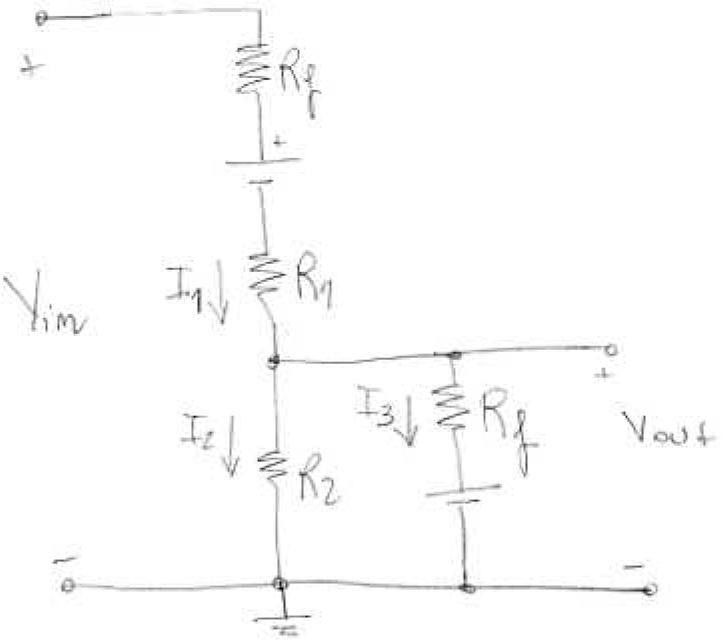
$$V_{out} = V_{D2} = (V_{in} - V_\gamma)\frac{R_2 /\!/ R_r}{R_2 /\!/ R_r + R_1 + R_f}$$

$$V_{D1} = V_\gamma + (V_{in} - V_\gamma)\frac{R_f}{R_f + R_1 + R_2 /\!/ R_r}$$

$$V_{D2} \simeq (V_{in} - V_\gamma)$$

$$V_{D1} \simeq V_\gamma + \frac{(V_{in} - V_\gamma) R_f}{R_r}$$

Sin $V_{in} = V_\gamma \Rightarrow V_{D2} = 0$ and $V_{D1} > V_\gamma$ hence this model is valid.
It will be valid until $V_{D2} \geq V_\gamma$ meaning $V_{in} - V_\gamma > V_\gamma \Rightarrow V_{in} > 2V_\gamma$. At this point also D_1 switches. We can then try the new state {ON, ON}



$$I_1 = \frac{V_{in} - V_{out} - V_f}{R_f + R_1}$$

$$I_2 = \frac{V_{out}}{R_2} \; ; \; I_3 = \frac{V_{out} - V_f}{R_f}$$

$$I_1 = I_2 + I_3 \Rightarrow \frac{V_{in} - V_{out} - V_f}{R_f + R_1} = \frac{V_{out}}{R_2} + \frac{V_{out} - V_f}{R_f}$$

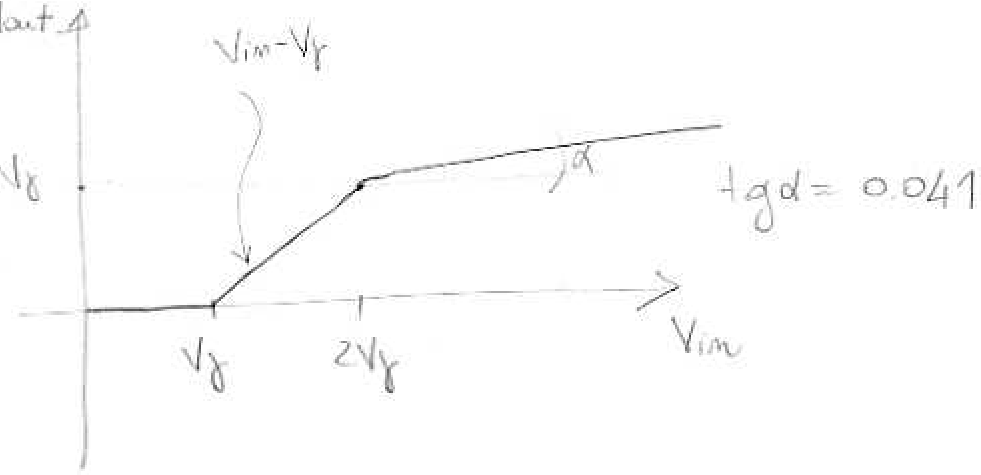
$$\frac{V_{in}}{R_f + R_1} = V_{out}\left(\frac{1}{R_2} + \frac{1}{R_f} + \frac{1}{R_f + R_1}\right) + V_f\left(\frac{1}{R_f + R_1} - \frac{1}{R_f}\right)$$

$$V_{out} = \frac{\dfrac{V_{in}}{R_f + R_1} + V_f\left(\dfrac{R_1}{(R_f + R_1)\,R_f}\right)}{\dfrac{R_f(R_f + R_1) + R_2(R_f + R_1) + R_2 R_f}{R_f R_2 (R_f + R_1)}} =$$

$$= V_{in} \frac{R_2 R_f}{(R_2 + R_f)(R_f + R_1) + R_2 R_f} + V_f \frac{R_1 R_2}{(R_2 + R_f)(R_1 + R_f) + R_2 R_f}$$

$V_{out} \simeq V_{in} \cdot 0.041 + V_\gamma \cdot 0.83$

So the transcharacteristic is:

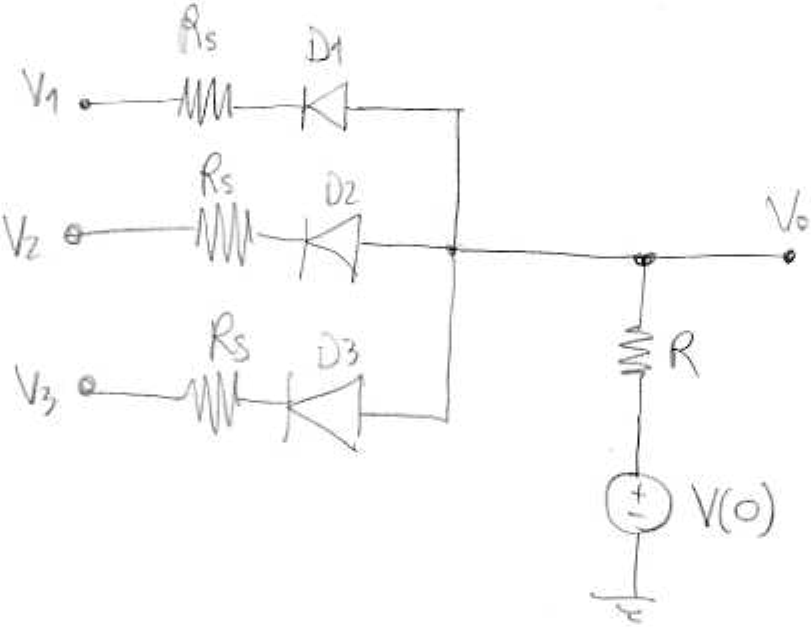


the solution is an approximate solution. In fact if you compute $V_{out} = 2V_\gamma\, 0.041 + V_\gamma\, 0.83$ the result is not exactly V_γ. The reason is that we have made a lot of approximation in our computation. For instance considering $R_2 /\!/ R_L = R_2$ and $R_L + R_f + R_1 = R_L$ and son on.

- APPLICATIONS

In your tex book you can find classical applications of diodes. Here I will give you other applications that you might like.

- LOGIC GATES



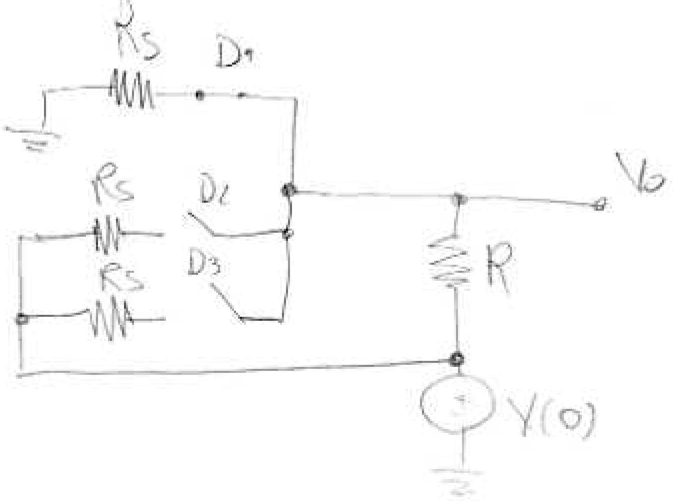
V_i can take on two values

$V(1)=0\ V \rightarrow$ binary 1

$V(0) > V_j \rightarrow$ binary 0

If $V_1 = V_2 = V_3 = V(0) \Rightarrow V_o = V(0)$

If $V_1 = V(1) = 0$ and $V_2 = V_3 = V(0)$ then

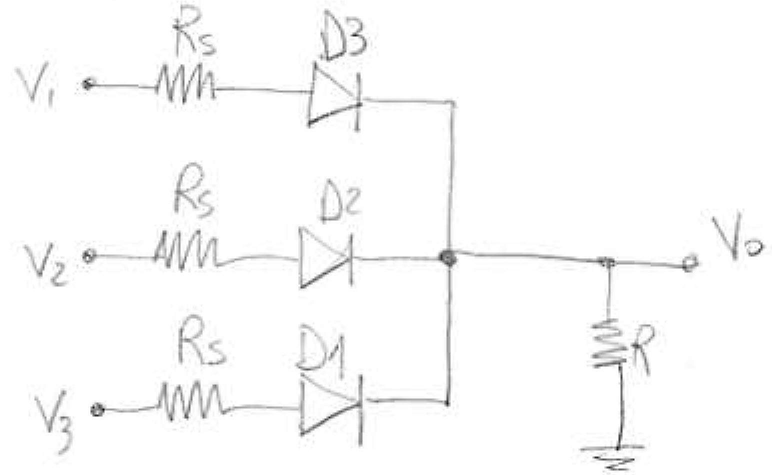


$$V_o = V(0) - \big(V(0) - V(1)\big)\frac{R}{R+R_s}$$

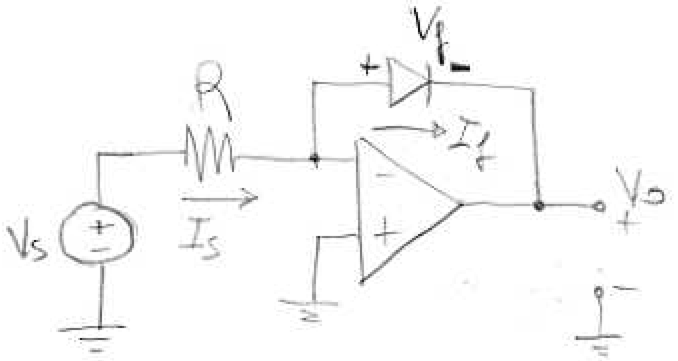
$$= V(0)\left(1 - \frac{R}{R+R_s}\right) + V(1)\frac{R}{R+R_s}$$

if $R \gg R_s \Rightarrow V_o \simeq V(1)$

This is an OR logic gate. Using the same method, we can build an AND logic gate:



— COMPUTING THE LOGARITHM



$$V_o = -V_f = -\eta V_T(\ln I_f - \ln I_o) =$$

$$= -\eta V_T\left(\ln \frac{V_s}{R} - \ln I_o\right)$$

$$I_s = \frac{V_s}{R} = I_f$$

$$I_f = I_o\left(e^{V_f/\eta V_T} \cancel{-1}\right)$$

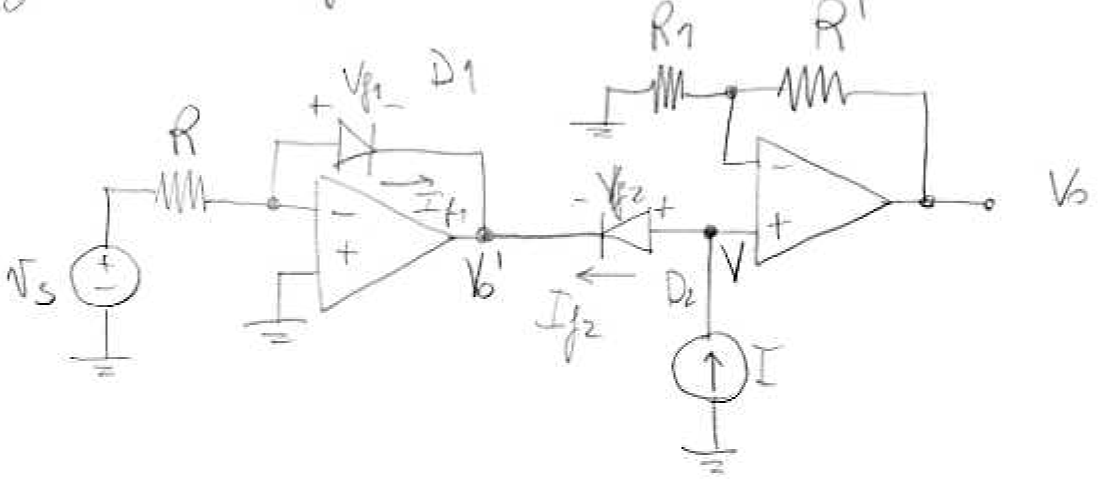
In forward polarization $e^{V_f/\eta V_T} \gg 1$ so:

$$I_f \simeq I_o e^{V_f/\eta V_T} \Rightarrow$$

$$\Rightarrow V_f = \eta V_T \ln \frac{I_f}{I_o}$$

$$= \eta V_T(\ln I_f - \ln I_o)$$

The problem with our log. is the constant factor $\ln I_o$ which we want to get rid of:



We use another diode that generate a V_{f2} that is going to be removed from V_{f1}:

$$V = V_{f2} + V_o' = \eta V_T \Big(\underbrace{\ln \frac{I}{I_o}}_{\text{from D2}} \underbrace{- \ln \frac{V_s}{R} + \ln I_o}_{V_o'} \Big) =$$

$$= -\eta V_T \ln \frac{V_s}{RI}$$

The output volteg is then: $V_o = \eta V_T \frac{R' + R_1}{R_1} \ln \frac{V_s}{RI}$

We can choose $\frac{R' + R_1}{R_1}$ and RI in such a way that $\boxed{V_o = \ln V_s}$

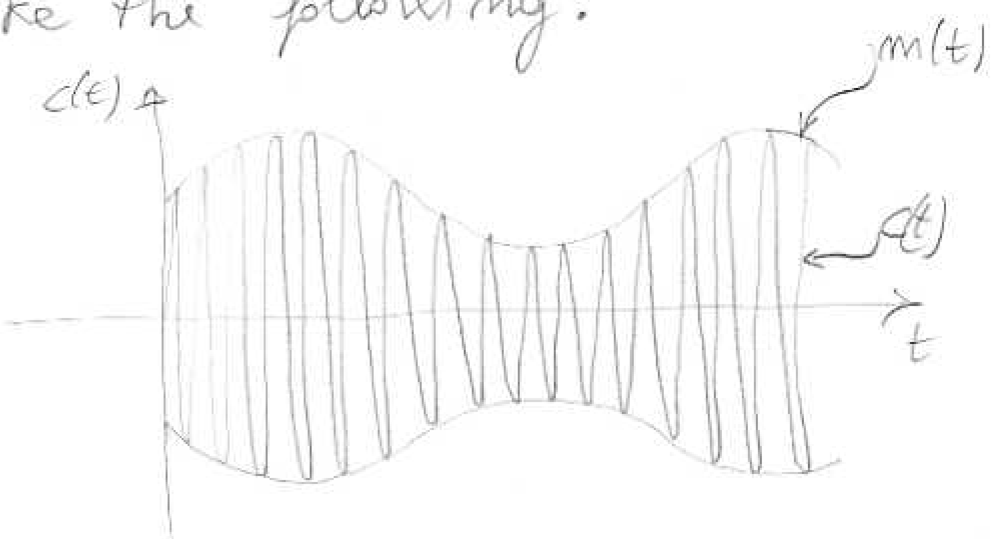
- AM DEMODULATOR

We first introduce the AM (Amplidute Modulation)
Consider a message $m(t)$ (for instance voice) that we want to send over the air. One technique that is used is the amplitude modulation. For simplicity we consider $m(t) = \sin(\omega_m t)$. We can use anothe sinusoidal signal at a much higher frequency, that we call carrier, and we change the carrier amplitude using $m(t)$:

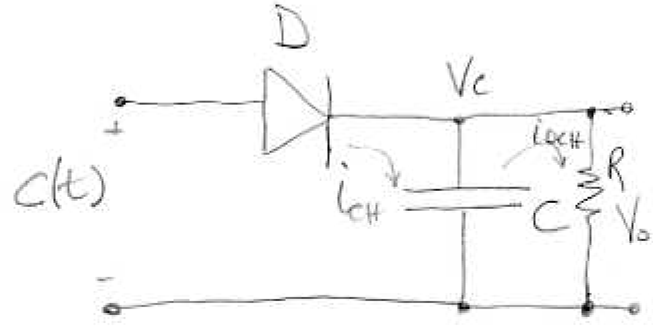
$$\boxed{c(t) = \left(C + m(t)\right)\sin(\omega_c t)}$$

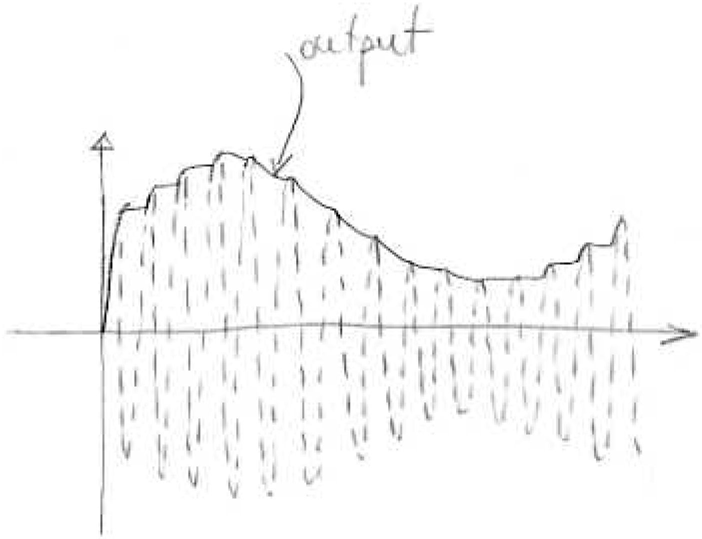
← thi is the modulated carrier

C is a constant, without going into details let's pick $C = 2$. The modulate signal looks like the following:



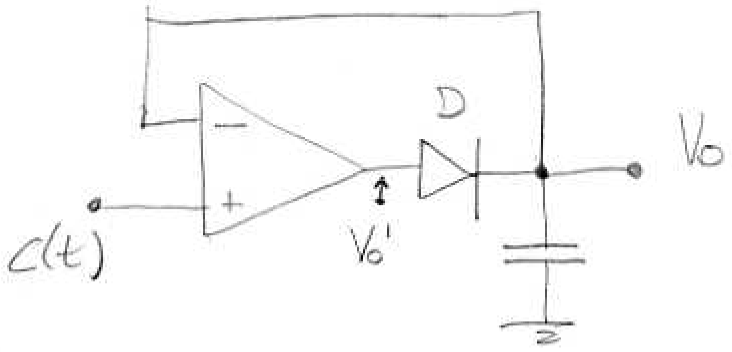
the amplitude of the signal $\sin(\omega_c t)$ is modulated by $m(t)$.
If we want to obtain the original signal $m(t)$ we have to follow the peak voltage of $c(t)$.
We can use a peak detector:





When $c(t) > V_c$ the current will charge the capacitor, when $c(t) < V_c$ the capacitor will be discharged by R.
The value of RC depend on ω_m and ω_c an it is chosen during the demodulator design. The problem with this circuit is that $c(t) \gg V_\gamma$ otherwise the diode will be always OFF.
But $c(t)$ is the signal received from an antenna an (even after amplification)

is usually very small.
To eliminate V_γ we can use a different peak detector that uses an op-amp:



$$V_o' = K(c(t) - V_o)$$

In order for D to be on, $V_o' - V_o > V_\gamma$

which means

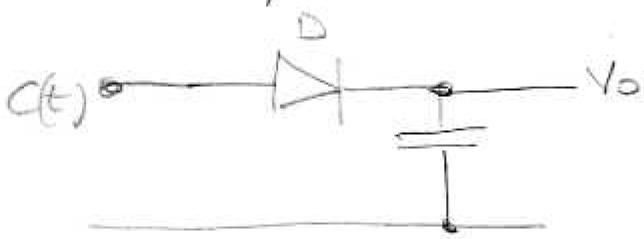
$$K c(t) - K V_o - V_o \geq V_\gamma$$

$$\Rightarrow \quad K c(t) - (K+1) V_o > V_\gamma$$

And since $K \gg 1$

$$K c(t) - K V_o > V_\gamma \Rightarrow c(t) - V_o > \frac{V_\gamma}{K}$$

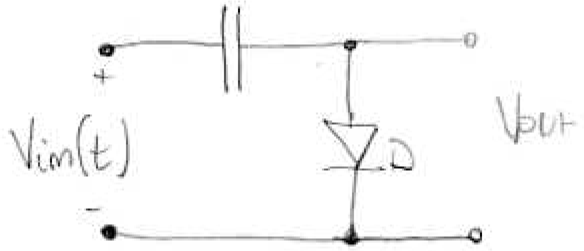
It is equivalent to



Where D has a threshold voltage $V_\gamma' = \frac{V_\gamma}{K} \simeq 0$ because K is very high

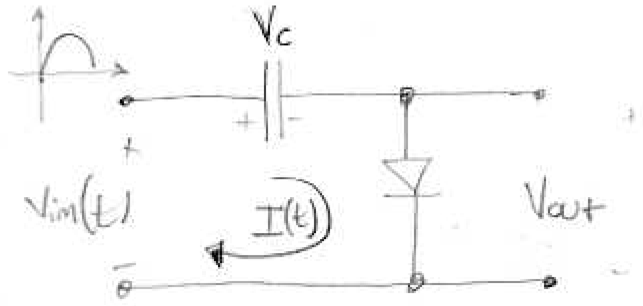
— V_{pp} DOUBLER

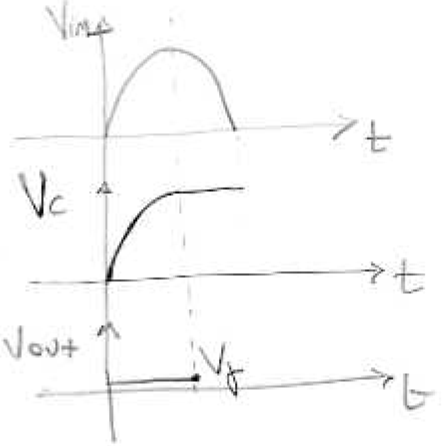
Diodes can be used to multiply the peak voltage of a sinusoidal waveform:



$V_{in}(t) = A\sin(\omega t)$

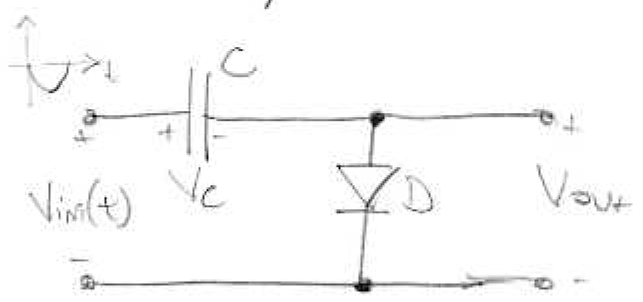
During the positive half of the wave the diod is ON and the capacitor charges to A:

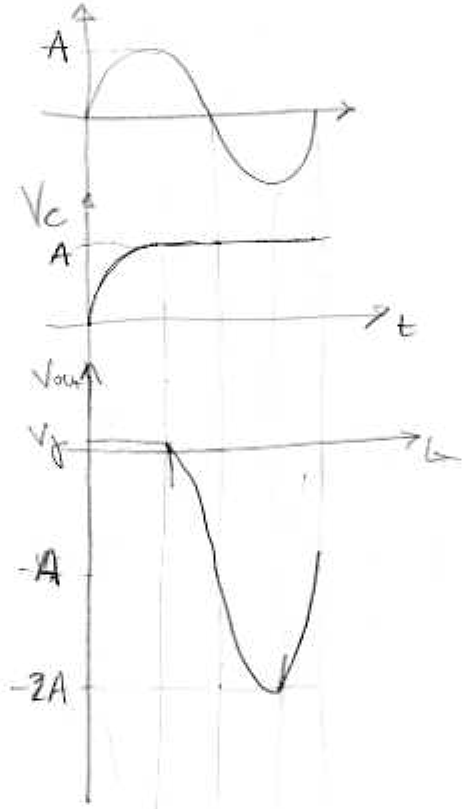




Vout is equal to the voltage across the diode that is V_γ since the diode is ON.

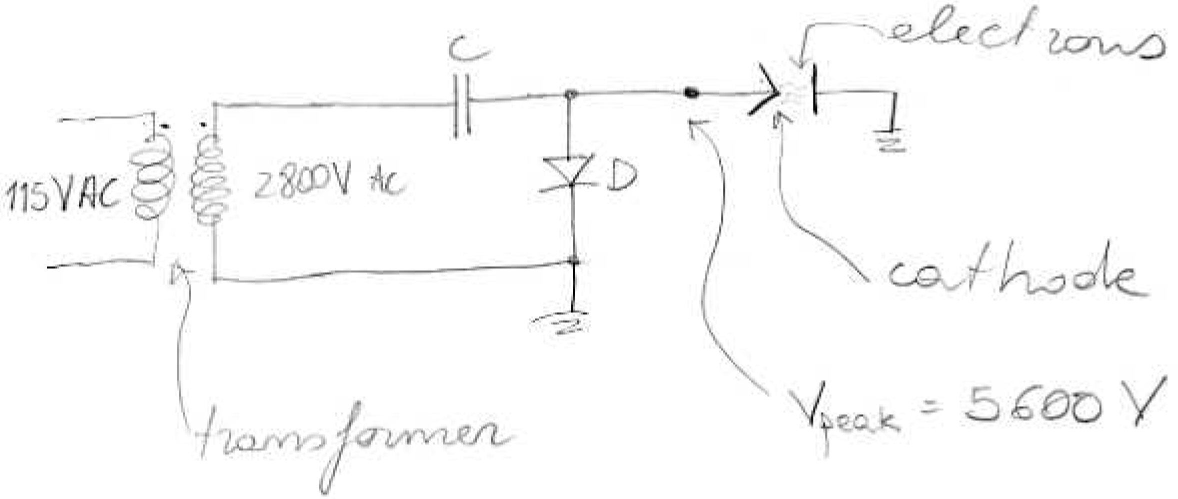
During the negative half the diode is OFF. $V_{out} = V_{in}(t) - V_C$ and since Vin is negative the output will reach a value equal to 2A





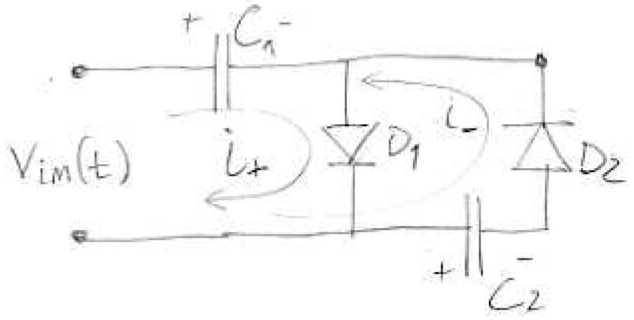
the maximum negative voltage is now −2A.

This circuit is used for instance in microwave ovens. Microwave ovens cook food using waves at very high frequency (2450 MHz). This device has a cathode which emits electrons that are then subject to a permanent magnetic field. The electrons will move in a resonant cavity and generate the microwave. The cathode requires a very high peak voltage to work. The circuit that is used is like the following.



Using the same strategy we can build a full wave doubler, a tripler a quadrupler and so on.

The following circuit is a full wave doubler



the simulation result for $V_{in}(t) = 15 \sin(2\pi 60 t)$ is shown in figure dbl on the following page.

During the first quarter of period D_1 is on and C_1 charges to 15.

Then V_{in} start decreasing and D_1 switches to OFF because the voltage $V_{D1} = V_{in} - V_{C1}$ 11.24

becomes negative.

At this point D_2 switches to on and C_2 charges to 15.

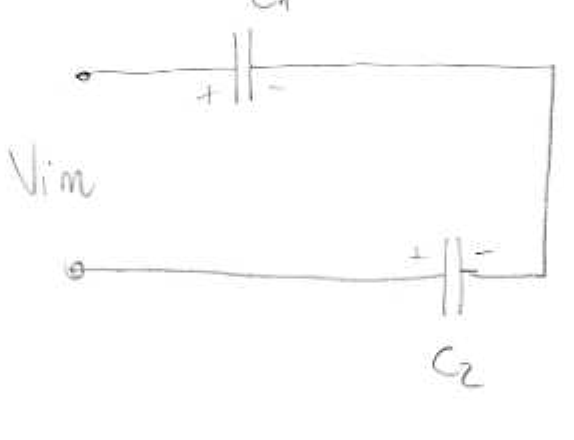
When V_{in} becomes positive again Then D_1 switches to on and C_1 charges to 15 again. Now when V_{in} becomes negative again we have the following situation:

$V_{C1} = 15$ $\qquad$ $V_{D1} < 0 \Rightarrow D_1 = \text{OFF}$

$V_{C2} = 15$ $\qquad$ $V_{D2} = -V_{in} \Rightarrow D_2 = \text{ON}$

$V_{in} < 0$

The circuit is like this:

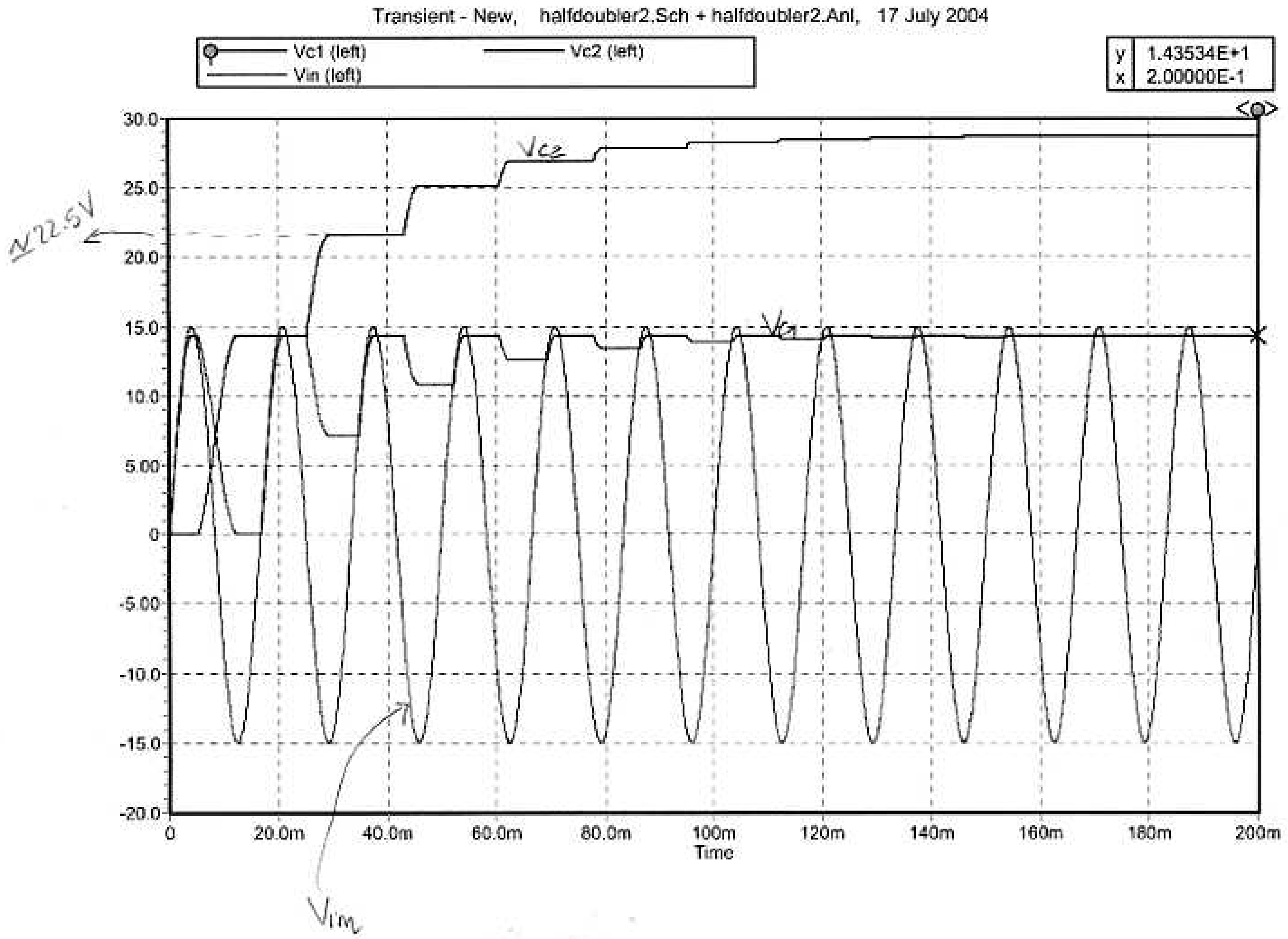


$V_{C1}(0) = 15 = \dfrac{Q_1}{C_1}$

$V_{C2}(0) = 15 = \dfrac{Q_2}{C_2}$

↑

those are the initial conditions


Transient - New, halfdoubler2.Sch + halfdoubler2.Anl, 17 July 2004
Vc1 (left)
Vc2 (left)
Vin (left)
y 1.43534E+1
x 2.00000E-1
30.0
25.0
20.0
15.0
10.0
5.00
0
-5.00
-10.0
-15.0
-20.0
0
20.0m
40.0m
60.0m
80.0m
100m
120m
140m
160m
180m
200m
Time
~22.5V
Vc2
Vc1
Vin

The final contitions are

$V_{C1} - V_{C2} = -15$

$\frac{Q_1}{C_1} - \frac{Q_2}{C_2} = -15$

but the total charge has to be the same to the beginning

$\Rightarrow Q_1 + Q_2 = 15 \cdot C_1 + 15 \cdot C_2$

$\Rightarrow Q_1 = 15C_1 + 15C_2 - Q_2$

$\Downarrow$

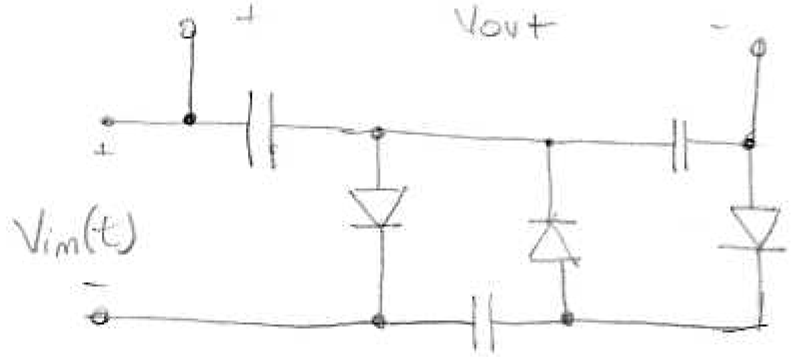
$$-\frac{Q_2}{C_1} + 15 + \frac{15C_2}{C_1} - \frac{Q_2}{C_2} = -15$$

$$-Q_2\left(\frac{1}{C_1} + \frac{1}{C_2}\right) = -30 - 15\frac{C_2}{C_1}$$

If $C_1 = C_2 \Rightarrow 2\frac{Q_2}{C} = +45 \Rightarrow V_{C2} = \frac{45}{2} = 22.5V$

You can go ahead and compute V_{C1} and then repeat the reasoning for the next cycle.

If we add another diode and capacitor pair we obtain a voltage tripler:



This circuit is used in CRT to drive the guides that are used to focus the electrons beam.