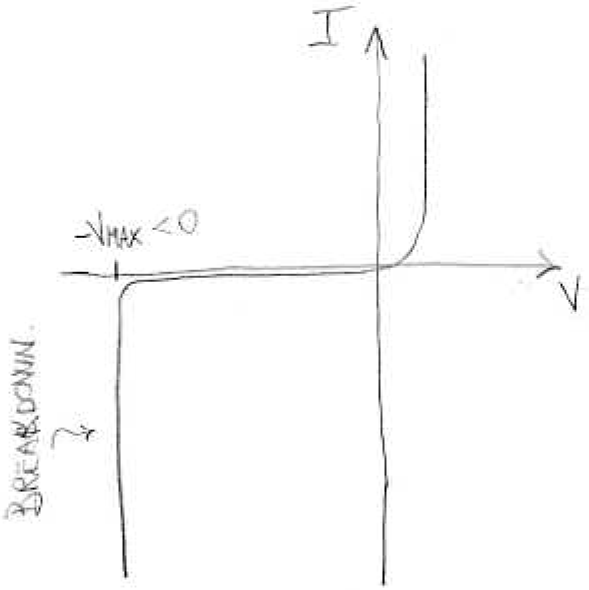
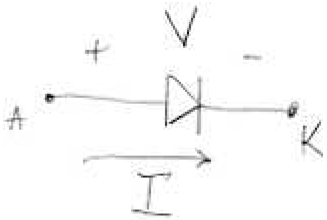
THE ZENER DIODE

We have never explored the behaviour of a diode for high negative voltage. A phenomenon called breakdown takes place where the electric field is so intense that the electrons velocity is very high. An electron, then, hitting a atom can rip another electron from it. The next free electron can do the same and an avalence phenomenon takes place:

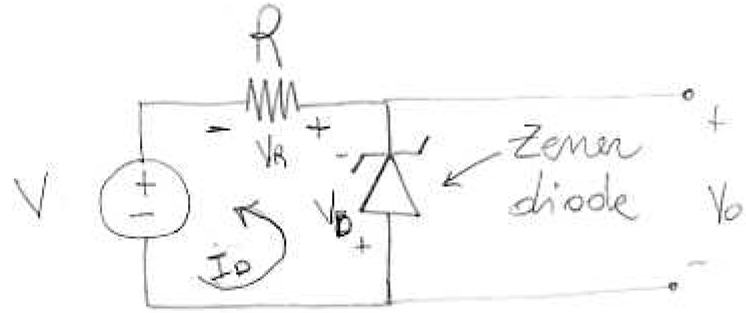




there are diodes that are built to work in the breakdow region. These diodes are called zener diode and are particularly suited to build power supplies.

To give you the intuition, in breakdown the V-I characteristic is almost vertical meaning that the voltage is a constant no matter what the current is. This is characteristic for a voltage source!

Let's use load line analysis for the following simple circuit:



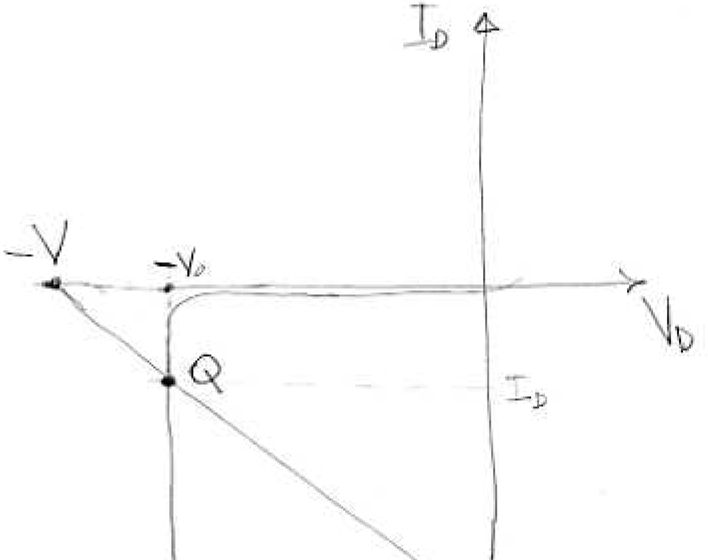
the load line equation is derived from the KVL equation:

$$V + V_D + V_R = 0$$

$$V + V_D + RI_D = 0$$

$$\Rightarrow I_D = \frac{-V - V_D}{R}$$

Consider $V > 0$

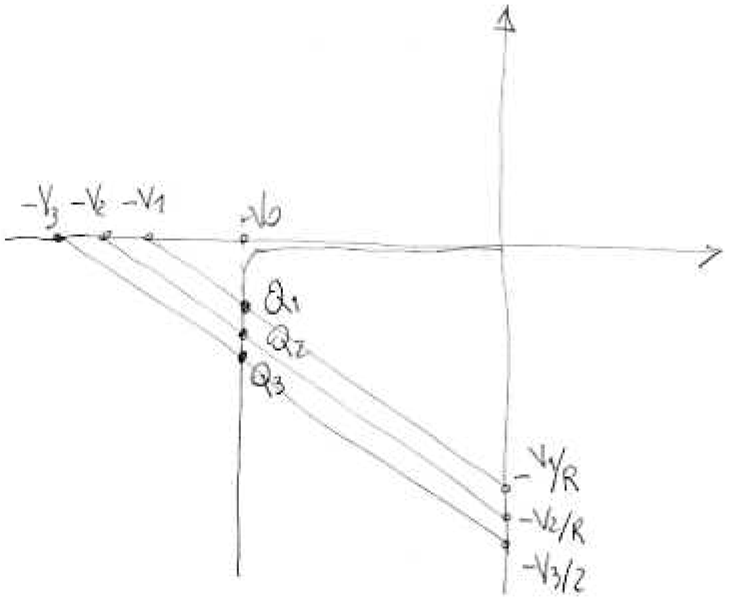


If $V_D = 0 \Rightarrow I_D = \frac{-V}{R}$

If $V_D = -V \Rightarrow I_D = 0$

The intersection of the load line with the diode characteristic gives us the quiscient point Q and hence the output voltage V_0 and the current in the diode.

If the input voltage changes, then the load line changes.



We obtain a set of parallel lines because R is fixed.

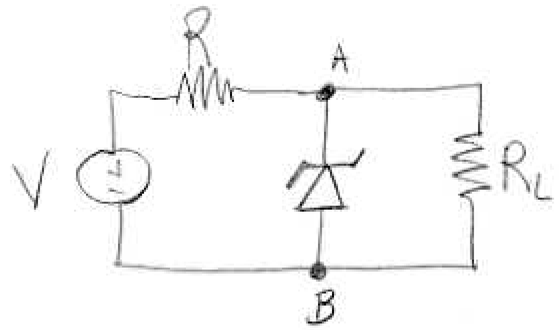
The output voltage ($V_0 = -V_D$) corresponding to the three quisient points Q_1, Q_2, Q_3 is always the same. We are regulating the input voltage. The output voltage is then stable and fixed to a specific value.

When you buy a zener dioded, it is characterized by a zener voltage which is the voltage at which the breakdown takes place.

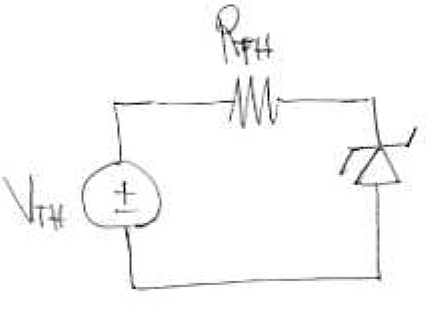
It is clear that, if the quiscient point is not in the breakdown region, then the output voltage depends on the input voltage. The diode then works as a regulator only if

$V > V_Z$ where V_Z is the zener voltage(V_Z is given on the datasheet as a positive voltage 4 but the breakdown happens at $-V_Z$ of course).

Let's consider the case where we want to use the output voltage for instance to power on a CD-player. Device can be caracterized by its Thevenin equivalent resistor. We denote the device with a load resistor R_L:



This case is not different from the previous one.
We can use the Thevenin equivalent circuit at A-B:

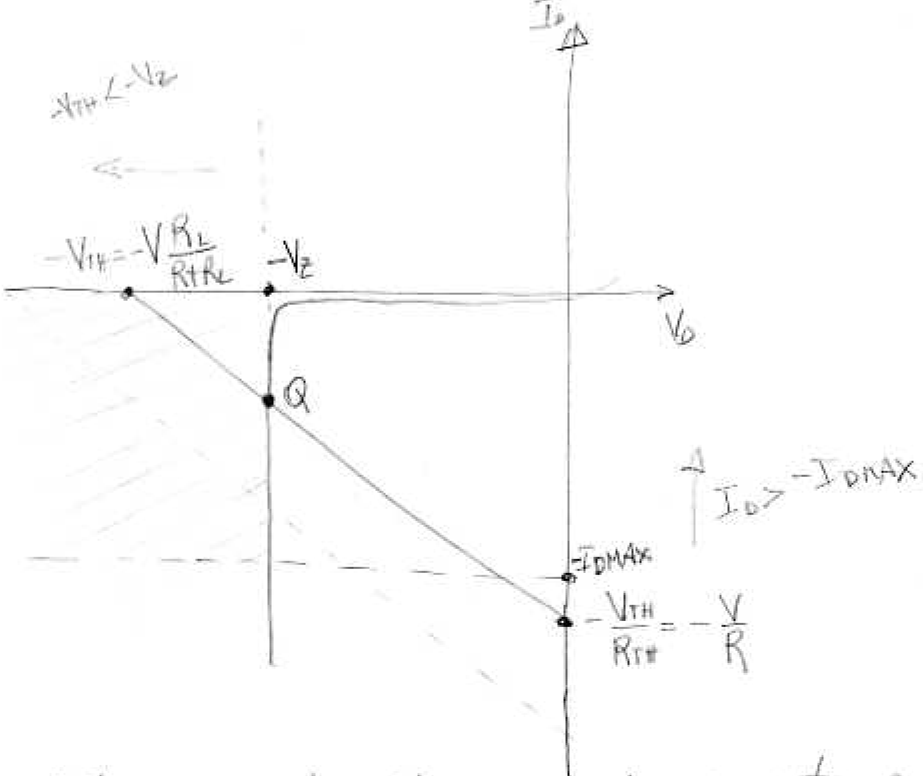


Where:

$$V_{TH} = V \frac{R_L}{R+R_L}$$

$$R_{TH} = R // R_L = \frac{R R_L}{R+R_L}$$

The new load line is then:



We want the quiescient point to be always in the breakdown region. So it must be

$$-V\frac{R_L}{R+R_L} < -V_Z \Rightarrow \quad V\frac{R_L}{R+R_L} > V_Z$$

On the other hand we have to make sure that the zener diode doesn't break. When you by a zener diode, it is also characterized by a maximum reverse current I_{DMAX}. So We want that $I_D > -I_{DMAX}$.

The previous inequalities have to hold in all operating conditions.

When we are desining a power supplier, the problem is the following:

- We have a voltage source that is not stable. Its value can swing betwen $V_{min} \leq V \leq V_{max}$.
- The load is not fixed, it depends on what kind of device we connect to the power supply. So $R_{L_{min}} \leq R_L \leq R_{L_{max}}$

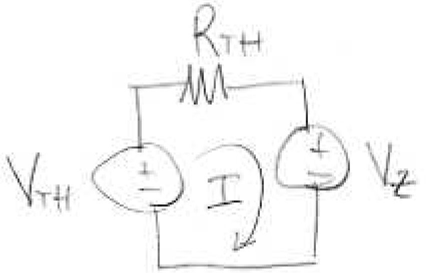
In all possible cases we have to make sure the the Q point is on the vertical line and doesn't cross the horizontal I_{DMAX} line.

V_{TH} is minimum when $V = V_{min}$ and $R_L = R_{L_{min}}$
So we have to have:

$$① \quad V_{min} \frac{R_{L_{min}}}{R + R_{L_{min}}} \geqslant V_Z$$

The diode current is maximum when $V = V_{max}$ and $R_L = R_{L_{max}}$

To understand this, whe notice that if the diode is in breakdown then the voltage across it is equal to $-V_Z$. We can then replace it wit a voltage source whose value is V_Z.



so $I = -I_D = \dfrac{V_{TH} - V_Z}{R_{TH}} =$

$$= \frac{V\frac{R_L}{R+R_L} - V_Z}{\frac{R R_L}{R+R_L}} = \frac{V R_L - V_Z(R+R_L)}{\cancel{R+R_L}} \cdot \frac{\cancel{R+R_L}}{R R_L}$$

$$= \frac{V}{R} - \frac{V_Z}{R /\!/ R_L}$$

I_D is maximize for $V = V_{max}$ and $R_L = R_{Lmax}$

so

$$I_D > -I_{DMAX} \Rightarrow I < I_{DMAX}$$

$$\frac{V_{max}}{R} - V_Z \frac{(R + R_{Lmax})}{R R_{Lmax}} < I_{Dmax} \quad (2)$$

Using ① and ② we can design our power supply.

For instance:

We want to design a power supply circuit To stabilize an input voltage that can change between 9V and 10V. Also the maximum load is ∞ (nothing is connected to the p.s.) and $10\,\Omega$. The output voltag has to be 5V.

So:

$V_Z = 5\text{ V}$ $\qquad V_{min} = 9\text{V}$ $\qquad V_{max} = 10\text{V}$

$R_{L\,min} = 10\,\Omega$ $\qquad R_{L\,max} = \infty$

from ①

$$\frac{9 \cdot 10}{R+10} \geqslant 5 \Rightarrow 90 \geqslant 5R + 50$$

$$\Rightarrow 40 \geqslant 5R \Rightarrow R \leq 8\,\Omega$$

from ②

$$\frac{10}{8} - \frac{5}{8} < I_{DMAX} \Rightarrow I_{DMAX} > \frac{5}{8}\text{ A}$$

We computed the properties of both R and the zener diode. 9

We havo to consider the power absorbed by the resistor.

The voltage across it is $V - V_Z$ so the power is:

$$P_R = \frac{(V - V_Z)^2}{R}$$

it is maximum when $V = V_{max}$:

$$P_{R_{max}} = \frac{(V_{max} - V_Z)^2}{R} = \frac{(10 - 5)^2}{8} = \frac{25}{8} \text{ W}$$

The maximum power shipped to the load is when the resistor R_L is minimum:

$$P_{L_{max}} = \frac{(V_Z)^2}{R_{L_{min}}} = \frac{25}{10} \text{ W}$$

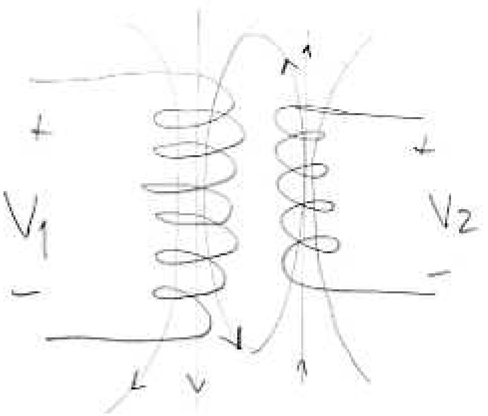
so notice that the power dissipated on R is not negligible. The efficiency of this p.s. is:

$$\eta = \frac{P_{L_{max}}}{P_{L_{max}} + P_{R_{max}} + P_{DZ_{max}}} = \frac{25/10}{25/10 + 25/8 + 5 \cdot 5/8} = 28.6\%$$

The reason is that the original constraints where pretty hard. Usually $V_{max} - V_{min}$ is of the order of millivolts.

We know now that this stage is useful to stabilize a voltage for different loads. Whe have to understand how to transform the 115 VAC coming out from the outlet to finally have a 12 VDC for example.

We first use a tranformer to decrease the amplitude of the 115 VAC. A transformer is built using two inductors L_1 and L_2. The Primary voltage (115 VAC for instance) is connected to L_1 that will generate a magnetic field. The magnetic field lines will pass through the second solenoid L_2 an will generate a voltage across L_2.



If L_1 has N_1 turns an L_2 N_2 turns, then: $\frac{V_1}{V_2} = \frac{N_1}{N_2}$

By designing N_1 and N_2 we can choose V_2 if we know V_1 (in our case 115 VAC).

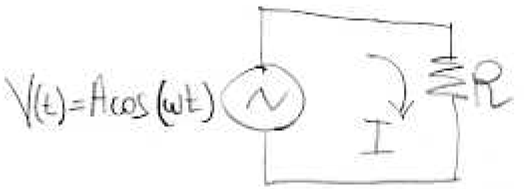
For instance if we want $V_2 = 11.5$ VAC

$\Rightarrow \frac{N_1}{N_2} = 10$, we could use $N_1 = 100$ and $N_2 = 10$.

We need a note here:

When talking about AC signals we use RMS (Root Mean Square) voltage.

Consider a circuit like the following



V and I are AC (sinusoidal signals). We want to compute the power that is tranferred to R:

$$p(t) = V(t)\, I(t) = \frac{V(t)^2}{R} = \frac{A^2 \cos^2(\omega t)}{R}$$

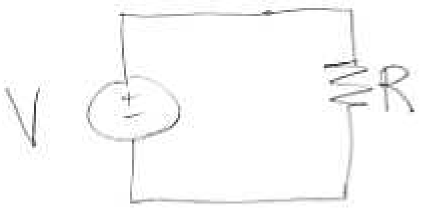
this power is instantaneous. We can compute the average tranfered to in a period $T = \frac{2\pi}{\omega}$

$$P_R = \frac{1}{R}\,\frac{1}{T}\int_0^T A^2 \cos^2(\omega t)\, dt = \frac{1}{R}\,\frac{1}{T}\int_0^T A^2\left(\frac{1}{2} + \cos(2\omega t)\right) dt$$

$$\cos^2(\omega t) = \frac{1 + \cos(2\omega t)}{2}$$

$P_R = \frac{A^2}{2R}$ because $\int_0^{2\pi/\omega} \cos(2\omega t)\, dt = 0$

Now we ask ourself what would be the value of a ~~A~~DC voltage source that will mak R absorb the same power? :



$P_R = \frac{V^2}{R}$

then $\frac{V^2}{\cancel{R}} = \frac{A^2}{2\cancel{R}} \Rightarrow V = \frac{A}{\sqrt{2}}$ this the Rms Voltage

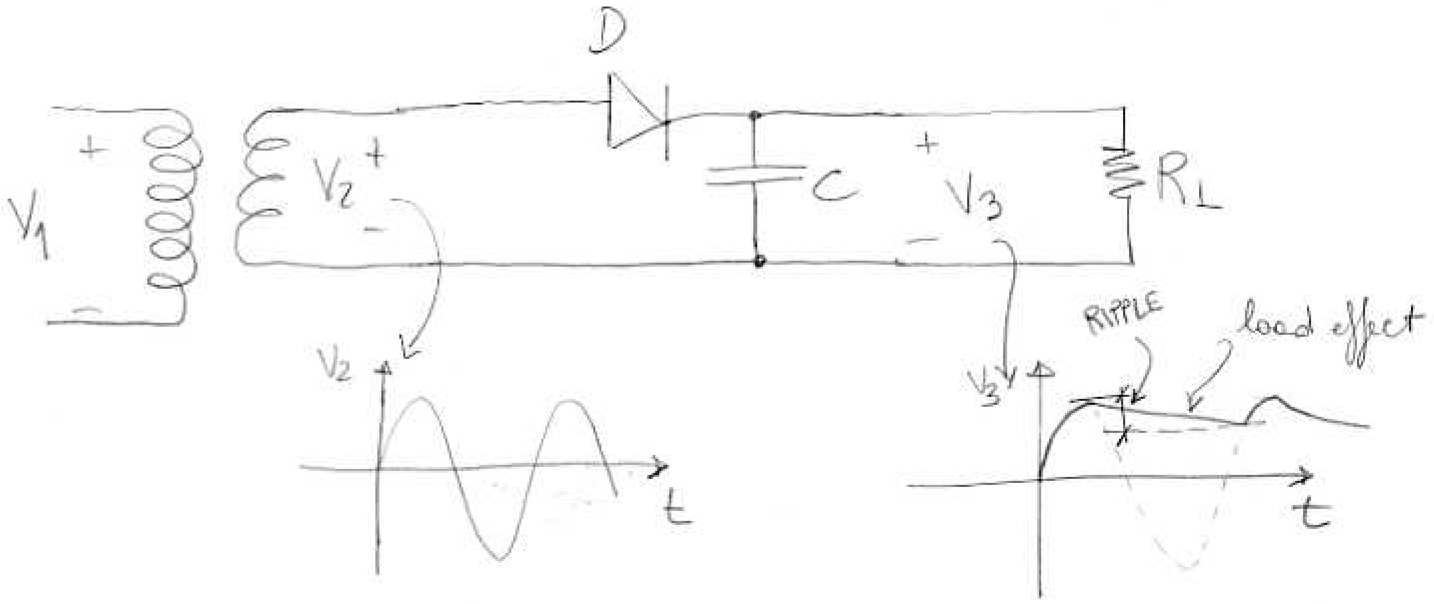
So an AC voltage whose peak value is A has an RMS voltage equal to $\frac{A}{\sqrt{2}}$.

When you buy a tranformer, the output is given in RMS voltage, so for instance an output of 12 V AC means a sinusoid whose peak value is $12 \cdot \sqrt{2}$ V.

So now we have a sinusoidal voltage whose amplitute is A.

Whe want to get something is closer to a DC voltage in order not to stress the regulator too much.

We can use a peak detector with a diode and a capacitor:



The diode acts as an half wave rectifier. The capacitor charges to the peak value of the voltage and ~~discharges~~ discharges when the diode is off (during the negative half of the sinusoid). The capacitor discharges with a time constant equal to $R_L C$

Where R_L model our regulator.
The residual voltage oscillation is called ripple. We can comput it very easily:

- the capacitor is charged at A and the time constant is $R_L C$. Also the input frequency is 60 Hz so we need to compute the voltage for $t \simeq \frac{1}{60}$

$$\Delta V = A - A e^{-\frac{1}{60\tau}} = A - A e^{-\frac{1}{60 R_L C}}$$

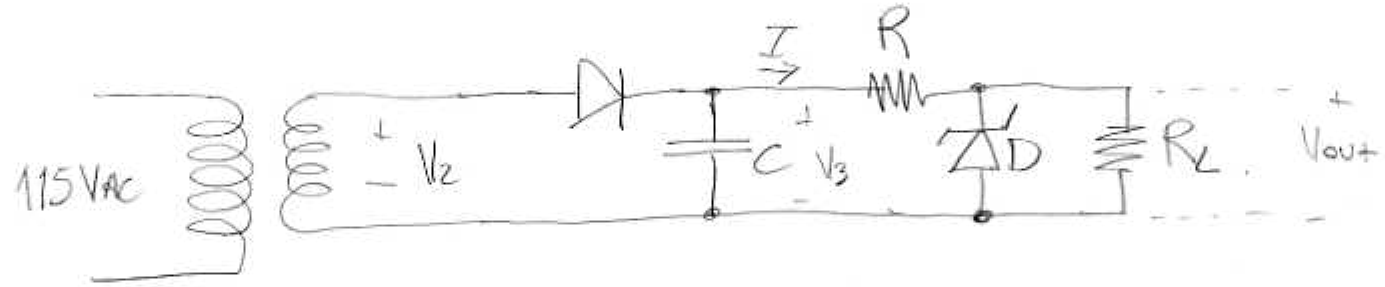
If $R_L = 10\,\Omega$ and we want a ripple whic is 1 % of the voltage A, then:

$$\frac{\Delta V}{A} = 0.01 = 1 - e^{-\frac{1}{600C}} \Rightarrow -0.99 = -e^{-\frac{1}{600C}}$$

$$\Rightarrow -\frac{1}{600C} = \ln(0.99) \Rightarrow C = -\frac{1}{600 \ln(0.99)} = 0.166\text{F}$$

which is a huge capacitor.

Whe can use a smaller capacitor an a voltage regulator using zener diode.



We have the following specifications:

- $R_{L_{min}} = 10\,\Omega$, $V_{out} = 5V$

assuming the zener diode ideal we can use the following method:

$V_z = 5V$ we fix $\eta = \dfrac{P_{Lmax}}{P_{Lmax} + P_{Rmax} + P_{DZmax}} = 80\%$

efficiency

$$\frac{\dfrac{(V_Z)^2}{R_{Lmin}}}{\dfrac{(V_{3max} - V_Z)^2}{R} + \dfrac{(V_Z)^2}{R_{Lmin}} + V_Z I_{DMAX}} = 0.8$$

$$2.5 = 0.8\left[\frac{(V_{3max} - V_Z)^2}{R} + 2.5 + 5 I_{Dmax}\right] \Rightarrow$$

$$\frac{0.8}{R}(V_{3max} - 5)^2 + 5 I_{Dmax} = 2.5 \times 0.2 = \frac{1}{2}$$

from ②

$$\frac{V_{3max}}{R} - \frac{V_Z}{R} < I_{Dmax}$$

from ①

$$V_{3min} \frac{10}{R+10} \geq 5$$

We have 3 eq. in 4 unknown. We could now fix the input ripple which is $V_{3max} - V_{3min}$ to obtain another eq.

We notice that $I = \frac{V_3 - V_Z}{R}$ and can be approximately considered constant so the voltage decreases as a ramp because $V = \frac{1}{C}\int i\,dt$. So:

$$V_{3max} - V_{3min} = \frac{V_3 - V_Z}{RC} \Delta t = \frac{V_3 - V_Z}{RC} \frac{1}{60}$$

this is also I_{Dmax} because when $R_L = \infty$ this current passes entirely through the diode

$$\begin{cases} \dfrac{V_{3max} - V_Z}{R} \leq I_{Dmax} \\[2ex] V_{3min} \dfrac{10}{R+10} \geq 5 \\[2ex] V_{3max} - V_{3min} = \dfrac{I_{Dmax}}{C} \dfrac{1}{60} \\[2ex] \dfrac{0.8}{R} \left(V_{3max} - 5\right)^2 + 5 I_{Dmax} = \dfrac{1}{2} \end{cases}$$

If we fix $C = 2200\,\mu F$

$$V_{3min} = V_{3max} - \frac{I_{Dmax}}{6 \cdot 2200 \cdot 10^{-5}} = V_{3max} - \frac{I_{Dmax}}{13.2 \cdot 10^{-2}}$$

$$\begin{cases} V_{3max} - 5 \leq R I_{DMAX} \\[2ex] 10 V_{3max} - \dfrac{I_{Dmax}}{13.2 \cdot 10^{-3}} \geq 5R + 50 \\[2ex] \dfrac{0.8}{R} \left(V_{3max} - 5\right)^2 + 5 I_{Dmax} = \dfrac{1}{2} \end{cases}$$

This is a non linear system of inequalities. 18

It can have of course one solution (rare) many solution (and you have to pick the most convenient) or $\emptyset$ solutitions (meaning that your choice for c of for η are bad choices).

After solving the system, you will find V_{3max} and $\frac{V_{3max}}{\sqrt{2}}$ will be the RMS value of your tranformer output.