— LINEARITY AND SUPERPOSITION

Def: Consider a function f
The function is linear if

$$f(ax+by) = a f(x) + b f(y)$$

> A circuit is linear if the effect due to the source X is proportional to X itself

A resistor is a linear component because $V=RI$ so if we double the current flowing through it, the voltage doubles (here we consider the current to be the source and the voltage to be the effect).

All circuit we have seen until now are linear. Given a circuit, solve it using for instance the node-Voltage analysis and obtain

$$\underbrace{[G]}_{\text{conductance matrix}}\underbrace{[V]}_{\text{node voltages}}=\underbrace{[I]}_{\text{current sources}}$$

([X] means the matrix X)

$[G]$ is non singular so it is invertible:

$$[V] = [G]^{-1}[I]$$

So each node voltage is a linear combination of currents sources:

$$V_i = g_{i1} I_1 + g_{i2} I_2 + \cdots + g_{iN} I_N$$

Which is a linear function.

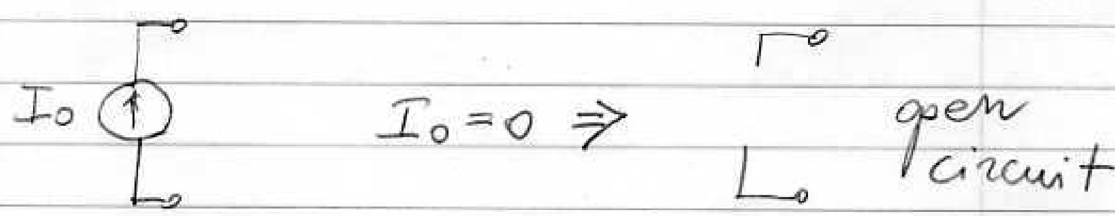
I_j is a current source (we can always reduce our circuit to have only current sources but nothing changes if you consider the indirect method because it adds other terms to that equation of the form αV_j when V_j is a voltage source).

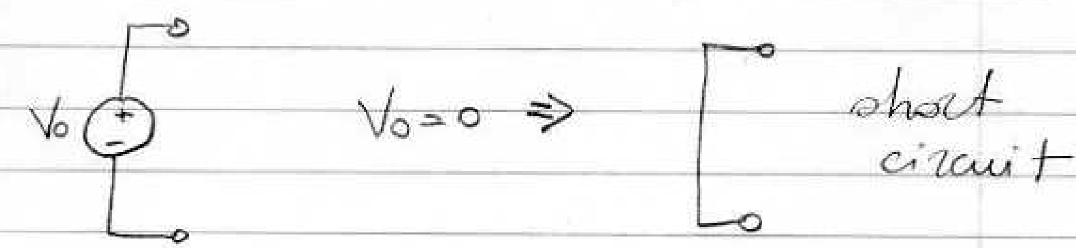
If you look at that expression it is the sum of the contribution of each current to the node voltage.

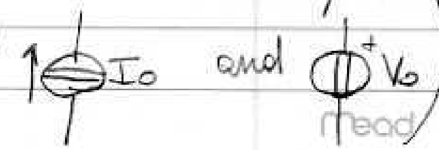
so we coult compute the contribution of I_j to V_i by setting all the other current generators to $\emptyset$ ($I_k = 0 \ \forall k \neq j$). If we do that for each current source, then we can combine the individual contributions to obtain the total voltage.

This method is called superposition of effects.

What does it mean to set a source to zero?

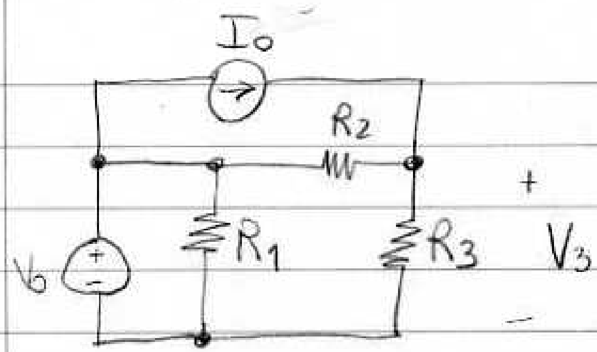




(this is why I like another 'set' of symbols for the sources: 

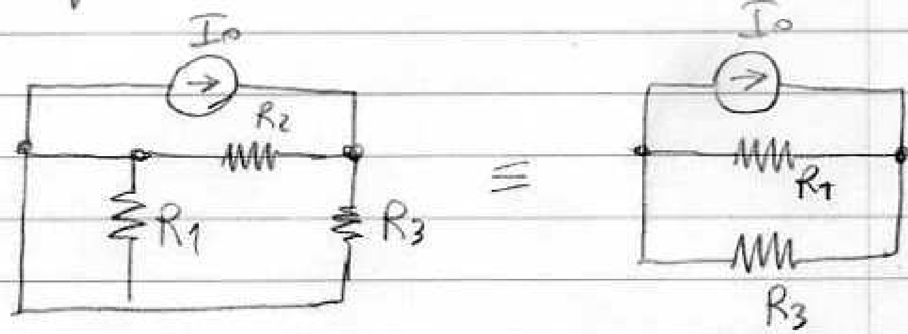
)

Example



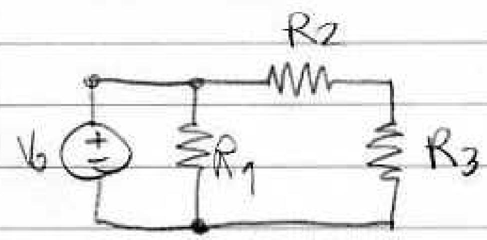
we want to compute V_3

We first set $V_0 = 0$



$$I_3\Big|_{V_0=0} = I_0 \frac{R_1}{R_1+R_3} \Rightarrow V_3\Big|_{V_0=0} = I_0 \frac{R_1 R_3}{R_1+R_3}$$

Then we set $I_0 = 0$



$$V_3\Big|_{I_0=0} = V_0 \frac{R_3}{R_2+R_3}$$

Finally we sum the two contributions

$$V_3 = V_3\Big|_{I_0=0} + V_3\Big|_{V_0=0} = V_0 \frac{R_3}{R_2+R_3} + I_0 \frac{R_1 R_3}{R_1+R_3}$$

This works only for linear circuits!!!

Leave dependent sources intact!!!

Superposition cannot be used to compute power: Consider the power on the element i:

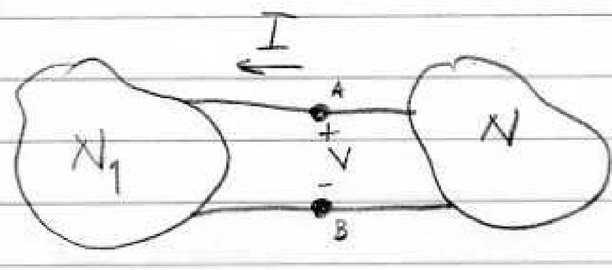
$$P_i = V_i I_i = (g_{i1}I_1 + \dots + g_{ii}I_i + \dots + g_{iN}I_N) I_i =$$

$$= g_{i1}I_1 I_i + \dots + g_{ii}I_i^2 + \dots + g_{iN}I_N I_i$$

This function is not linear. If you double I_i the power is going to be 4 times bigger.

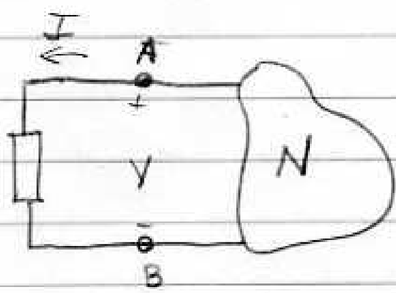
— SUBSTITUTION THEOREM

Consider a circuit an split it into two parts



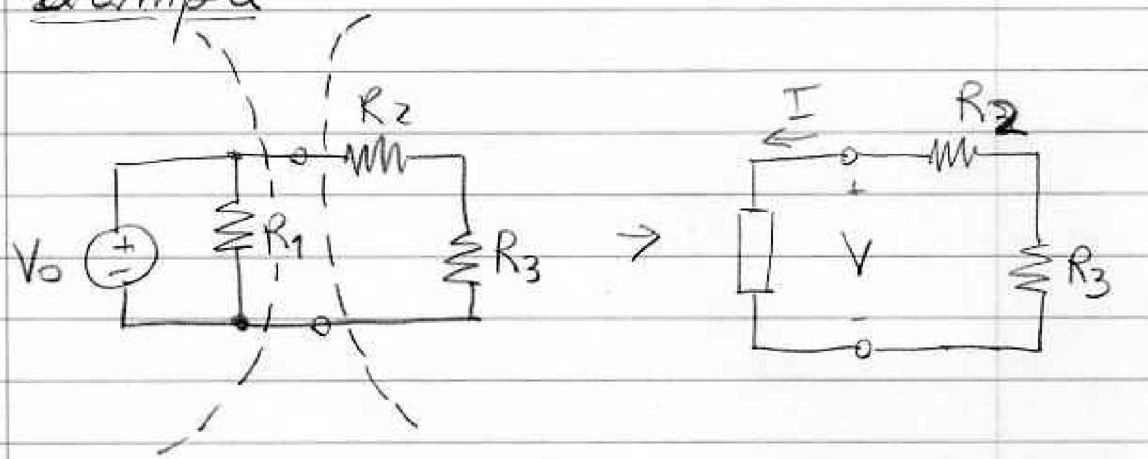
here we can compute v and I

We want to find an element (a blackbox) to substitute to N_1



Since we don't know this element, in solving this circuit we have to consider both v and I unknown. So basically we have more unknown than equations.

Example



$$V = -I(R_2 + R_3)$$

Anyway, in general, if N is a complicated circuit you will find a system of equations where I and V are present and unknown.
By applying substitution you can reduce your circuit to only one equation in the variables I and V (like in the small example above).

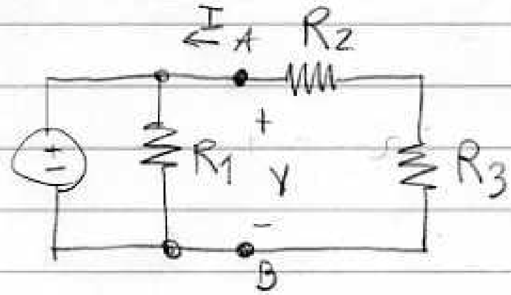
$$V = f(I) \quad \text{and} \quad I = f^{-1}(V)$$

So if we fix one of the two quantities, the other is going

to be determined as well.
So it is sufficient to impose one of the two quantities to be equal to the original circuit and the other will be the same as a consequence.

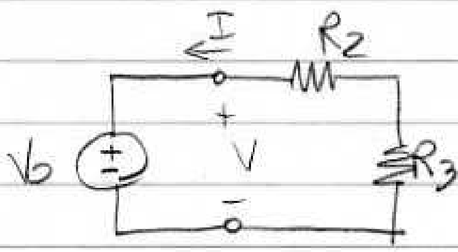
It means that we can set the element to be a current source or a voltage source (substitution theorem)

Example

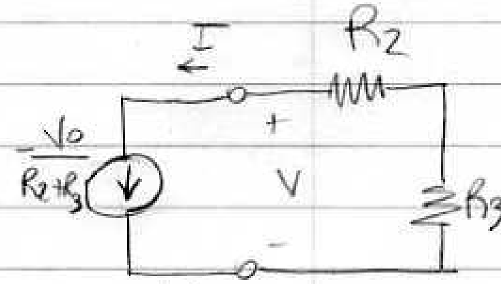


$$V = V_0$$
$$I = -\frac{V_0}{R_2+R_3}$$

$\equiv$



or



$$V = V_0$$
$$I = -\frac{V_0}{R_2+R_3}$$

$$I = -\frac{V_0}{R_2+R_3}$$
$$V = -I(R_2+R_3) = V_0$$

There are exeptions:

After substitution you could obtain

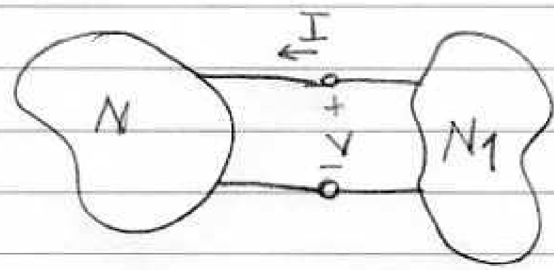
① I = constant or
② V = constant

it means that N behaves as a current source in ① and a voltage source in ②.

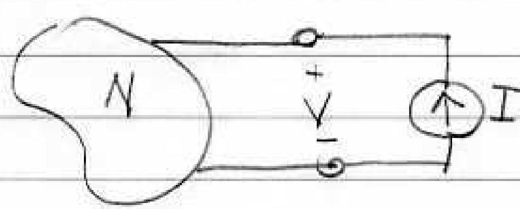
Then the only thing you can do is to set the element to a voltage source in ① and a current source in ② whose values are equal to the original I or V.

– NORTON AN THEVENIN EQ.

Consider again a circuit which is divided into two parts:



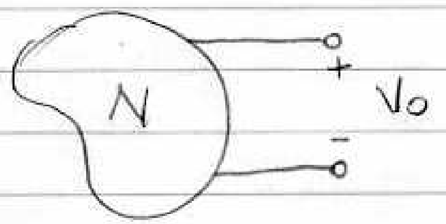
the substitution theorem says that we can substitute N_1 with a current source:



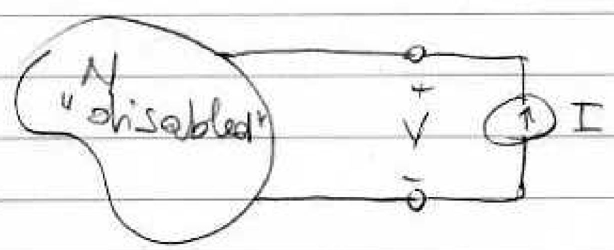
What is V ? How can we compute it ?

We can use the superposition principle considering two sets of sources: I and all the sources in N then $V = V\Big|_{I=0} + V\Big|_{\substack{N \text{ is} \\ \text{"disabled"}}}$

"N is disabled" means that all current sources are considered open circuits and all voltage sources are considered short-circuits.
So we first set $I=0$

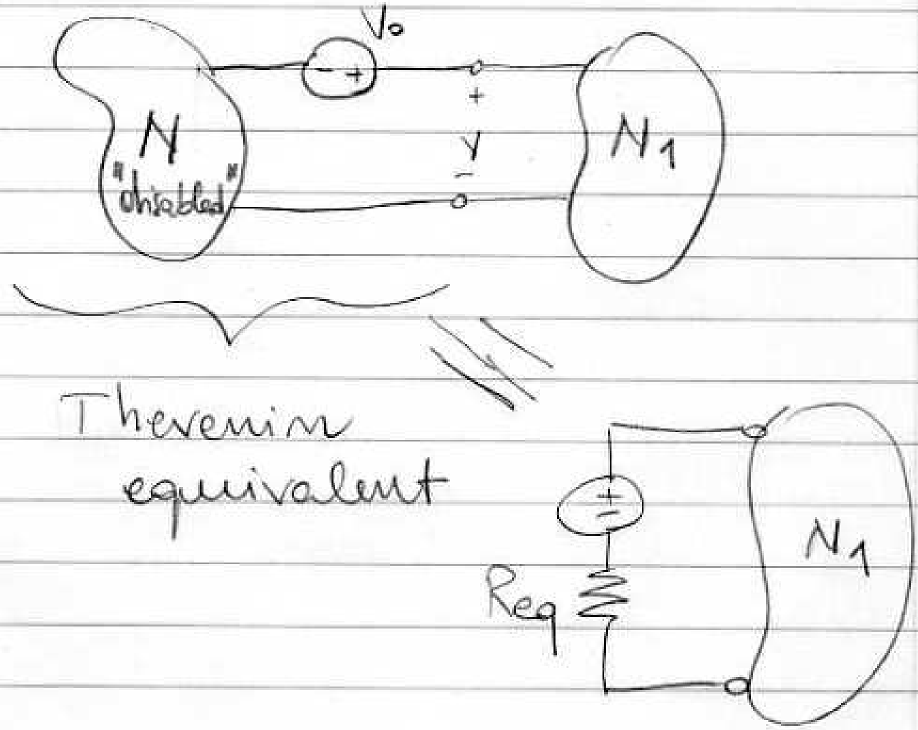


this is the voltage that we observe when N is disconnected from the rest of the circuit.
Then we "disable" N:



this look lik a resistor.

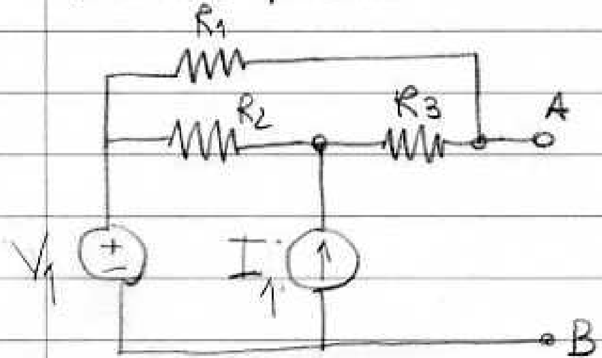
Since the voltage is the sum, the situation is like this



Now you help me in finding the Norton equivalent.

The exceptions to the applications of equivalences are for the same cases of the substitution method.

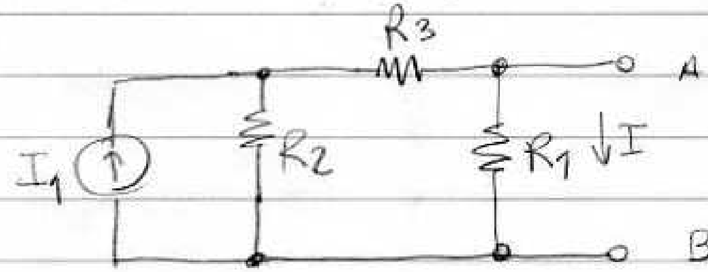
Example



use Thevenin eq. to characterize the circuit at A-B.

Compute Vo (voltage V_{AB} when the circuit is open, nothing is connected)

$$V_0 = V_1 + V_{AB}\Big|_{V_1=0}$$

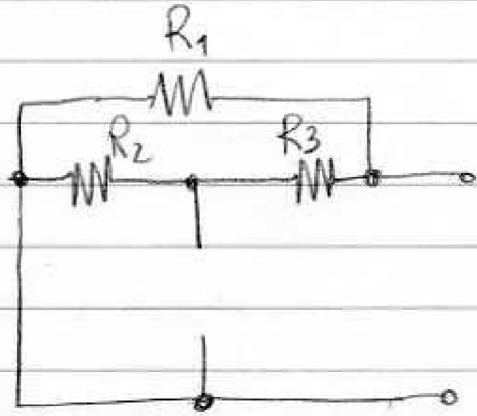


$$I = I_1 \frac{R_2}{R_1+R_2+R_3}$$

$$V_{AB}\Big|_{V_1=0} = I_1 \frac{R_1 R_2}{R_1+R_2+R_3}$$

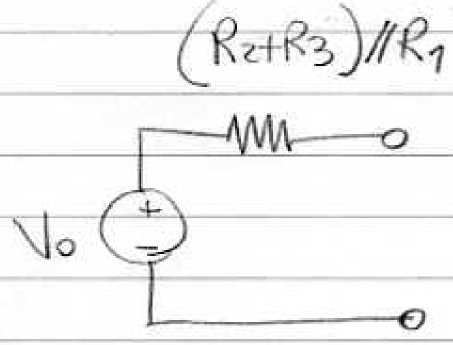
$$V_0 = V_1 + I_1 \frac{R_1 R_2}{R_1 + R_2 + R_3}$$

We have to compute Req :



$$R_{eq} = (R_2 + R_3) \,/\!/\, R_1$$

the Th. equivalent circuit is

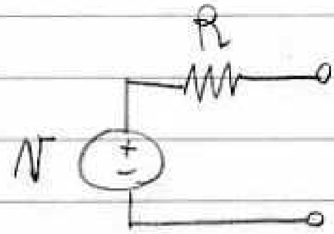


Find now the Norton eq.

– VOLTAGE AMLIFIER

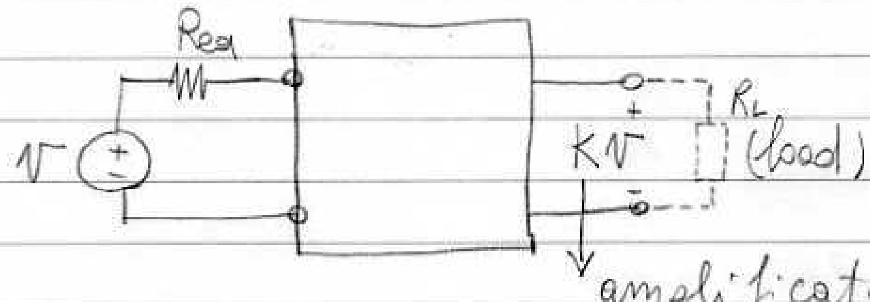
We need a device that amplifies our voltogl signal

A moder for our source is:



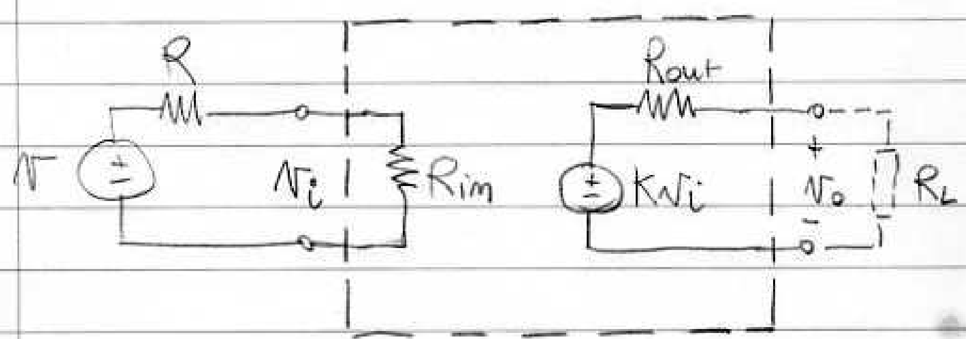
(Why? a little Hint, only two letters T. E.)

We need a box that multiplies the input signal by a constant



A simple model for the amplifier is then a controlled voltage source. It is actually a little bit more complicated because we cannot buil an'

ideal controlled voltage source.
It looks like this:



If you look at the black box
the amplification is:

$$K' = \frac{v_o}{v_i} = \frac{Kv_i \dfrac{R_L}{(R_{out}+R_L)}}{v \dfrac{R_{in}}{R+R_{in}}}$$

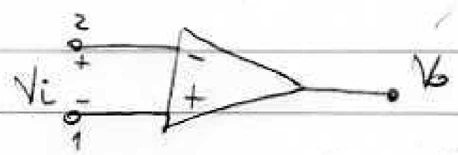
In order to have $K' \simeq K$
we need $R_{in} \gg R$ and $R_{out} \ll R_L$.

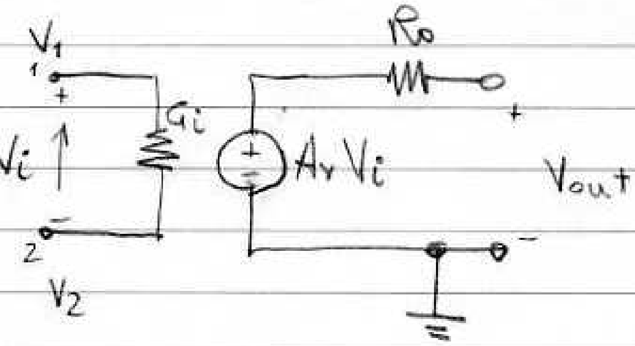
So, to make it closer to ideal we
have to make R_{in} very big
and R_{out} very small.

- THE OPERATIONAL AMPLIFIER

It is a component that we define, we play with. Then there are many way of implementing it but all of them can just approximate our mathematical definition (well, the approximation is not that bad).

Symbol:





1) $G_i = 0$

2) $R_o = 0$

3) $A_v = -\infty$

4) $B = \infty$ (bandwidth)

5) If $V_1 = V_2 \Rightarrow V_o = 0$

6) All the characteristic are temperature invariant