EECS 42 Introduction to Electronics for Computer Science

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Lecture #8 Node Equations

- Basic concept
- Generalization to supernodes

http://inst.EECS.Berkeley.EDU/~ee42/

Game Plan 02/19/03

Wednesday 02/19/03

- ☐ EC Response: Sketch/Trend, Exponential Solution
  Schwarz and Oldham: 2.3, 2.5,2.6

Next (6th) Week

- ☐ Monday Node Equation App.; Midterm Review
- ☐ Wednesday Quiz and Basic Logic: Sheila Ross

Next Next (7^{th}) Week:

- ☐ Monday Logic
- ☐ Wednesday: Midterm In Class, Closed Book

Problem Set #5 – Out 2/19/03 - Due 2/26/03 2:30 in box in 240 Cory

Node Analysis: basic, supernode, advanced; review: circuit analysis, transients

FORMAL CIRCUIT ANALYSIS

Systematic approaches to writing down KCL and KVL: Section 2.3 of Text - In particular use of KCL gives NODAL ANALYSIS

Mathematical foundation is rigorous: EECS 104

Nodal Analysis: Node voltages are the unknowns
Mesh Analysis: Branch currents are the unknowns

} Use one **or** the other for circuit analysis

We will do only nodal analysis – (because voltages make more convenient variables than currents) Thus omit Text Section 2.4 ; it is redundant.

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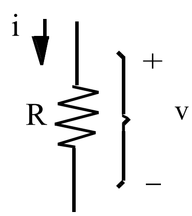
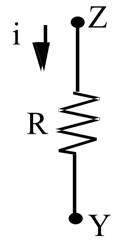
Version Date 02/19/03

REVIEW OF IV CHARACTERISTICS OF A RESISTOR



If we use associated current and voltage (i.e., i is defined as into + terminal), then $v = iR$ (Ohm's law)



Another version of the same statement, and the one most important to us:

$i = (V_Z - V_Y)/R$ (Ohm's law)

NOTE ORDER OF NODES: $V_Z - V_Y$!

FORMAL CIRCUIT ANALYSIS USING KCL: NODAL ANALYSIS

(Memorize these steps and apply them rigorously!)

1 Choose a Reference Node

2 Define unknown node voltages (those not fixed by voltage sources)

3 Write KCL at each unknown node, expressing current in terms of the node voltages (using the constitutive relationships of branch elements*)

4 Solve the set of equations (N equations for N unknown node voltages)

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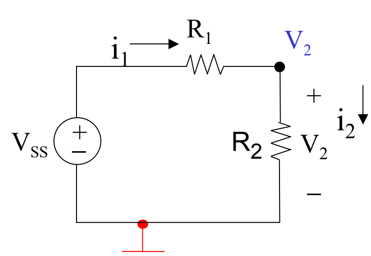
* With inductors or floating voltages we will use a modified Step 3: The Supernode Method – see slide 10

NODAL ANALYSIS USING KCL –Example: The Voltage Divider –

1 Choose reference node

2 Define unknown node voltages

3 Write KCL at unknown nodes



$$\frac{V_{SS}-V_2}{R_1}=\frac{V_2-0}{R_2}$$

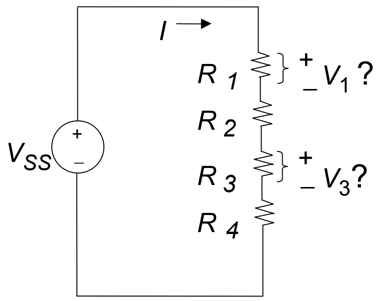
4 Solve:

$$V_2 = V_{SS}\cdot\frac{R_2}{R_1+R_2}$$

This is of course the voltage divider formula and is by itself very useful.

GENERALIZED VOLTAGE DIVIDER (solved without Nodal Analysis)

Circuit with several resistors *in series*



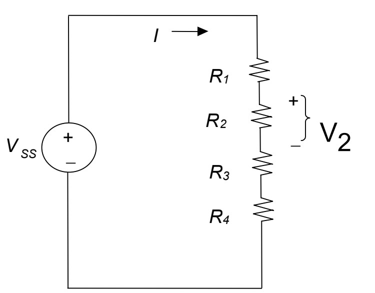
- We know
  $$I = V_{SS}/(R_1 + R_2 + R_3 + R_4)$$
- Thus,
  $$V_1 = \frac{R_1}{R_1 + R_2 + R_3 + R_4} \cdot V_{SS}$$

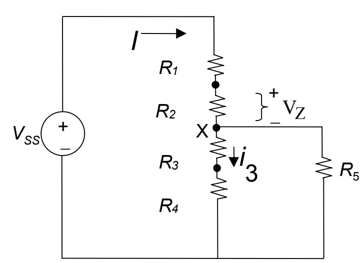
and

$$V_3 = \frac{R_3}{R_1 + R_2 + R_3 + R_4} \cdot V_{SS}$$

etc.. etc..

WHEN IS VOLTAGE DIVIDER FORMULA CORRECT?





$$V_2 = \frac{R_2}{R_1 + R_2 + R_3 + R_4} \cdot V_{SS}$$

Correct if nothing else connected to nodes

$$V_Z \neq \frac{R_2}{R_1 + R_2 + R_3 + R_4} \cdot V_{SS}$$

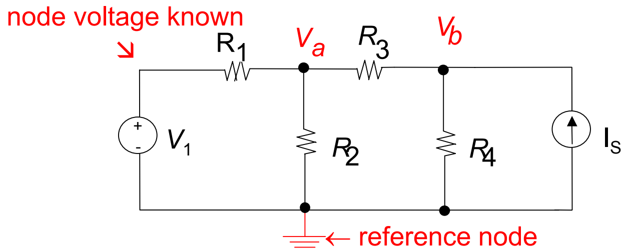
because R_5 removes condition of resistors in series – i.e. $i_3 \neq I$

What is V_Z?

Answer: $\frac{R_2}{R_1 + R_2 + R_5 \| (R_3 + R_4)} \cdot V_{SS}$

ANOTHER EXAMPLE OF NODAL ANALYSIS

Choose a reference node and define the node voltages (except reference node and the one set by the voltage source);



Apply KCL:

$$\frac{V_a - V_1}{R_1} + \frac{V_a - V_b}{R_3} + \frac{V_a}{R_2} = 0$$

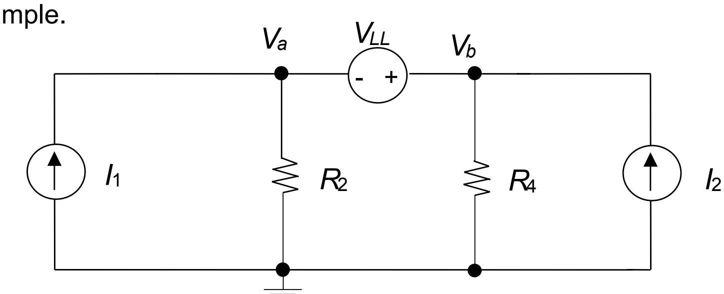
$$\frac{V_b - V_a}{R_3} + \frac{V_b}{R_4} - I_S = 0$$

You can solve for V_a, V_b.

What if we used different ref node?

NODAL ANALYSIS WITH "FLOATING" VOLTAGE SOURCES

A "floating" voltage source is a voltage source for which neither side is connected to the reference node. V_{LL} in the circuit below is an example.



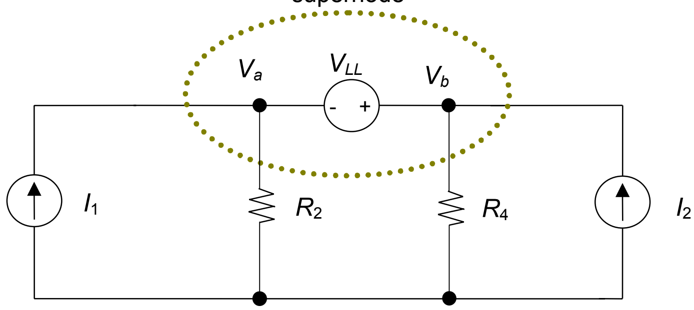
What is the problem? → We cannot write KCL at node a or b because there is no way to express the current through the voltage source in terms of $V_a - V_b$.

Solution: Define a "supernode" – that chunk of the circuit containing nodes a and b. Express KCL at this supernode.

FLOATING VOLTAGE SOURCES (cont.)

Version Date 02/19/03

Use a Gaussian surface to enclose the floating voltage source; write KCL for that surface



We have two unknowns: V_a and V_b.

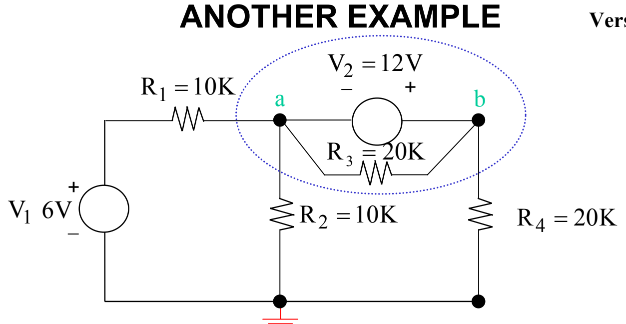
We obtain one equation from KCL at supernode: $I_1 - \frac{V_a}{R_2} - \frac{V_b}{R_4} + I_2 = 0$

We obtain a second "auxiliary" equation from the property of the voltage source: $V_{LL} = V_b - V_a$ (often called the "constraint")

$\Rightarrow$ 2 Equations & 2 Unknowns

ANOTHER EXAMPLE

Version Date 02/19/03



1 Choose reference node (can it be chosen to avoid floating voltage source?)

2 Label unknowns V_a and V_b

3 Equation at supernode: $\frac{V_1 - V_a}{R_1} = \frac{V_b}{R_4} + \frac{V_a}{R_2}$ $\rightarrow$ $V_a\left(\frac{1}{R_1} + \frac{1}{R_2}\right) + \frac{V_b}{R_4} = \frac{V_1}{R_1}$

4 Auxiliary equation: $V_b - V_a = V_2$ $\longrightarrow$ $V_a \quad - V_b = -V_2$

Solve: $V_a\left(\frac{R_4}{R_1} + \frac{R_4}{R_2} + 1\right) = V_1\frac{R_4}{R_1} - V_2$

$V_b = V_a + V_2$

SOLUTION: $V_a = 0$

$V_b = 12$