EECS 42 Introduction to Electronics for Computer Science

Andrew R. Neureuther

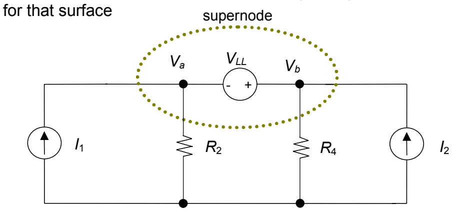
Lecture #9 Node Equations

- Recap and Checking Solutions
- Applications to parallel and bridge
- Midterm Exam Topics
- Thevenin/Norton Eq. Cir. Review

http://inst.EECS.Berkeley.EDU/~ee42/

FLOATING VOLTAGE SOURCES (cont.)

Use a Gaussian surface to enclose the floating voltage source; write KCL for that surface



We have two unknowns: V_a and V_b.

We obtain one equation from KCL at supernode: $I_1 - \frac{V_a}{R_2} - \frac{V_b}{R_4} + I_2 = 0$

We obtain a second "auxiliary" equation from the property of the voltage source: $V_{LL} = V_b - V_a$ (often called the "constraint")

$\Rightarrow$ 2 Equations & 2 Unknowns

Game Plan 02/24/03

Monday 02/24/03

- ☐ **Node Equations: S&O 2.3, 2.5,2.6; Exam Topics; Thevenin Review**

Wednesday 02/26/03: Sheila Ross instructor

- ☐ **Quiz on Basic Circuit Analysis and Transients**
- ☐ **Logic – Functions, Tables, Circuit Symbols 391-406**

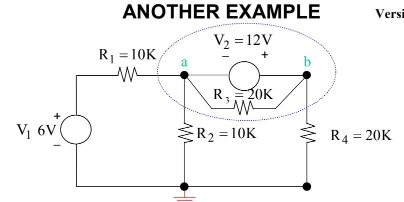
Next (7th) Week:

- ☐ **Monday 3/3: Brief Exam Review; Logic Synthesis**
- ☐ **Monday 3/3: TA Exam Review Session (247 Cory?)**
- ☐ **Wednesday: Midterm In Class, Closed Book**

Problem Set #5 – Out 2/19/03 - Due 2/26/03 2:30 in box in 240 Cory; Node Analysis: basic, supernode, advanced; review: circuit analysis, transients

No Problem Set Due 7th week, Problem set #6 out Monday 3/3 and due at 2:30 3/10 in box in 240 Cory

ANOTHER EXAMPLE



1 Choose reference node (can it be chosen to avoid floating voltage source?)

2 Label unknowns V_a and V_b

3 Equation at supernode: $\frac{V_1 - V_a}{R_1} = \frac{V_b}{R_4} + \frac{V_a}{R_2}$ → $V_a(\frac{1}{R_1} + \frac{1}{R_2}) + \frac{V_b}{R_4} = \frac{V_1}{R_1}$

4 Auxiliary equation: $V_b - V_a = V_2$ → $V_a - V_b = -V_2$

Solve: $V_a(\frac{R_4}{R_1} + \frac{R_4}{R_2} + 1) = V_1 \frac{R_4}{R_1} - V_2$

$V_b = V_a + V_2$

SOLUTION: $V_a = 0$

$V_b = 12$

FORMAL CIRCUIT ANALYSIS USING KCL: NODAL ANALYSIS

(Memorize these steps and apply them rigorously!)

1 Choose a Reference Node ⏚

2 Define unknown node voltages (those not fixed by voltage sources)

3 Write KCL at each unknown node, expressing current in terms of the node voltages (using the constitutive relationships of branch elements*)

4 Solve the set of equations (N equations for N unknown node voltages)

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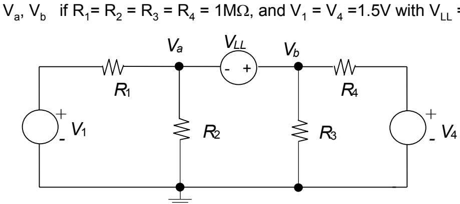
* With inductors or floating voltages we will use a modified Step 3: The Supernode Method – see slide 10

NODAL ANALYSIS EXAMPLE

Find V_a, V_b if $R_1 = R_2 = R_3 = R_4 = 1M\Omega$, and $V_1 = V_4 = 1.5V$ with $V_{LL} = 1V$



Solution: At supernode enclosing nodes a and b :

$(V_1 - V_a)/R_1 - V_a/R_2 = V_b/R_3 + (V_b - V_4)/R_4$ *and*

$V_b = V_a + V_{LL}$ *Thus:* $V_a = 0.25$, $V_b = 1.25$ Be sure to check answer with KCL!

CHECK ANSWER WITH KCL

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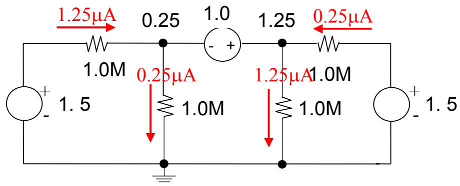
Is V_a= 1.25 and V_b = 0.25 if R_1= R_2 = R_3 = R_4 = 1MΩ, and V_1 = V_4 =1.5V with V_{LL} = 1V ????



KCL at the Supernode: 0.25 -1.25 + 1.25 - 0.25 =0

Clearly the current into the supernode is zero and we have verified that the solution is correct. :

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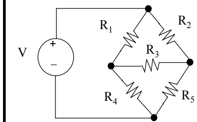
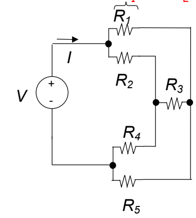
IDENTIFYING SERIES AND PARALLEL COMBINATIONS (cont.)

Some circuits *must* be analyzed (not amenable to simple inspection)

R_1 and R_2 are **not** in ||





R_1 and R_5 are **not** in series

Special cases:
$R_3 = 0$ **OR** $R_3 = \infty$

Example: $R_3 = 0 \Rightarrow R_1 \| R_2$; $R_4 \| R_5$ in series; $R_{eq} = R_1 \| R_2 + R_4 \| R_5$

OR IF $R_3 = \infty \Rightarrow (R_1 + R_5) \| (R_2 + R_4)$

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RESISTORS IN PARALLEL

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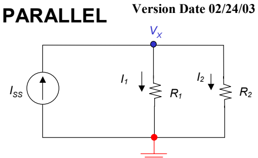
1 Select Reference Node

2 Define unknown node voltages



Note: $I_{ss} = I_1 + I_2$, i.e.,

$$I_{SS} = \frac{V_X}{R_1} + \frac{V_X}{R_2} \Rightarrow V_X = I_{SS} \cdot \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = I_{SS} \cdot \frac{R_1 R_2}{R_1 + R_2}$$

RESULT 1 EQUIVALENT RESISTANCE: $R_{\|} \equiv R_1 \| R_2 = \frac{R_1 R_2}{R_1 + R_2}$

RESULT 2 CURRENT DIVIDER: $I_1 = \frac{V_X}{R_1} = I_{SS} \times \frac{R_2}{R_1 + R_2}$

$$I_2 = \frac{V_X}{R_2} = I_{SS} \times \frac{R_1}{R_1 + R_2}$$

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First Midterm Exam: Topics

- Basic Circuit Analysis (KVL, KCL)
- Equivalent Circuits and Graphical Solutions for Nonlinear Loads
- Transients in Single Capacitor Circuits
- Node Analysis Technique and Checking Solutions

Exam is in class 3:10-4:03 PM, Closed book, Closed notes, Bring a calculator, Paper provided

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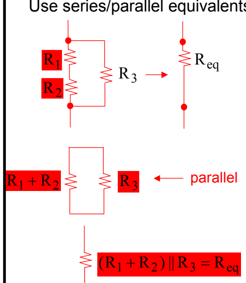
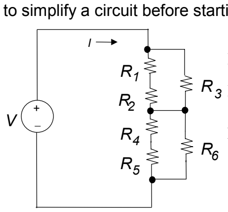
IDENTIFYING SERIES AND PARALLEL COMBINATIONS

Use series/parallel equivalents to simplify a circuit before starting KVL/KCL



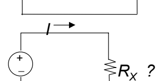


$R_1 = R_2 = 10\,K$

$R_3 = 20\,K$

$R_4 = 5 = 5\,K$

$R_6 = 10\,K$



$$R_X = (R_1 + R_2) \| R_3 + (R_4 + R_5) \| R_6 = 15\,K$$

Please note the order of math operators here!

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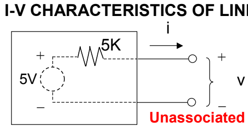
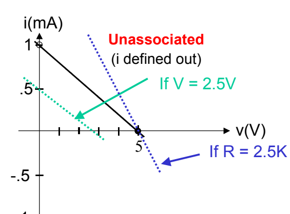
I-V CHARACTERISTICS OF LINEAR TWO-TERMINAL NETWORKS



Apply v, measure i, or vice versa

Consider how the graph changes with differences in V and R.



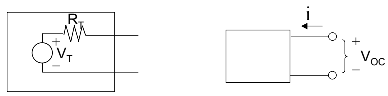
First consider change in V, eg V= 2.5V, not 5V

Now consider change in R (with V back at 5V)

Clearly by varying V and R we can produce an arbitrary linear graph … in other words this circuit can produce *any* linear graph

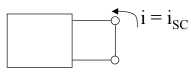
FINDING V_T, R_T BY MEASUREMENT

1 V_T is the open-circuit voltage V_{OC} (i.e., i = 0)



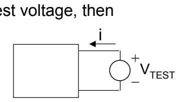
2a) If we short the output clearly $I = -V_T/ R_T$ thus
R_T is the ratio of V_{OC} to - i_{SC}, the short-circuit current

$$R_T = -\frac{V_{OC}}{I_{SC}}$$



2b) If V_T = 0, you need to apply test voltage, then

$$R_T = \frac{V_{TEST}}{i}$$



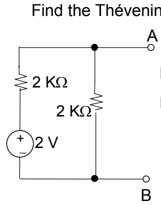
FINDING V_T, R_T BY ANALYSIS

1 Calculate V_{OC}. $V_T = V_{OC}$

2 Turn off all independent sources and find equivalent R at terminals

EXAMPLE 1

Find the Thévenin and Norton equivalents of:



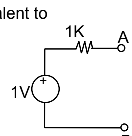
Find $V_{AB} = V_{OC}$ from voltage divider. :

If open circuit, $V_{AB} = V_{OC}$ = 2 X 2/4 = 1V

If A-B is shorted, I_{SC} = - 2V/2K = -1 mA (into A)

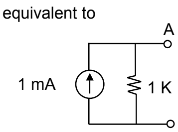
$$R_{TH} = \frac{1}{10^{-3}} = 1K$$

equivalent to



Thévenin

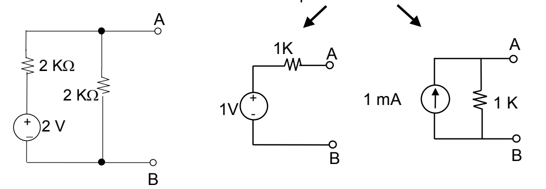
and equivalent to



Norton

EXAMPLE 1, Continued

In what sense is this circuit equivalent to these?



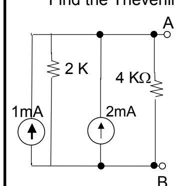
They have identical I-V characteristics and therefore have

The same open circuit voltage

The same short circuit current

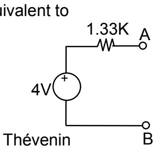
EXAMPLE 2

Find the Thévenin and Norton equivalents of:



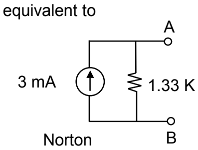
Find Norton equivalent of circuit leg on the left:
(1mA and 2K) ⇒ Two Norton circuits in parallel
Thus combine current sources (2+1=3) = I_N
and combine resistors in parallel: 8/6 K = R_N
So, $V_{TH} = R_N I_N$ = 4V
$R_{TH} = R_N$ = 4/3 K

equivalent to



Thévenin

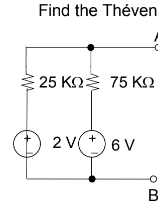
and equivalent to



Norton

EXAMPLE 3

Find the Thévenin and Norton equivalents of:



Find $V_{AB} = V_{OC}$ from voltage divider. Left to right:

(4 V rise across 25K + 75K) ⇒

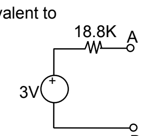
3 V across 75K, 1 V across 25k.

So, $V_{AB} = -3 + 6 = 3V = V_{OC}$

I_{SC} = - 6V/75K+2V/25K = -0.16 mA

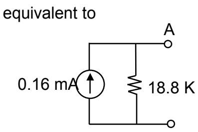
$$R_{TH} = \frac{3}{.16 \times 10^{-3}} = 18.8K$$

equivalent to



Thévenin

and equivalent to



Norton