Charging and Discharging RC Circuits
Handout for EECS 42

Developed by Professor W.G. Oldham to provide understanding of transient issues in computer logic.

Extensions by Professor A.R. Neureuther in Spring 2003 to include sequential switching of logic gates as occurs in the EECS 43 logic gate experiment.

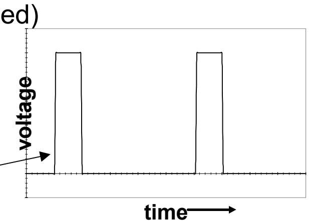
Charging and discharging in RC Circuits
(an enlightened approach)

- *Before* we analyze real electronic circuits - lets study RC circuits
- Rationale: Every node in a circuit has capacitance to ground, like it or not, and it's the charging of these capacitances that limits real circuit performance (speed)

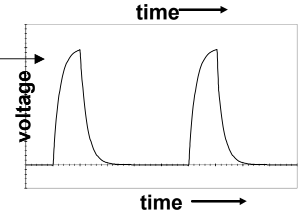
Relevance to digital circuits:

We communicate with pulses

We send beautiful pulses out



But we receive lousy-looking pulses and must restore them

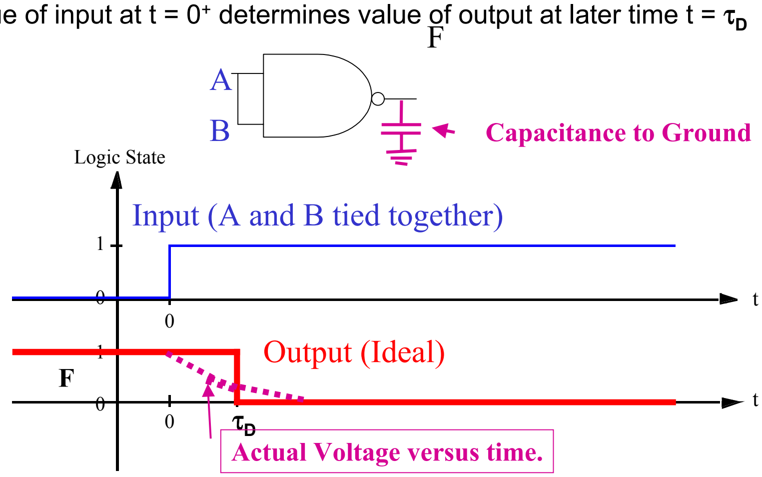


RC charging effects are responsible …. So lets review them.

LOGIC GATE DELAY τ_D

Time delay τ_D occurs between input and output: "computation" is not instantaneous

Value of input at $t = 0^+$ determines value of output at later time $t = \tau_D$



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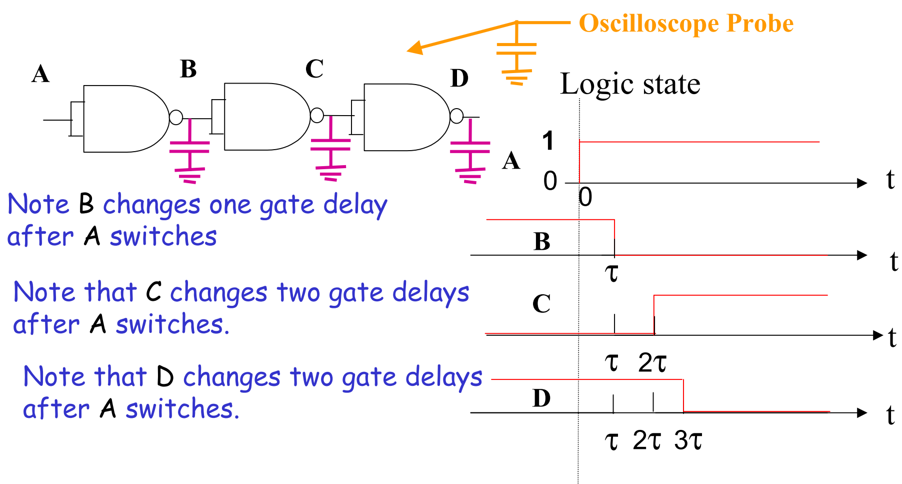
SIGNAL DELAY: TIMING DIAGRAMS

Show transitions of variables vs time

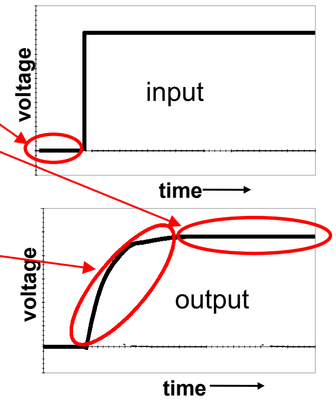


Simplification for time behavior of RC Circuits

Before any input change occurs we have a dc circuit problem (that is we can use dc circuit analysis to relate the output to the input).

Long after the input change occurs things "settle down" …. Nothing is changing …. So again we have a dc circuit problem.

We call the time period during which the output changes the *transient*



We can predict a lot about the transient behavior from the pre- and post-transient dc solutions

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What environment do pulses face?

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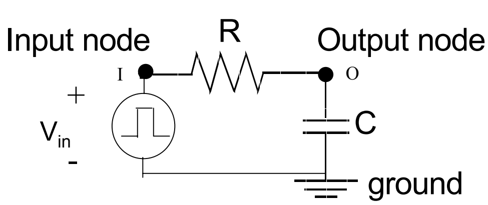
- Every real wire in a circuit has resistance.
- Every junction (*node*) has capacitance to ground and to other nodes.
- The active circuit elements (transistors) add additional resistance in series with the wires, and additional capacitance in parallel with the node capacitance.

Thus the most basic model circuit for studying transients consists of a resistor driving a capacitor.

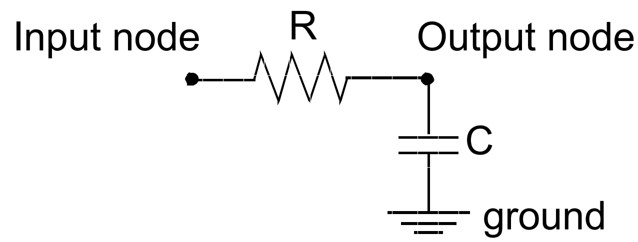


A pulse originating at **node I** will arrive delayed and distorted at **node O** because it takes time to charge C through R

If we focus on the circuit which distorts the pulses produced by V_{in}, its most simple form consists simply of an R and a C. (V_{in} represents the time-varying source which produces the input pulse.)

The RC Circuit to Study

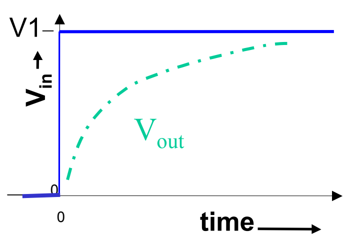
(All single-capacitor circuits reduce to this one)

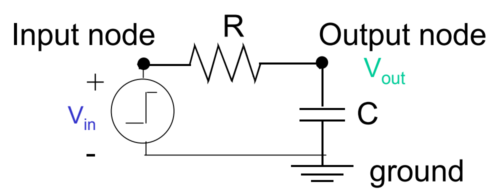


- R represents total resistance (wire plus whatever drives the input node)
- C represents the total capacitance from node to the outside world (from devices, nearby wires, ground etc)

RC RESPONSE

Case 1 – Rising voltage. Capacitor uncharged: Apply + voltage step





- V_{in} "jumps" at t=0, but V_{out} cannot "jump" like V_{in}. Why not?

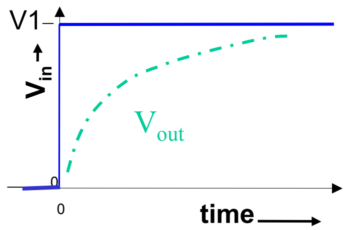
☞ Because an instantaneous change in a capacitor voltage would require instantaneous increase in energy stored ($1/2CV^2$), that is, infinite power. (Mathematically, V must be differentiable: I=CdV/dt)

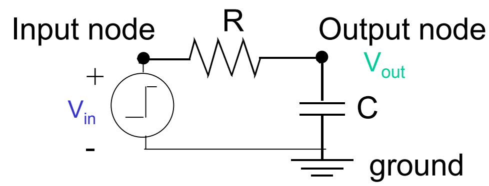
V does not "jump" at t=0 , i.e. $V(t=0^+) = V(t=0^-)$

Therefore the dc solution before the transient tells us the capacitor voltage at the beginning of the transient.

RC RESPONSE

Case 1 Continued – Capacitor uncharged: Apply voltage step





- V_{out} approaches its final value asymptotically (It never actually gets exactly to V1, but it gets arbitrarily close). Why?

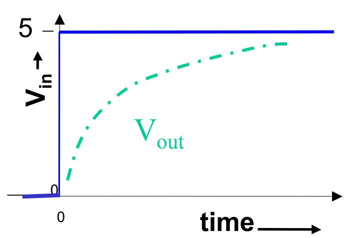
After the transient is over (nothing changing anymore) it means d(V)/dt = 0 ; that is all currents must be zero. From Ohm's law, the voltage across R must be zero, i.e. $V_{in} = V_{out}$.

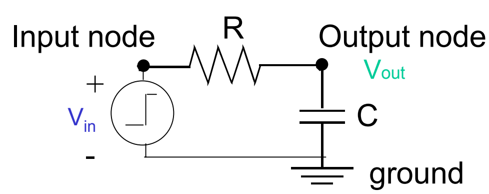
☞ That is, $V_{out} \rightarrow$ V1 as $t \rightarrow \infty$. (Asymptotic behavior)

Again the dc solution (after the transient) tells us (the asymptotic limit of) the capacitor voltage during the transient.

RC RESPONSE

Example – Capacitor uncharged: Apply voltage step of 5 V



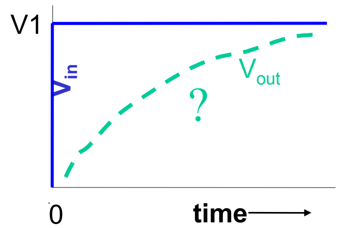


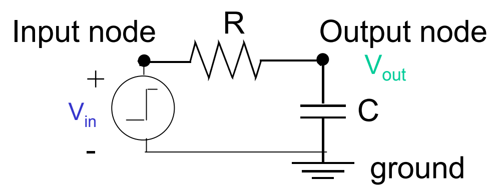
- Clearly V_{out} starts out at 0V (at $t = 0^+$) and approaches 5V.
- We know this because of the pre-transient dc solution (V=0) and post-transient dc solution (V=5V).

So we know a lot about V_{out} during the transient - namely its initial value, its final value , *and we know the general shape* .

We even know the initial slope from **I** = C(dV/dt) as (dV/dt) = (1/C)**I** = $(1/C)(V_{in} – 0)/R = (V_{in} – 0)/(RC)$

RC RESPONSE: Case 1 (cont.)

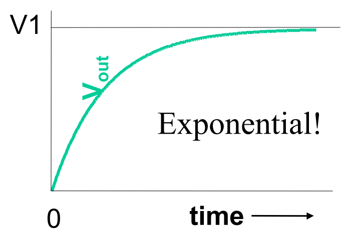


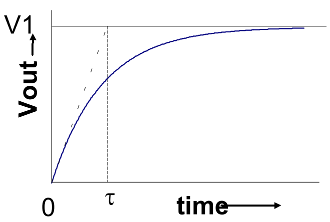


Exact form of V_{out}?

Equation for Vout: Do you remember general form?

$$V_{out} = V_1(1-e^{-t/\tau})$$





Review of simple exponentials.

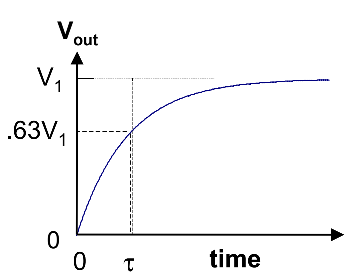
Rising Exponential from Zero

$$V_{out} = V_1(1-e^{-t/\tau})$$

at $t = 0$, $V_{out} = 0$, and

at $t \rightarrow \infty$, $V_{out} \rightarrow V_1$ also

at $t = \tau$, $V_{out} = 0.63\ V_1$



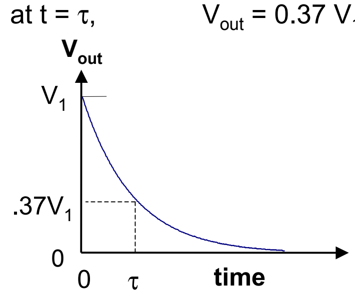
Falling Exponential to Zero

$$V_{out} = V_1 e^{-t/\tau}$$

at $t = 0$, $V_{out} = V_1$, and

at $t \rightarrow \infty$, $V_{out} \rightarrow 0$, also

at $t = \tau$, $V_{out} = 0.37\ V_1$



Further Review of simple exponentials:

Rising Exponential from Zero

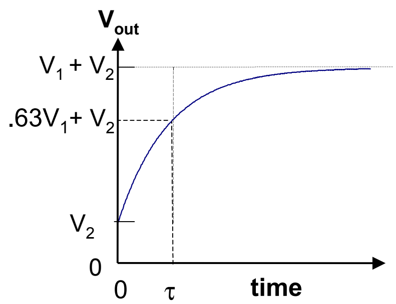
$V_{out} = V_1(1-e^{-t/\tau})$

Falling Exponential to Zero

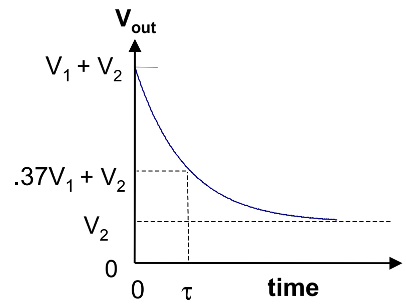
$V_{out} = V_1 e^{-t/\tau}$

We can add a constant (positive or negative)

$V_{out} = V_1(1-e^{-t/\tau}) + V_2$



$V_{out} = V_1 e^{-t/\tau} + V_2$



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Further Review of simple exponentials:

Rising Exponential

$V_{out} = V_1(1-e^{-t/\tau}) + V_2$

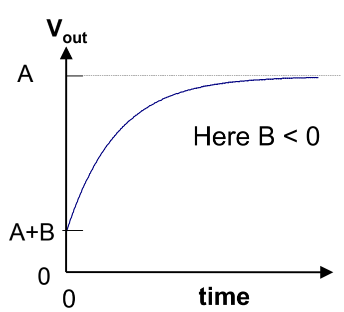
Falling Exponential

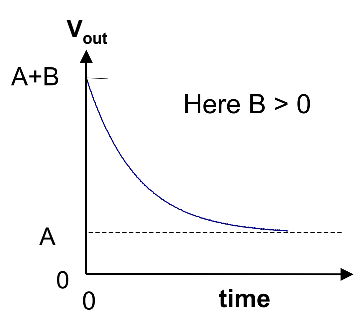
$V_{out} = V_1 e^{-t/\tau} + V_2$

Both equations can be written in one simple form: $V_{out} = A + Be^{-t/\tau}$

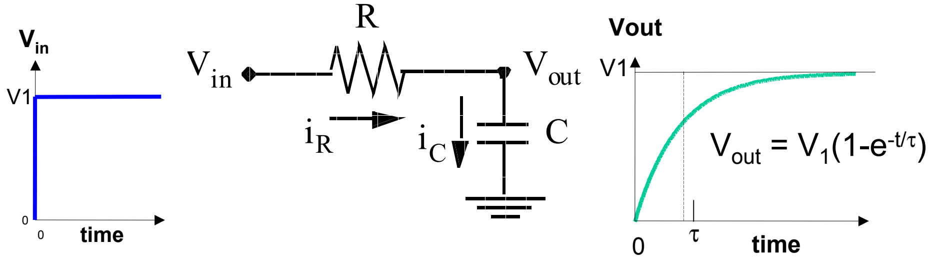
Initial value (t=0) : $V_{out} = A + B$. Final value ($t >> \tau$): $V_{out} = A$

Thus: if $B < 0$, rising exponential; if $B > 0$, falling exponential





RC RESPONSE: Case 1 (Rising exponential)



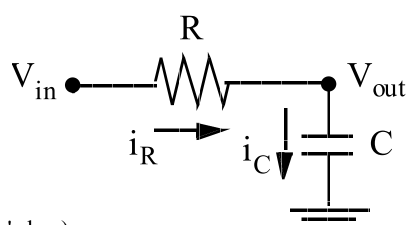
- How is τ related to R and C ?
  - If C is bigger, it takes longer ($\tau\uparrow$).
  - If R is bigger, it takes longer ($\tau\uparrow$).
  - Thus, τ is proportional to RC.

In fact, τ = RC !

Thus, $V_{out} = V_1(1-e^{-t/RC})$

RC RESPONSE: Case 1 (cont.)

Proof that $V_{out} = V_1(1-e^{-t/RC})$



$$i_R = \frac{V_{in} - V_{out}}{R} \quad \text{(Ohm's law)}$$

$$i_C = C\frac{dV_{out}}{dt} \quad \text{(capacitance law)}$$

But $i_R = i_C$!

$$\text{Thus,} \quad \frac{V_{in} - V_{out}}{R} = C\frac{dV_{out}}{dt}$$

or

$$\boxed{\frac{dV_{out}}{dt} = \frac{1}{RC}(V_{in} - V_{out})}$$

RC RESPONSE Case 1 (cont.)

Proof that $V_{out} = V_1(1-e^{-t/RC})$

We have: $\frac{dV_{out}}{dt} = \frac{1}{RC}(V_{in} - V_{out})$

But $V_{in} = V_1 = \text{constant}$

and $V_{out} = 0$ at $t = 0^+$

I claim that the solution to this first-order linear differential equation is:

$$V_{out} = V_1(1-e^{-t/RC})$$

Proof by substitution:

$$\frac{dV_{out}}{dt} \stackrel{?}{=} \frac{1}{RC}(V_{in} - V_{out})$$

$$\downarrow$$

$$\frac{V_1}{RC}e^{-t/RC} \stackrel{?}{=} \frac{1}{RC}(V_1 - V_1(1-e^{-t/RC}))$$

clearly

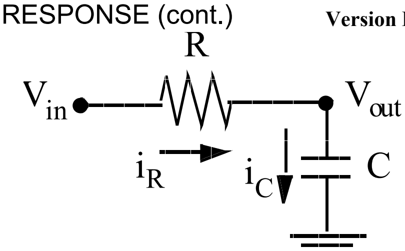
$$\frac{V_1}{RC}e^{-t/RC} = \frac{V_1}{RC}e^{-t/RC}$$

and

$$V_{out} = 0 \text{ at } t = 0^+ \quad \text{OK}$$

RC RESPONSE (cont.)

Generalization



Vin switches at t = 0; then for any time interval t > 0, in which Vin is a constant, Vout is ***always*** of the form: $V_{out} = A + Be^{-t/RC}$

We determine A and B from the initial voltage on C, and the value of Vin. Assume Vin "switches" at t=0 from Vco to V1:

First, at $t = 0$ $V_C \equiv V_{Co}$ initial voltage

☞ Thus, $A + B = V_{Co}$

as $t \to \infty, V_C \to V_1$

☞ Thus, $A = V_1 \Rightarrow B = V_{Co} - V_1$

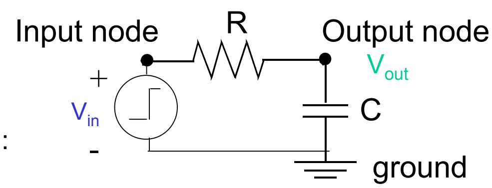
You may choose to solve RC problems using this "A and B" formulation, but in the next lecture we show you an easier way.

Re-Cap: Charging and discharging in RC Circuits

Last Time:

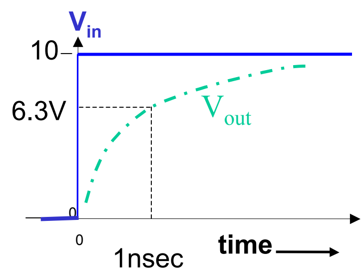
We learned that simple the simple RC circuit with a step input has a universal exponential solution of the form:

$$V_{out} = A + Be^{-t/RC}$$



Example 0: R = 1K, C = 1pF, V_{in} steps from zero to 10V at t=0:

1) Initial value of V_{out} is 0
2) Final value of V_{out} is 10V
3) Time constant is RC = 10^{-9} sec
4) V_{out} reaches 0.63 X 10 in 10^{-9} sec



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Charging and discharging in RC Circuits

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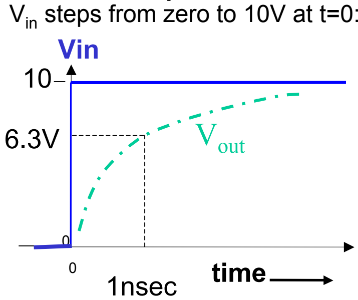
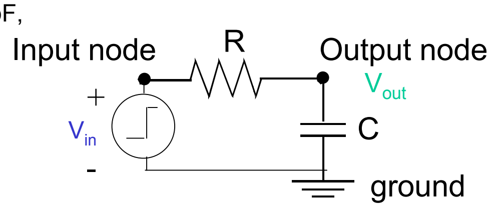
- Example 1 (rising exponential) continued -

For this example: R = 1K, C = 1pF, V_{in} steps from zero to 10V at t=0:





V_{out} starts at 0, ends at 10 and has time constant of 1nsec

$$V_{out} = 10 - 10e^{-t/1nsec}$$

Note that we found this graph without even using the equation

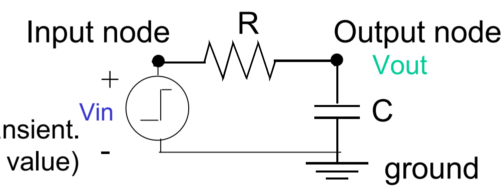
$V_{out} = A + Be^{-t/RC}$ (That is we did not try to evaluate A and B).

We simply used the dc solution for t<0 and the dc solution for t>>0 to get the limits and we used the time constant to get the horizontal scale. We only need the equation to remind us the solution is an exponential. So this will be the basis of our **easy method**.

Charging and discharging in RC Circuits
(The official EE42 Easy Method)

Method of solving for any node voltage in a single capacitor circuit.

1) Simplify the circuit so it looks like one resistor, a source, and a capacitor (it will take another two weeks to learn all the tricks to do this.) But then the circuit looks like this:



2) The time constant of the transient is τ = RC.

3) Solve the dc problem for the capacitor voltage before the transient. This is the starting value (initial value) for the transient voltage.

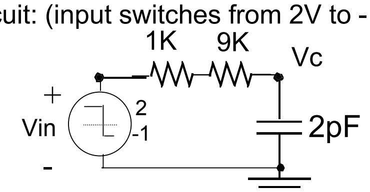
4) Solve the dc problem for the capacitor voltage after the transient is over. This is the asymptotic value.

5) Sketch the Transient. It is 63% complete after one time constant.

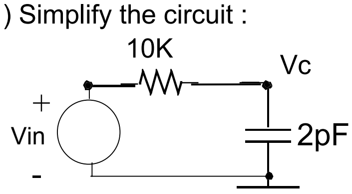
6) Write the equation by inspection.

Charging and discharging in RC Circuits
(Example 1 of the EE42 Easy Method)

Find Vc(t) for the following circuit: (input switches from 2V to -1V at t = 0)

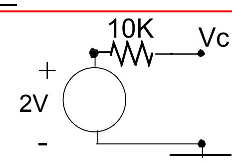


1) Simplify the circuit :

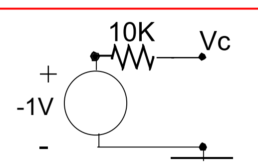


2) The time constant of the transient is τ = RC = 20nsec

3) Before the transient Vin = 2V so Vc =2V



4) After the transient is over Vin = -1V so Vc = -1V. This is the asymptotic value.



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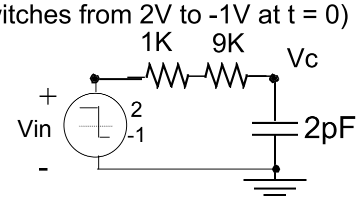
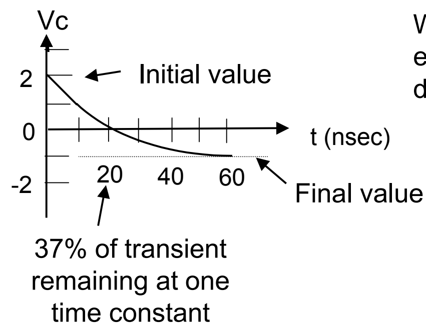
Charging and discharging in RC Circuits (Example 1 of the EE42 Easy Method)

Find Vc(t) for the following circuit: (input switches from 2V to -1V at t = 0)

We have : Initial value of Vc is 2V, final value is -1V and τ = 20nsec



5) Sketch Vc (t) :



What is the equation for an exponential beginning at 2V, decaying to -1V, with τ = 20nsec?

$$V_c(t) = -1 + 3e^{-t/20nsec}$$

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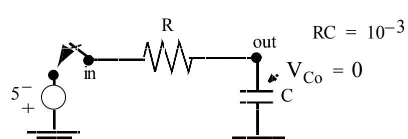
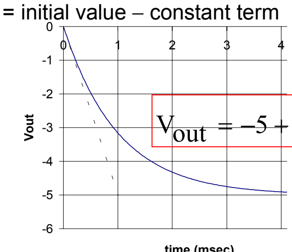
2 MORE EXAMPLES ---OUR METHOD AVOIDS ALL MATH!

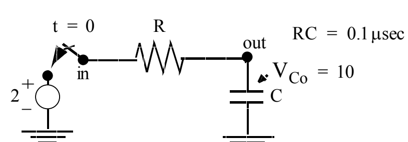
① Sketch waveform (starts at Vco, ends asymptotically at V1, initial slope intersects at t = RC or transient is 63% complete at t=RC)

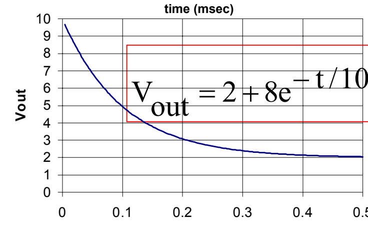
② Write equation: **2a.** constant term A = limit of V as t $\rightarrow \infty$
2b. pre-exponent B = initial value – constant term





$$V_{out} = -5 + 5e^{-t/.001}$$





$$V_{out} = 2 + 8e^{-t/10^{-7}}$$

4 Examples

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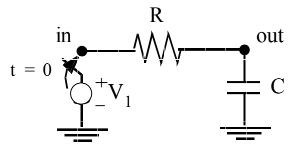
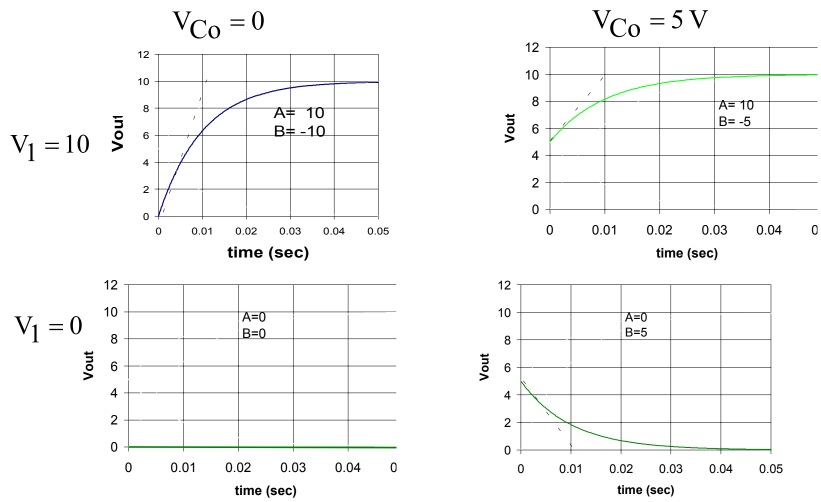
$$V_{out} = A + Be^{-t/RC}$$



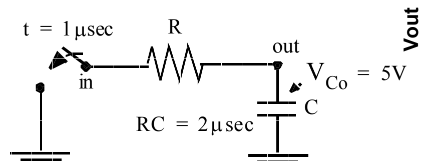
RC = 0.01 sec

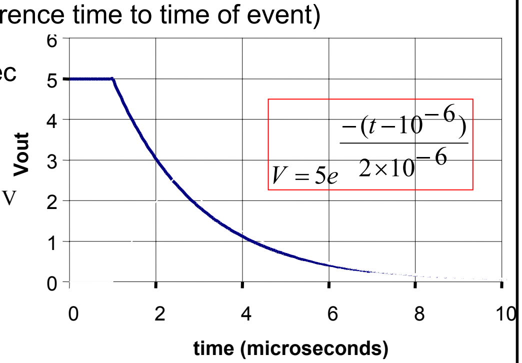


COMPLICATION: Event Happens at t ≠ 0

(Solution: Shift reference time to time of event)

Example: switch closes at 1µsec





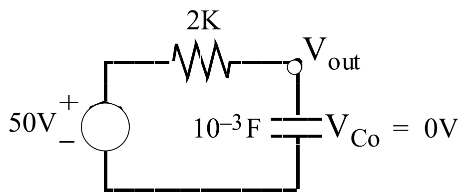
$$V = 5e^{\frac{-(t-10^{-6})}{2\times10^{-6}}}$$

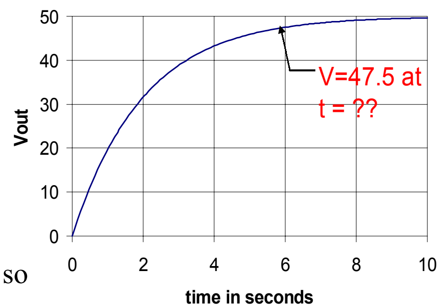
We shift the time axis here by one microsecond, i.e. imagine a new time coordinate t* = t-1µsec so that in the new time domain, the event happens at t* = 0 and our standard solution applies. Of course we replace t* by t -1µsec in the equations and plots. Thus instead of t* =0 we have t = 1µsec, etc.

EXAMPLE of CHARGING to 95%

Your photo flash charges a 1000µF capacitor from a 50V source through a 2K resistor. If the capacitor is initially uncharged, how long must you wait for it to reach 95% charged (47.5 V)?

Solution: $RC = 2K \times 10^{-3} = 2\ \text{sec}$





By inspection: $V_o = 50 - 50e^{-t/2}$, so

$$47.5 = 50(1 - e^{-\frac{t_x}{2}}) \Rightarrow e^{-\frac{t_x}{2}} = (1 - \frac{47.5}{50}) \Rightarrow \boxed{t_x = 6\ \text{sec}}$$

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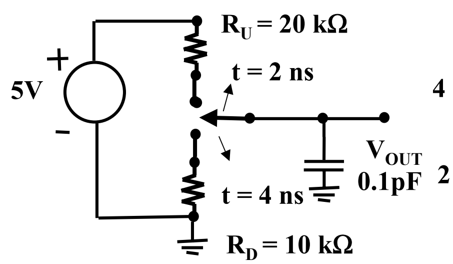
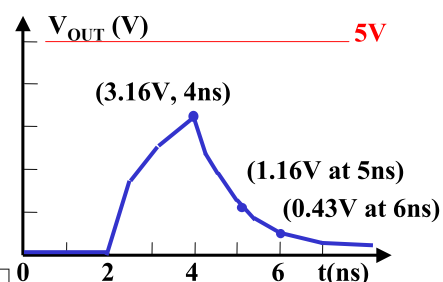
EXAMPLE of a SWITCHING LOGIC GATE





Prior to t = 2ns Switch has been down a long time.
$V_{OUT} = 0$

At 2 ns Switch goes up: heads for 5V with RC = 20kΩ 0.1pF = 2 ns
$V_{OUT} = 5 - 5e^{-(t-2ns)/2ns}$

At 4 ns Switch goes down: starts from present value of 3.16V and heads down to zero.
V_{OUT} = 3.16e-(t-4ns)/1ns