The Beauty and Joy of Computing

Lecture #10
Recursion II

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RECURSIVE DRAWING

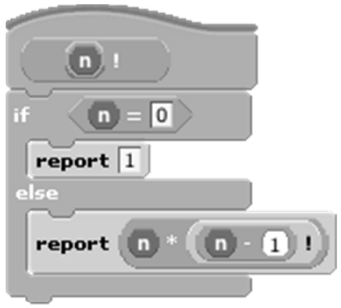
Toby Shachman created this amazing spatial programming language called "Recursive Drawing" that allows you to create drawings (even recursive ones) without typing a line of code. It's a great example of a next-generation interface…

recursivedrawing.com

How the Computer Works … n!

- **Factorial(n) = n!**
  Inductive definition:
  - **n! = 1 , n = 0**
  - **n! = n * (n-1)!, n > 0**
- **Let's act it out…**
  - **"contractor" model**
  - **5!**

n	n!
0	1
1	1
2	2
3	6
4	24
5	120



Order of growth of # of calls of n!

a) Constant
b) Logarithmic
c) Linear
d) Quadratic
e) Exponential

(source: FallingFifth.com)

How the Computer Works … fib(n)

en.wikipedia.org/wiki/Fibonacci_number
www.ics.uci.edu/~eppstein/161/960109.html

- **Inductive definition:**
  - **fib(n) = n , n < 2**
  - **fib(n) = fib(n-1)+fib(n-2), n > 1**

$$F(n) := \begin{cases} 0 & \text{if } n = 0; \\ 1 & \text{if } n = 1; \\ F(n-1) + F(n-2) & \text{if } n > 1. \end{cases}$$

- **Let's act it out…**
  - **"contractor" model**
  - **fib(5)**

n	fib(n)
0	0
1	1
2	1
3	2
4	3
5	5

Leonardo de Pisa aka, Fibonacci

Let's now: trace… (gif from Ybungalobill@wikimedia)

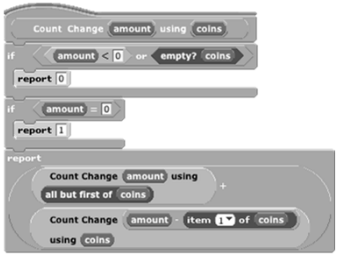
Order of growth of # of calls of fib(n)

a) Constant
b) Logarithmic
c) Linear
d) Quadratic
e) Exponential

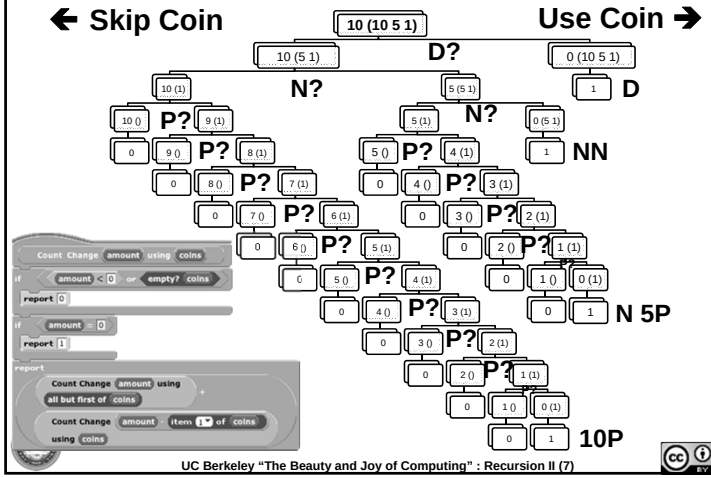
Chimney of Turku Energia, Turku, Finland featuring Fibonacci sequence in 2m high neon lights. By Italian artist Mario Merz for an environmental art project. (Wikipedia)

Counting Change (thanks to BH)

- Given coins {50, 25, 10, 5, 1} how many ways are there of making change?
  - 5
    - 2 (N, 5P)
  - 10
    - 4 (D, 2N, N 5P, 10P)
  - 15
    - 6 (DN, D5P, 3N, 2N5P, 1N10P, 15P)
  - 100?



Call Tree for "Count Change 10 (10 5 1)"



Summary

- It's important to understand the machine model
- It's often the cleanest, simplest way to solve many problems
  - Esp those recursive in nature!
- Recursion is a very powerful idea, often separates good from great (you're great!)

Menger Cube by Dan Garcia