The Beauty and Joy of Computing

Lecture #23
Limits of Computing

UC Berkeley EECS
Sr Lecturer SOE
Dan Garcia

NEIL AI LEARNS BY ITSELF, 24/7

Researchers at CMU have built a system which searches the Web for images constantly and tries to decide how the images relate to each other. The goal is to "recreate common sense".

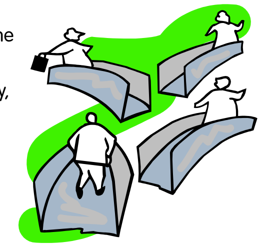
www.cs.cmu.edu/news/carnegie-mellon-computer-searches-web-247-analyze-images-and-teach-itself-common-sense

---

www.csprinciples.org/docs/APCSPrinciplesBigIdeas20110204.pdf

Let's revisit algorithm complexity

- Problems that…
  - are tractable with efficient solutions in reasonable time
  - are intractable
  - are solvable approximately, not optimally
  - have no known efficient solution
  - are not solvable



Tractable with efficient sols in reas time

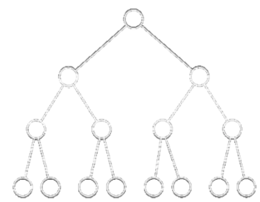
- Recall our algorithm complexity lecture, we've got several common orders of growth
  - Constant
  - Logarithmic
  - Linear
  - Quadratic
  - Cubic
  - Exponential
- Order of growth is polynomial in the size of the problem
- E.g.,
  - Searching for an item in a collection
  - Sorting a collection
  - Finding if two numbers in a collection are same
- These problems are called being "in P" (for polynomial)

---

en.wikipedia.org/wiki/Intractability_(complexity)#Intractability

Intractable problems

- Problems that can be solved, but not solved fast enough
- This includes exponential problems
  - E.g., $f(n) = 2^n$
    - as in the image to the right
- This also includes poly-time algorithm with a huge exponent
  - E.g, $f(n) = n^{10}$
- Only solve for small n

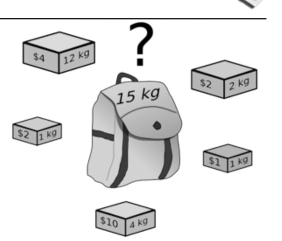


Imagine a program that calculated something important at each of the bottom circles. This tree has height n, but there are 2^n bottom circles!

Peer Instruction

What's the most you can put in your knapsack?

a) $10
b) $15
c) $33
d) $36
e) $40



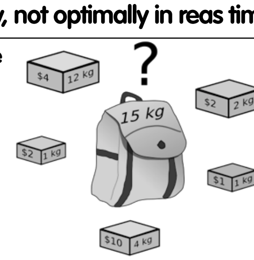
Knapsack Problem
You have a backpack with a weight limit (here **15kg**), which boxes (with weights and values) should be taken to maximize value?
(any # of each box is available)

---

en.wikipedia.org/wiki/Knapsack_problem

Solvable approximately, not optimally in reas time

- A problem might have an optimal solution that cannot be solved in reasonable time
- BUT if you don't need to know the perfect solution, there might exist algorithms which could give pretty good answers in reasonable time



Knapsack Problem
You have a backpack with a weight limit (here 15kg), which boxes (with weights and values) should be taken to maximize value?

en.wikipedia.org/wiki/P_%3D_NP_problem

Have no known efficient solution

- **Solving one of them would solve an entire class of them!**
  - We can transform one to another, i.e., reduce
  - A problem P is "hard" for a class C if <u>every</u> element of C can be "reduced" to P
- **If you're "in NP" and "NP-hard", then you're "NP-complete"**

-2 -3 15
14 7 -10

Subset Sum Problem
Are there a handful of these numbers (at least 1) that add together to get 0?

- **If you guess an answer, can I <u>verify it</u> in polynomial time?**
  - Called being "in NP"
  - Non-deterministic (the "guess" part) Polynomial

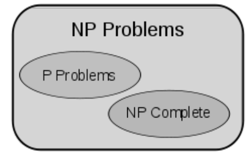
Garcia

en.wikipedia.org/wiki/P_%3D_NP_problem

The fundamental question. Is P = NP?

- **This is THE major unsolved problem in Computer Science!**
  - One of 7 "millennium prizes" w/a $1M reward
- **All it would take is solving ONE problem in the NP-complete set in polynomial time!!**
  - Huge ramifications for cryptography, others

If P $\neq$ NP, then



- **Other NP-Complete**
  - Traveling salesman who needs most efficient route to visit all cities and return home

Garcia

www.cgl.uwaterloo.ca/~csk/halt/

Problems NOT solvable

- **<u>Decision problems</u> answer YES or NO for an infinite # of inputs**
  - E.g., is N prime?
  - E.g., is sentence S grammatically correct?
- **An algorithm is a <u>solution</u> if it correctly answers YES/NO in a finite amount of time**
- **A problem is <u>decidable</u> if it has a solution**

June 23, 2012 was his 100th birthday celebration!!

Alan Turing
He asked:
"Are all problems decidable?"
(people used to believe this was true)
Turing proved it wasn't for CS!

Garcia

Review: Proof by Contradiction

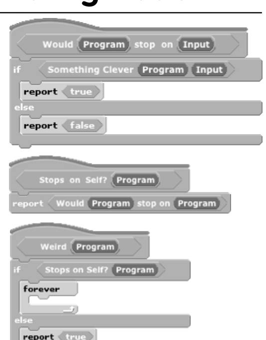
- **Infinitely Many Primes?**
- **Assume the contrary, then prove that it's impossible**
  - Only a finite set of primes, numbered p_1, p_2, …, p_n
  - Consider $q=(p_1 \bullet p_2 \bullet \ldots \bullet p_n)+1$
  - Dividing q by p_i has remainder 1
  - q either prime or composite
    - If prime, q is not in the set
    - If composite, since no p_i divides q, there must be another p that does that is not in the set.
  - So there's infinitely many primes

Euclid
www.hisschemoller.com/wp-content/uploads/2011/01/euclides.jpg

Garcia

Turing's proof : The Halting Problem

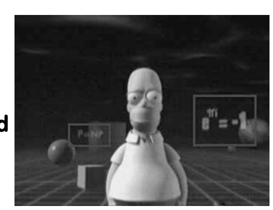
- **Given a program and some input, will that program eventually stop? (or will it loop)**
- **Assume we could write it, then let's prove a contradiction**
  - 1. write Stops on Self?
  - 2. Write Weird
  - 3. Call Weird on itself



Garcia

Conclusion

- **Complexity theory important part of CS**
- **If given a hard problem, rather than try to solve it yourself, see if others have tried similar problems**
- **If you don't need an exact solution, many approximation algorithms help**
- **Some not solvable!**



P=NP question even made its way into popular culture, here shown in the Simpsons 3D episode!

Garcia