# CS10 The Beauty and Joy of Computing 

## Lecture \#11 : Recursion II

2011-03-02

## IPAD 2 ANNOUNCED TODAY

Will Apple continue to thrall its users with outstanding technology amidst tons of competition from its Android Rivals? Will Steve Jobs make an appearance? Will you buy one?


## How the Computer Works ... n!

- Factorial(n) = n! Inductive definition:
- $n!=1 \quad, n=0$
- $\boldsymbol{n !}=\mathbf{n}^{*}(\mathbf{n}-7)!, n>0$
- Let's act it out...
" "Little people", or "subcontractor" model
- 5!

| $\mathbf{n}$ | $\mathrm{n}!$ |
| :---: | :---: |
| 0 | 1 |
| 1 | 1 |
| 2 | 2 |
| 3 | 6 |
| 4 | 24 |
| 5 | 120 |



## How the Computer Works ... fib(n)

- Inductive detinifion:
- $\mathbf{f i b}(\mathrm{n})=\mathrm{n}$

$$
F(n):= \begin{cases}0 & \text { if } n=0 \\ 1 & \text { if } n=1 \\ F(n-1)+F(n-2) & \text { if } n>1\end{cases}
$$

- Let's act it out...
- "contractor" model
- fib(5)
n fib(n)

| 0 | 0 |
| :--- | :--- |
| 1 | 1 |
| 2 | 1 |
| 3 | 2 |
| 4 | 3 |
| 5 | 5 |

,$n<2$

- fib(n) $=$ fib( $n-1)+f i b(n-2), n>1$


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## Order of growth of \# of calls of fib(n)

## a) Constant

b) Logarithmic
c) Linear
d) Quadratic
e) Exponential

Chimney of Turku Energia, Turku, Finland featuring Fibonacci sequence in 2 m high neon lights. By Italian artist Mario Merz for an environmental art project.


## Counting Change (thanks to BH )

- Given coins $\{50,25$,
$10,5,1\}$ how many
ways are there of making change?
- 5: 2 (N,5 P)
- 10
- 4 (D, 2N, N 5P, 10P)
- 15
- 6 (DN,D5P,3N,2N5P,IN10P, 15P)

- 100?


## Call Tree for "Count Change 10 (105 1)"

## Skip Coin



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## Summary

- If's important to understand the machine model
- Recursion is a very powerful idea, and one way to separate good from great

Menger Cube by Dan Garcia


