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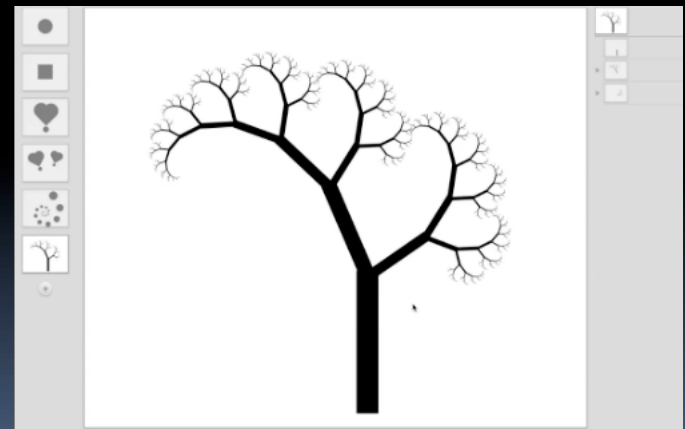
# The Beauty and Joy of Computing

## Lecture #10 Recursion II



### RECURSIVE DRAWING

Toby Shachman created this amazing spatial programming language called "Recursive Drawing" that allows you to create drawings (even recursive ones) without typing a line of code. It's a great example of a next-generation interface...



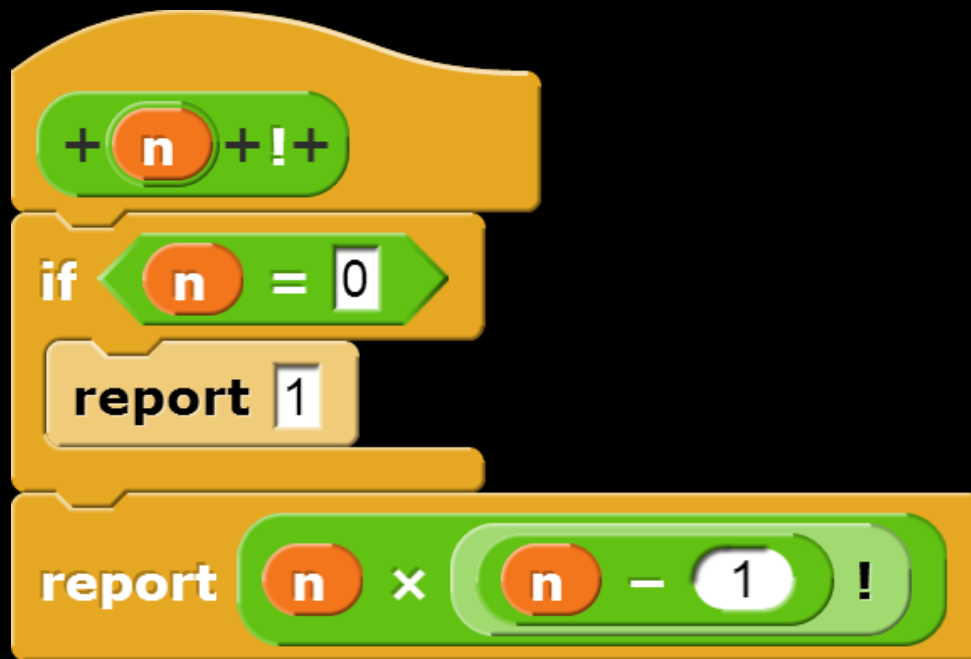
[recursivedrawing.com](http://recursivedrawing.com)



# How the Computer Works ... n!

- **Factorial(n) = n!**  
Inductive definition:
  - $n! = 1$  ,  $n = 0$
  - $n! = n * (n-1)!$ ,  $n > 0$
- **Let's act it out...**
  - "contractor" model
  - **5!**

n	n!
0	1
1	1
2	2
3	6
4	24
5	120





# Order of growth of # of calls of $n!$

(source: FallingFifth.com)

- a) Constant
- b) Logarithmic
- c) Linear
- d) Quadratic
- e) Exponential





# How the Computer Works ... fib(n)

- Inductive definition:

- fib(n) = n, n < 2
- fib(n) = fib(n-1)+fib(n-2), n > 1

$$F(n) := \begin{cases} 0 & \text{if } n = 0; \\ 1 & \text{if } n = 1; \\ F(n-1) + F(n-2) & \text{if } n > 1. \end{cases}$$

- Let's act it out...

- "contractor" model
- fib(5)

```

+fib + n +
if n < 2
  report n
report fib n - 1 + fib n - 2
  
```

n	fib(n)
0	0
1	1
2	1
3	2
4	3
5	5



Leonardo de Pisa  
aka, Fibonacci



Let's now: trace... (gif from Ybungalobill@wikimedia)





# Order of growth of # of calls of fib(n)

Chimney of Turku Energia, Turku, Finland featuring Fibonacci sequence in 2m high neon lights. By Italian artist [Mario Merz](#) for an environmental art project. (Wikipedia)

- a) Constant
- b) Logarithmic
- c) Linear
- d) Quadratic
- e) Exponential





# Counting Change (thanks to BH)

- Given coins {50, 25, 10, 5, 1} how many ways are there of making change?

- 5
  - 2 (N, 5P)
- 10
  - 4 (D, 2N, N5P, 10P)
- 15
  - 6 (DN, D5P, 3N, 2N5P, 1N10P, 15P)
- 100?

```
+Count+Change+ amount +Using+ coins : +
if amount < 0 or empty? coins
  report 0
if amount = 0
  report 1
report Count Change amount Using all but first of coins +
  Count Change amount - item 1 of coins Using coins
```

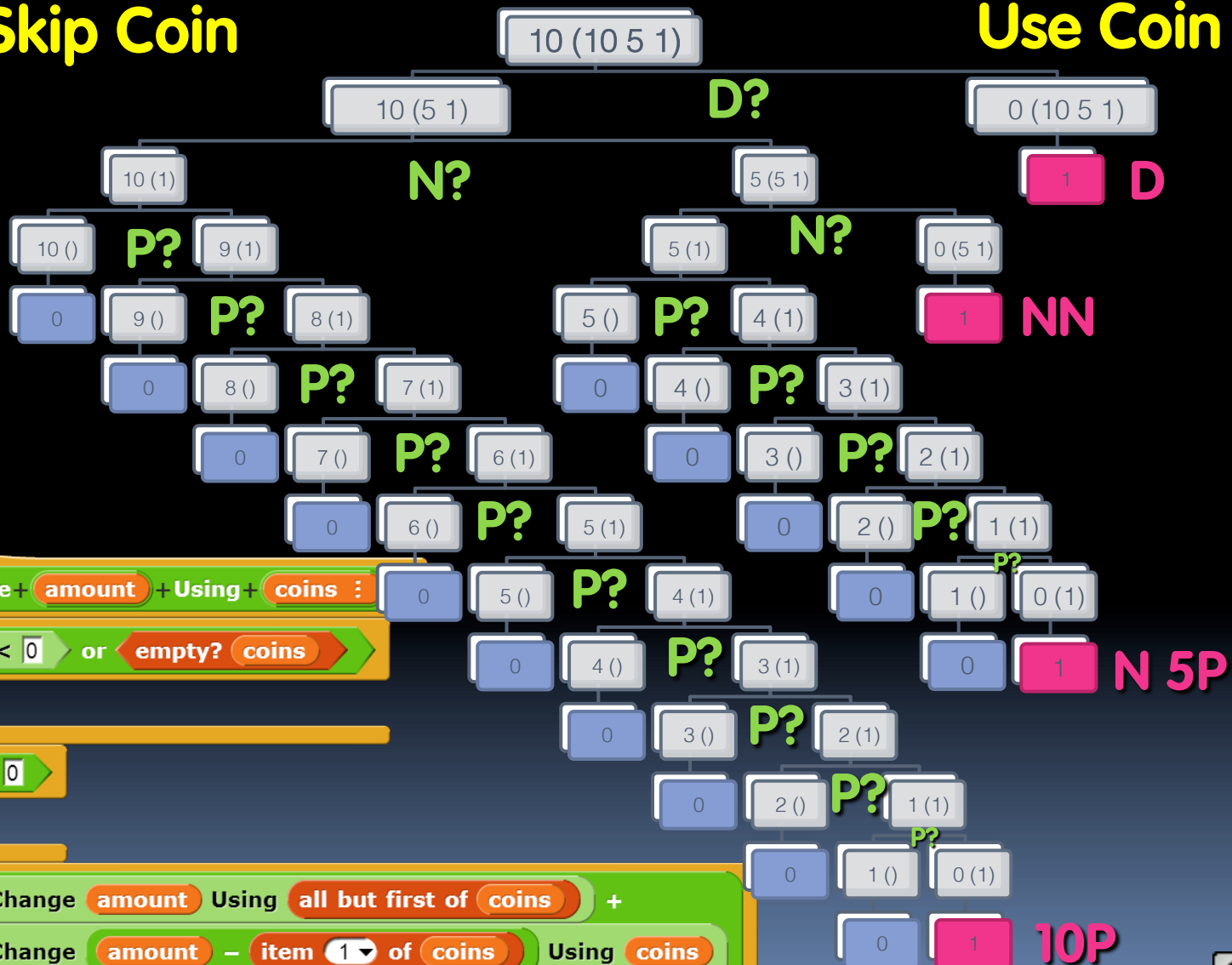




# Call Tree for "Count Change 10 (10 5 1)"

← Skip Coin

Use Coin →



```

+Count+ Change+ amount +Using+ coins :
if amount < 0 or empty? coins
  report 0
if amount = 0
  report 1
report
  Count Change amount Using all but first of coins +
  Count Change amount - item 1 of coins Using coins

```

Garcia





# "I understood Count Change"

- a) Strongly disagree
- b) Disagree
- c) Neutral
- d) Agree
- e) Strongly agree



[img4.joyreactor.com/pics/post/drawing-recursion-girl-275624.jpeg](http://img4.joyreactor.com/pics/post/drawing-recursion-girl-275624.jpeg)







# Summary

- It's important to understand the machine model
- It's often the cleanest, simplest way to solve many problems
  - Esp those recursive in nature!
- **Recursion is a very powerful idea, often separates good from great (you're great!)**

Menger Cube by Dan Garcia

