How the Computer Works … n!

- Factorial: \( n! = n \)
- Informal Definition:
  \( n! = \{ 1 \cdot 2 \cdot 3 \cdot \ldots \cdot n \} \)
- Inductive Definition:
  \( n! = \{ \begin{align*} 
  1, & \quad \text{if } n = 0 \\
  n \cdot (n-1)!, & \quad \text{if } n > 0 
\end{align*} \)

How the Computer Works … \( \text{fib}(n) \)

- Inductive definition:
  \( \text{fib}(n) = \{ \begin{align*} 
  n, & \quad \text{if } n \leq 1 \\
  \text{fib}(n-1) + \text{fib}(n-2), & \quad \text{if } n > 1 
\end{align*} \)
- Let’s act it out…
  - subcontractor model
  - \( \text{fib}(5) \)

Order of growth of # of calls of \( n! \)

- Constant
- Logarithmic
- Linear
- Quadratic
- Exponential

Fibonacci

- Inductive definition:
  \( \text{fib}(n) = \{ \begin{align*} 
  n, & \quad \text{if } n \leq 1 \\
  \text{fib}(n-1) + \text{fib}(n-2), & \quad \text{if } n > 1 
\end{align*} \)
- Let’s act it out…
  - subcontractor model
  - \( \text{fib}(5) \)

Back in 2008, researchers at UCSD demonstrated that they could make a working copy of a key based on a photo taken from 195 feet away. The shape of a key is actually a secret that can be stolen and needs to be protected!

http://vision.ucsd.edu/~blaxton/sneakey.html
Order of growth of # of calls of fib(n)

- a) Constant
- b) Logarithmic
- c) Linear
- d) Quadratic
- e) Exponential

Counting Change (thanks to BH)

- Given coins {50, 25, 10, 5, 1} how many ways are there of making change?
  - 5: 2 (N, 5 P)
  - 10: 4 (D, 2N, N 5P, 10P)
  - 15: 6 (2D, 3N, 2N 5P, N10P, 15P)
  - 100?

Call Tree for “Count Change 10 (10 5 1)”

Summary

- It’s important to understand the machine model.
- It’s often the cleanest, simplest way to solve many problems.
  - Especially those recursive in nature.
- Recursion is a very powerful idea, and one way to separate good from great.