The problem sets are intended to help you learn the material, and we encourage you to collaborate with other students and to ask questions in discussion sections and office hours to understand the problems. However, each student must turn in their own solutions to the problems.

The problem sets also provide essential background material for the quizzes. The problem sets will be graded primarily on an effort basis, but if you do not work through the problem sets you are unlikely to succeed at the quizzes! We will distribute solutions to the problem sets on the day the problem sets are due to give you feedback. Homework assignments are due at the beginning of class on the due date. Homework will not be accepted once solutions are handed out.
Problem P4.1: Trace Scheduling

Trace scheduling is a compiler technique that increases ILP by removing control dependencies, allowing operations following branches to be moved up and speculatively executed in parallel with operations before the branch. It was originally developed for statically scheduled VLIW machines, but it is a general technique that can be used in different types of machines and in this question we apply it to a single-issue MIPS processor.

Consider the following piece of C code (% is modulus) with basic blocks labeled:

```c
A    if (data % 8 == 0)
B      X = V0 / V1;
    else
C      X = V2 / V3;
D    if (data % 4 == 0)
E      Y = V0 * V1;
    else
F      Y = V2 * V3;
G
```

Assume that `data` is a uniformly distributed integer random variable that is set sometime before executing this code.

The program’s control flow graph is

![Control Flow Graph]

The decision tree is

![Decision Tree]

A control flow graph and the decision tree both show the possible flow of execution through basic blocks. However, the control flow graph captures the static structure of the program, while the decision tree captures the dynamic execution (history) of the program.
Problem P4.1.A

On the decision tree, label each path with the probability of traversing that path. For example, the leftmost block will be labeled with the total probability of executing the path ABDEG. (Hint: you might want to write out the cases). Circle the path that is most likely to be executed.

Problem P4.1.B

This is the MIPS code (no delay slots):

A:  lw  r1, data  
     andi r2, r1, 7  ;; r2 <- r1%8  
     bnez r2, C  
B:  div r3, r4, r5  ;; X <- V0/V1  
     j  D  
C:  div r3, r6, r7  ;; X <- V2/V3  
D:  andi r2, r1, 3  ;; r2 <- r1%4  
     bnez r2, F  
E:  mul r8, r4, r5  ;; Y <- V0*V1  
     j  G  
F:  mul r8, r6, r7  ;; Y <- V2*V3  
G:  

This code is to be executed on a single-issue processor without branch speculation. Assume that the memory, divider, and multiplier are all separate, long latency, unpipelined units that can be run in parallel. Rewrite the above code using trace scheduling. Optimize only for the most common path. Just get the other paths to work. Don’t spend your time performing any other optimizations. Ignore the possibility of exceptions. (Hint: Write the most common path first then add fix-up code.)

Problem P4.1.C

Assume that the load takes x cycles, divide takes y cycles, and multiply takes z cycles. Approximately how many cycles does the original code take? (ignore small constants) Approximately how many cycles does the new code take in the best case?
Problem P4.2: VLIW machines

The program we will use for this problem is listed below (In all questions, you should assume that arrays \( A \), \( B \) and \( C \) do not overlap in memory).

```c
for (i=0; i<328; i++) {
    C[i] = C[i] + A[i];
}
```

In this problem, we will deal with the code sample on a VLIW machine. Our machine will have six execution units:
- two ALU units, latency one cycle, also used for branch operations
- two memory units, latency three cycles, fully pipelined, each unit can perform either a store or a load
- two FPU units, latency four cycles, fully pipelined, one unit can perform \texttt{fadd} operations, the other \texttt{fmul} operations.

Our machine has no interlocks. The result of an operation is written to the register file immediately after it has gone through the corresponding execution unit: one cycle after issue for ALU operations, three cycles for memory operations and four cycles for FPU operations. The old values can be read from the registers until they have been overwritten.

Below is a diagram of our VLIW machine:
The program for this problem translates to the following VLIW operations:

```
loop: 1. ld f1, 0(r1) ; f1 = A[i]
  2. ld f2, 0(r2) ; f2 = B[i]
  3. fmul f4, f2, f1 ; f4 = f1 * f2
  4. st f4, 0(r1) ; A[i] = f4
  5. ld f3, 0(r3) ; f3 = C[i]
  6. fadd f5, f4, f3 ; f5 = f4 + f3
  7. st f5, 0(r3) ; C[i] = f5
  8. add r1, r1, 4 ; i++
  9. add r2, r2, 4
10. add r3, r3, 4
11. add r4, r4, -1
12. bnez r4, loop ; loop
```

**Problem P4.2.A**

Table P4.2-1, on the next page, shows our program rewritten for our VLIW machine, with some operations missing (instructions 2, 6 and 7). We have rearranged the instructions to execute as soon as they possibly can, but ensuring program correctness. Please fill in missing operations. (Note, you may not need all the rows)

**Problem P4.2.B**

How many cycles are required to complete one iteration of the loop in steady state? What is the performance (flops/cycle) of the program?

**Problem P4.2.C**

How many VLIW instructions would the smallest software pipelined loop require? Explain briefly. Ignore the prologue and the epilogue. Note: You do not need to write the software pipelined version. (You may consult Table P4.2-1 for help)

**Problem P4.2.D**

What would be the performance (flops/cycle) of the program? How many iterations of the loop would we have executing at the same time?
<table>
<thead>
<tr>
<th>ALU1</th>
<th>ALU2</th>
<th>MU1</th>
<th>MU2</th>
<th>FADD</th>
<th>FMUL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Add r1, r1, 4</td>
<td>add r2, r2, 4</td>
<td>ld f1, 0(r1)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Add r3, r3, 4</td>
<td>add r4, r4, -1</td>
<td>ld f3, 0(r3)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
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<td></td>
<td></td>
<td></td>
<td>fmul f4, f2, f1</td>
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<td></td>
<td></td>
<td></td>
<td>st f4, -4(r1)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>bnez r4, loop</td>
</tr>
</tbody>
</table>

Table P4.2-1: VLIW Program
Problem P4.2.E

If we unrolled the loop once, would that give us better performance? How many VLIW instructions would we need for optimal performance? How many flops/cycle would we get? Explain.

Problem P4.2.F

What is the maximal performance in flops/cycle for this program on this architecture? Explain.

Problem P4.2.G

If our machine had a rotating register file, could we use fewer instructions than in Question P4.2.F and still achieve optimal performance? Explain.

Problem P4.2.H

Imagine that memory latency has just increased to 100 cycles. Circle how many instructions (approximately) an optimal loop would require. (no rotating register file, ignoring prologue/epilogue). Explain briefly.

5 50 100 200

Problem P4.2.I

Now our processor still has memory latency of up to 100 cycles when it needs to retrieve data from main memory, but only 3 cycles if the data comes from the cache. Thus a memory operation can complete and write its result to a register anywhere between 3 and 100 cycles after being issued. Since our processor has no interlocks, other instructions will continue being issued. Thus, given two instructions, it is possible for the instruction issued second to complete and write back its result first. Circle how many instructions (approximately) are required for an optimal loop. Explain briefly.

5 50 100 200
Problem P4.3: VLIW & Vector Coding

Ben Bitdiddle has the following C loop, which takes the absolute value of elements within a vector.

```c
for (i = 0; i < N; i++) {
    if (A[i] < 0)
        A[i] = -A[i];
}
```

**Problem P4.3.A**

Ben is working with an in-order VLIW processor, which issues two MIPS-like operations per instruction cycle. Assume a five-stage pipeline with two single-cycle ALUs, memory with one read and one write port, and a register file with four read ports and two write ports. Also assume that there are no branch delay slots, and loads and stores only take one cycle to complete. Turn Ben’s loop into VLIW code. A and N are 32-bit signed integers. Initially, R1 contains N and R2 points to A[0]. You do not have to preserve the register values. Optimize your code to improve performance but do not use loop unrolling or software pipelining. What is the average number of cycles per element for this loop, assuming data elements are equally likely to be negative and non-negative?

**Problem P4.3.B**

Ben wants to remove the data-dependent branches in the assembly code by using predication. He proposes a new set of predicated instructions as follows:

1) Augment the ISA with a set of 32 predicate bits $P_0$-$P_{31}$.
2) Every standard non-control instruction now has a predicated counterpart, with the following syntax:

   ```
   (pbit1) OPERATION1 ; (pbit2) OPERATION2
   ```

   (Execute the first operation of the VLIW instruction if pbit1 is set and execute the second operation of the VLIW instruction if pbit2 is set.)

3) Include a set of compare operations that conditionally set a predicate bit:

   ```
   CMPLTZ  pbit,reg ; set pbit if reg < 0
   CMPGEZ  pbit,reg ; set pbit if reg >= 0
   CMPEQZ  pbit,reg ; set pbit if reg == 0
   CMPNEZ  pbit,reg ; set pbit if reg != 0
   ```
Eliminate all forward branches from Question P4.3.A with the new predicated operations. Try to optimize your code but do not use software pipelining or loop unrolling.

What is the average number of cycles per element for this new loop? Assume that the predicate-setting compares have single cycle latency (i.e., behave similarly to a regular ALU instruction including full bypassing of the predicate bit).

**Problem P4.3.C**

Unroll the predicated VLIW code to perform two iterations of the original loop before each backwards branch. You should use software pipelining to optimize the code for both performance and code density. What is the average number of cycles per element for large N?
Problem P4.4: Vector Machines

In this problem, we analyze the performance of vector machines. We start with a baseline vector processor with the following features:

- 32 elements per vector register
- 8 lanes
- One ALU per lane: 1 cycle latency
- One load/store unit per lane: 4 cycle latency, fully pipelined
- No dead time
- No support for chaining
- Scalar instructions execute on a separate 5-stage pipeline

To simplify the analysis, we assume a magic memory system with no bank conflicts and no cache misses.

We consider execution of the following loop:

```
for (i=0; i<320; i++) {
}
```

<table>
<thead>
<tr>
<th>C code</th>
<th>assembly code</th>
</tr>
</thead>
</table>
| for (i=0; i<320; i++) {  
} | # initial conditions:  
    # R1 points to A[0]  
    # R2 points to B[0]  
    # R3 points to C[0]  
    # R4 = 1  
    # R5 = 320  

loop:
LV V1, R1       # load A  
LV V2, R2       # load B  
ADDV V3, V1, V2 # add A+B  
SUBVS V4, V3, R4 # subtract 1  
SV R3, V4       # store C  
ADDI R1, R1, 128 # incr. A pointer  
ADDI R2, R2, 128 # incr. B pointer  
ADDI R3, R3, 128 # incr. C pointer  
SUBI R5, R5, 32  # decr. count  
BNEZ R5, loop   # loop until done |
Problem P4.4.A

Complete the pipeline diagram of the baseline vector processor running the given code.

The following supplementary information explains the diagram:
Scalar instructions execute in 5 cycles: fetch (F), decode (D), execute (X), memory (M), and writeback (W).
A vector instruction is also fetched (F) and decoded (D). Then, it stalls (—) until its required vector functional unit is available. With no chaining, a dependent vector instruction stalls until the previous instruction finishes writing back all of its elements. A vector instruction is pipelined across all the lanes in parallel. For each element, the operands are read (R) from the vector register file, the operation executes on the load/store unit (M) or the ALU (X), and the result is written back (W) to the vector register file.
A stalled vector instruction does not block a scalar instruction from executing.
LV1 and LV2 refer to the first and second LV instructions in the loop.

| instr. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
|--------|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| LV1    | F | D | R | M1 | M2 | M3 | M4 | W |
| LV1    |   |   | R | M1 | M2 | M3 | M4 | W |
| LV1    |   |   | R | M1 | M2 | M3 | M4 | W |
| LV2    | F | D | — | — | R | M1 | M2 | M4 | W |
| LV2    |   |   | R | M1 | M2 | M3 | M4 | W |
| LV2    |   |   | R | M1 | M2 | M3 | M4 | W |
| LV2    |   |   | R | M1 | M2 | M3 | M4 | W |
| ADDV   | F | D | — | — | — | — | — | — | R | X1 | W |
| ADDV   |   |   | R | X1 | W |
| ADDV   |   |   | R | X1 | W |
| ADDV   |   |   | R | X1 | W |
| SUBVS  | F | D | — |   |
| SUBVS  |   |   |   |   |
| SUBVS  |   |   |   |   |
| SUBVS  |   |   |   |   |
| SV     | F | D | — |   |
| SV     |   |   |   |   |
| SV     |   |   |   |   |
| SV     |   |   |   |   |
| ADDI   | F | D | X | M | W |
| ADDI   | F | D | X | M | W |
| ADDI   | F | D | X | M | W |
| SUBI   | F | D | X | M | W |
| BNEZ   | F | D | X | M | W |
| LV1    | F | D | — |   |
| LV1    |   |   |   |   |
| LV1    |   |   |   |   |
| LV1    |   |   |   |   |
In this question, we analyze the performance benefits of chaining and additional lanes. Vector chaining is done through the register file and an element can be read (R) on the same cycle in which it is written back (W), or it can be read on any later cycle (the chaining is flexible). For this question, we always assume 32 elements per vector register, so there are 2 elements per lane with 16 lanes, and 1 element per lane with 32 lanes.

To analyze performance, we calculate the total number of cycles per vector loop iteration by summing the number of cycles between the issuing of successive vector instructions. For example, in Question P4.4.A, LV₁ begins execution in cycle 3, LV₂ in cycle 7 and ADDV in cycle 16. Therefore, there are 4 cycles between LV₁ and LV₂, and 9 cycles between LV₂ and ADDV.

Complete the following table. The first row corresponds to the baseline 8-lane vector processor with no chaining. The second row adds flexible chaining to the baseline processor, and the last two rows increase the number of lanes to 16 and 32. (Hint: You should consider each pair of vector instructions independently, and you can ignore the scalar instructions.)
Problem P4.4.C

Even with the baseline 8-lane vector processor with no chaining (used in Question P4.4.A), we can improve performance using software loop-unrolling and instruction scheduling. As a first step, we unroll two iterations of the loop and rename the vector registers in the second iteration:

```
loop:
I1:    LV    V1, R1       # load A
I2:    LV    V2, R2       # load B
I3:    ADDV V3, V1, V2   # add A+B
I4:    SUBVS V4, V3, R4  # subtract 1
I5:    SV    R3, V4       # store C
I6:    ADDI R1, R1, 128  # incr. A pointer
I7:    ADDI R2, R2, 128  # incr. B pointer
I8:    ADDI R3, R3, 128  # incr. C pointer
I9:    SUBI R5, R5, 32   # decre. count
I10:   LV    V5, R1      # load A
I11:   LV    V6, R2      # load B
I12:   ADDV V7, V5, V6   # add A+B
I13:   SUBVS V8, V7, R4  # subtract 1
I14:   SV    R3, V8      # store C
I15:   ADDI R1, R1, 128  # incr. A pointer
I16:   ADDI R2, R2, 128  # incr. B pointer
I17:   ADDI R3, R3, 128  # incr. C pointer
I18:   SUBI R5, R5, 32   # decre. count
I19:   BNEZ  R5, loop    # loop until done
```

Reorder the instructions in the unrolled loop to improve performance on the baseline vector processor (your solution does not need to be optimal). Provide a valid ordering by listing the instruction numbers below (a few have already been filled in for you). Filling in the “Instruction” field is optional. You may assume that the A, B and C arrays do not overlap.

<table>
<thead>
<tr>
<th>Instr. Number</th>
<th>Instruction</th>
</tr>
</thead>
<tbody>
<tr>
<td>I1</td>
<td>LV V1, R1</td>
</tr>
<tr>
<td>I2</td>
<td>LV V2, R2</td>
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<td></td>
</tr>
<tr>
<td>I15</td>
<td>ADDI R1, R1, 128</td>
</tr>
<tr>
<td>I16</td>
<td>ADDI R2, R2, 128</td>
</tr>
<tr>
<td>I17</td>
<td>ADDI R3, R3, 128</td>
</tr>
<tr>
<td>I9</td>
<td>SUBI R5, R5, 32</td>
</tr>
<tr>
<td>I18</td>
<td>SUBI R5, R5, 32</td>
</tr>
<tr>
<td>I19</td>
<td>BNEZ R5, loop</td>
</tr>
</tbody>
</table>
Problem P4.5: Multithreading

This problem evaluates the effectiveness of multithreading using a simple database benchmark. The benchmark searches for an entry in a linked list built from the following structure, which contains a key, a pointer to the next node in the linked list, and a pointer to the data entry.

```c
struct node {
    int key;
    struct node *next;
    struct data *ptr;
}
```

The following MIPS code shows the core of the benchmark, which traverses the linked list and finds an entry with a particular key. Assume MIPS has no delay slots.

```
; ; R1: a pointer to the linked list
; R2: the key to find
;
loop:  LW      R3, 0(R1)   ; load a key
      LW      R4, 4(R1)   ; load the next pointer
      SEQ     R3, R3, R2 ; set R3 if R3 == R2
      BNEZ    R3, End    ; found the entry
      ADD     R1, R0, R4
      BNEZ    R1, Loop   ; check the next node
End:
      ; R1 contains a pointer to the matching entry or zero
      ; if not found
```

We run this benchmark on a single-issue in-order processor. The processor can fetch and issue (dispatch) one instruction per cycle. If an instruction cannot be issued due to a data dependency, the processor stalls. Integer instructions take one cycle to execute and the result can be used in the next cycle. For example, if `SEQ` is executed in cycle 1, `BNEZ` can be executed in cycle 2. We also assume that the processor has a perfect branch predictor with no penalty for both taken and not-taken branches.

Problem P4.5.A

Assume that our system does not have a cache. Each memory operation directly accesses main memory and takes 100 CPU cycles. The load/store unit is fully pipelined, and non-blocking. After the processor issues a memory operation, it can continue executing instructions until it reaches an instruction that is dependent on an outstanding memory operation. How many cycles does it take to execute one iteration of the loop in steady state?
Problem P4.5.B

Now we add zero-overhead multithreading to our pipeline. A processor executes multiple threads, each of which performs an independent search. Hardware mechanisms schedule a thread to execute each cycle.

In our first implementation, the processor switches to a different thread every cycle using fixed round robin scheduling (similar to CDC 6600 PPU). Each of the N threads executes one instruction every N cycles. What is the minimum number of threads that we need to fully utilize the processor, i.e., execute one instruction per cycle?

Problem P4.5.C

How does multithreading affect throughput (number of keys the processor can find within a given time) and latency (time processor takes to find an entry with a specific key)? Assume the processor switches to a different thread every cycle and is fully utilized. Check the correct boxes.

<table>
<thead>
<tr>
<th>Throughput</th>
<th>Latency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Better</td>
<td></td>
</tr>
<tr>
<td>Same</td>
<td></td>
</tr>
<tr>
<td>Worse</td>
<td></td>
</tr>
</tbody>
</table>

Problem P4.5.D

We change the processor to only switch to a different thread when an instruction cannot execute due to data dependency. What is the minimum number of threads to fully utilize the processor now? Note that the processor issues instructions in-order in each thread.
Problem P4.6: Multithreading

Consider a single-issue in-order multithreading processor that is similar to the one described in Problem P4.5.

Each cycle, the processor can fetch and issue one instruction that performs any of the following operations:

- load/store, 12-cycle latency (fully pipelined)
- integer add, 1-cycle latency
- floating-point add, 5-cycle latency (fully pipelined)
- branch, no delay slots, 1-cycle latency

The processor does not have a cache. Each memory operation directly accesses main memory. If an instruction cannot be issued due to a data dependency, the processor stalls. We also assume that the processor has a perfect branch predictor with no penalty for both taken and not-taken branches.

You job is to analyze the processor utilizations for the following two thread-switching implementations:

**Fixed Switching:** the processor switches to a different thread every cycle using fixed round robin scheduling. Each of the N threads executes an instruction every N cycles.

**Data-dependent Switching:** the processor only switches to a different thread when an instruction cannot execute due to a data dependency.

Each thread executes the following MIPS code:

```mips
loop:    L.D F2, 0(R1) ; load data into F2
         ADDI R1, R1, 4 ; bump source pointer
         FADD F3, F3, F2 ; F3 = F3 + F2
         BNE F2, F4, loop ; continue if F2 != F4
```

**Problem P4.6.A**

What is the minimum number of threads that we need to fully utilize the processor for each implementation?

**Fixed Switching:** _________________ Thread(s)

**Data-dependent Switching:** _________________ Thread(s)
Problem P4.6.B

What is the minimum number of threads that we need to fully utilize the processor for each implementation if we change the load/store latency to 1-cycle (but keep the 5-cycle floating-point add)?

Fixed Switching: _________________ Thread(s)

Data-dependent Switching: _________________ Thread(s)

Problem P4.6.C

Consider a Simultaneous Multithreading (SMT) machine with limited hardware resources. Circle the following hardware constraints that can limit the total number of threads that the machine can support. For the item(s) that you circle, briefly describe the minimum requirement to support $N$ threads.

(A) Number of Functional Unit:

(B) Number of Physical Registers:

(C) Data Cache Size:

(D) Data Cache Associatively: