Random Number Generation and Electronic Cash

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Random Number Generation

- **Many crypto protocols require parties to generate random numbers**
  - **Key generation**
  - **Generating nonces**
- **How to generate random numbers?**
  - **Step 1: how to generate truly random bits?**
  - **Step 2: crypto methods to stretch a little bit of true randomness into a large stream of pseudorandom values that are indistinguishable from true random bits (PRNG)**

Case Study

- **Random number generation is easy to get wrong**
- **Can you spot the problems in this example?**

```
unsigned char key[16];

srand(time(null));
 for (i=0; i<16; i++)
    key[i] = rand() & 0xFF;

where

   static unsigned int next = 0;
   void srand(unsigned int seed) {
      next = seed;
   }

   int rand(void) {
       next = next * 1103515245 + 12345;
       return next % 32768;
   }
```

Real-world Examples

- X Windows “magic cookie” was generated using rand()
- Netscape browsers generated SSL session keys using time & process ID as seed (1995)
- Kerberos
  - First discover to be similarly flawed
  - 4 yrs later, discovered flaw with memset()
- PGP used return value from read() to seed its PRNG, rather than the contents of buffer
- On-line poker site used insecure PRNG to shuffle cards

Lessons Learned

- Seeds must be unpredictable
- Algorithm for generating pseudorandom bits must be secure

Generating Pseudorandom Numbers

- True random number generator (TRNG)
  - Generates bits that are distributed uniformly at random, so that all outputs are equally likely, with no patterns, correlations, etc.
- Cryptographically secure pseudorandom number generator (CS-PRNG)
  - Taking a short true-random seed, and generates long sequence of bits that is computationally indistinguishable from true random bits

CS-PRNG

- **CS-PRNG: cryptographically secure pseudorandom number generator**
  - **G: maps a seed to an output G(S)**
    - » **E.g., G: $\{0,1\}^{128}$ -> $\{0,1\}^{1000000}$**
  - **Let K denote a random variable distributed uniformly at random in domain of G**
  - **Let U denote a random variable distributed uniformly at random in range of G**
  - **G is secure if output G(K) is computationally indistinguishable from U**
- **Sample construction**
  - **Use the seed as a key k, and compute AES-CBC(k, 0^n)**

TRNG (I)

- **TRNG should be random and unpredictable**
- **Good or bad choices?**
  - **IP addresses**
  - **Contents of network packets**
  - **Process IDs**
  - **High-speed clock**
  - **Soundcard**
  - **Keyboard input**
  - **Disk timings**

TRNG (II)

- **How to convert non-uniform sources of randomness into TRNG?**
  - **Use a cryptographic hash function, such as SHA1**
  - **Suppose x is a value from an imperfect source, or a concatenation of values from multiple sources, and it is impossible for an attacker to predict the exact value x except with probability $1/2^n$**
  - **Then hash(x) truncated to n bits should provide a n-bit value that is uniformly distributed, if hash() is secure**

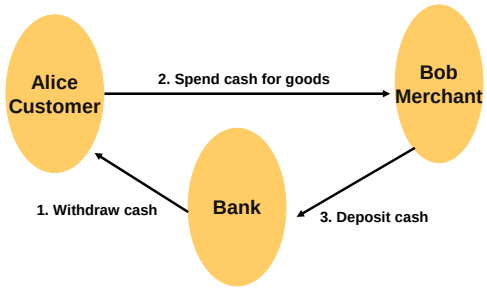
Administrative Matters

- **HW2 graded**

Mean: 41.7
Standard deviation: 13.2
1st quartile: 39.8
2nd quartile (median): 44.5
3rd quartile: 50.0
Maximum: 57.0

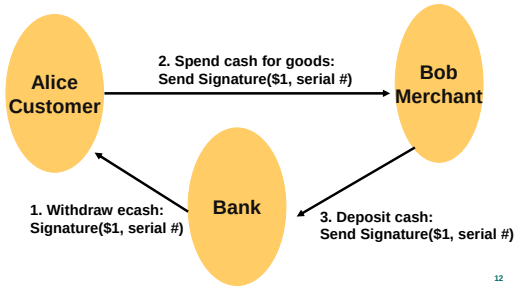
Ecash

- **Example for how crypto helps e-commerce**
- **Traditional cash**



Ecash

- **Digital form of cash**
- **First attempt, what's the problem?**

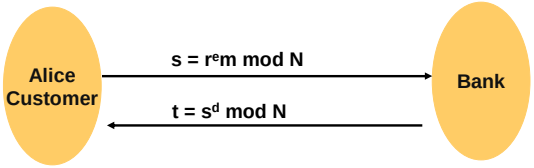


Desired Properties for Ecash

- **Anonymous: bank should not know how Alice spends her money**
- **Prevent forging**
- **Prevent double spending**

Building Block: Blind Signatures

- **Blind signature: achieve anonymity**
  - **How can Alice get a signature from the Bank without the Bank knowing what message is being signed?**
- **Protocol:**
  - **generating blind signature on message m in RSA setting**
  - **Bank's private key (d, p, q), public key (e, N)**



  - **Alice computes t/r mod N = m^d mod N**

Ecash Using Blind Signature

- **How to use blind signature to build ecash?**
- **A valid $1 bill is a pair (x,y), where $y = \text{hash}(x)^d \bmod N$, hash() is one-way function**
- **How does the ecash protocol work?**
- **Why do we need hash()?**
- **How to prevent double spending?**
- **What to do for different denominations?**
  - **Nickles, dimes, dollars**

Other Methods for Ecash

- **Use zero-knowledge proofs (out of scope)**
  - **More building blocks of ZKP**
  - **Support many properties**
    - » **Identifying double spenders**

Conclusion

- **Random number generator**
  - **CS-PRNG**
    - » **Definition**
    - » **How to construct it?**
  - **TRNG**
- **Ecash**
  - **Example of the power of crypto**
  - **Blind signatures**