2. (25 points) Synchronization primitives: Consider a machine with hardware support for a single thread synchronization primitive, called Compare-And-Swap (CAS).
Compare-and-swap is an atomic operation, provided by the hardware, with the following pseudocode:
```
int compare_and_swap(int *a, int old, int new) {
    if (*a == old) {
        *a = new;
        return 1;
        } else {
            return 0;
        }
}
```

Your first task is to implement the code for a simple spinlock using compare-andswap. You are not allowed to assume any other hardware or kernel support exists (e.g., disabling interrupts). You may assume your spinlock will be used correctly (i.e., no double release or release by a thread not holding the lock)
a. (3 points) Fill in the code for the spinlock data structure.

```
struct spinlock { /* Fill in */
```

\}
b. (4 points) Fill in the code for the acquire data function.
void acquire(struct spinlock *lock) \{ /* Fill in */
\}
c. (4 points) Fill in the code for the release data function.
void release(struct spinlock *lock) \{ /* Fill in */
\}
After completing your implementation, you realize that using a spinlock is inefficient for applications that may hold the lock for a long time. You consider using the following two primitives to implement more efficient locks: atomic_sleep and wake.
atomic_sleep is an atomic operation, provided by the hardware, with the following pseudocode:

```
void atomic_sleep(struct *lock, int *val1, int val2){
            *val1 = val2; /* set vall to val2 */
            enqueue(lock); /* put current thread on a
                                    lock's wait queue*/
            sleep(); /* put current thread to sleep */
}
```

wake is non-atomic with the following pseudocode:

```
void wake(struct lock *lock) {
    dequeue(); /* remove a thread (if any) from lock's
                wait queue and add it to the
                scheduler's ready queue */
    }
```

Your second task is to reimplement your lock code more efficiently using atomic_sleep and wake. You may use Compare-And-Swap if you want. You are not allowed to assume any other hardware or kernel support exists (e.g., disabling interrupts).
d. (4 points) Fill in the code for the new lock data structure.
struct lock \{ /* Fill in */
\}
e. (5 points) Fill in the code for the new acquire data function. void acquire(struct lock *lock) \{ /* Fill in */
\}
\}
f. (5 points) Fill in the code for the new release data function.
void release(struct lock *lock) \{ /* Fill in */
\}
3. (12 points) Synchronization: A common parallel programming pattern is to perform processing in a sequence of parallel stages: all threads work independently during each stage, but they must synchronize at the end of each stage at a synchronization point called a barrier. If a thread reaches the barrier before all other threads have arrived, it waits. When all threads reach the barrier, they are notified and can begin the execution on the next phase of the computation.

Example:

```
while (true) {
    Compute stuff;
    BARRIER();
    Read other threads results;
}
```

a) (4 points) The following implementation of Barrier is incomplete and has two lines missing. Fill in the missing lines so that the Barrier works according to the prior specifications.

```
class Barrier() {
    int numWaiting = 0; // Initially, no one at barrier
    int numExpected = 0; // Initially, no one expected
    Lock L = new Lock();
    ConditionVar CV = new ConditionVar();
    void threadCreated() {
        L.acquire();
        numExpected++;
        L.release();
    }
    void enterBarrier() {
            L.acquire();
            numWaiting++;
            if (numExpected == numWaiting) { // If we are the last
                numWaiting = 0; // Reset barrier and wake threads
                                    / / Fill me in
            } else { // Else, put me to sleep
```

$\qquad$

```
                        / / Fill me in
            }
            L.release() ;
    }
}
```

b) (5 points) Now, let us use Barrier in a parallel algorithm. Consider the linked list below:

Node 4


Node 3

| value $=1$ <br> next <br> updated_value $=$ null <br> updated_next $=$ null |
| :--- |

Node 2

| value $=1$ <br> next <br> updated_value = null <br> updated_next = null | $>$value $=1$ <br> next = null <br> updated_value $=$ null <br> updated_next = null |
| :--- | :--- |

head
In our parallel algorithm, there are four threads (Thread 1, Thread 2, Thread 3, Thread 4). Each thread has its own instance variable node, and all threads share the class variable barrier. Initially, Thread 1's node references Node 1, Thread 2's node references Node 2, Thread 3's node references Node 3, and Thread 4's node references Node 4.

In the initialization steps, barrier.threadCreated() is called once for each thread created, so we have barrier.numExpected $==4$ as a starting condition.

Once all four threads are initialized, each thread calls its run() method. The run() method is identical for all threads:

```
void run() {
    boolean should_print = true;
    while (true) {
        if (node.next != null) {
            node.updated_value = node.value +
                    node.next.value;
            node.updated_next = node.next.next;
        } else if (should_print) {
            System.out.println(node.value);
            should_print = false;
        }
        barrier.enterBarrier();
        node.value = node.updated_value;
        node.next = node.updated_next;
        barrier.enterBarrier();
    }
}
```

List all the values that are printed to stdout along with the thread that prints each value. For example, "thread 1 prints 777".
c) (3 points) In an attempt to speed-up the parallel algorithm from the previous part (2c), you notice that the line barrier.enterBarrier() occurs twice in run()'s while loop. Can one of these two calls to barrier.enterBarrier() be removed while guaranteeing that the output of the previous part (2c) remains unchanged? If your answer is "yes", specify whether you would remove the first or second occurrence of barrier.enterBarrier().
4. (22 points) Deadlock:

A restaurant would like to serve four dinner parties, P1 through P4. The restaurant has a total of 8 plates and 12 bowls. Assume that each group of diners will stop eating and wait for the waiter to bring a requested item (plate or bowl) to the table when it is required. Assume that the diners don't mind waiting. The maximum request and current allocation tables are shown as follows:

| Maximum <br> Request | Plates | Bowls |
| :--- | :--- | :--- |
| P1 | 7 | 7 |
| P2 | 6 | 10 |
| P3 | 1 | 2 |
| P4 | 2 | 4 |


| Current <br> Allocation | Plates | Bowls |
| :--- | :--- | :--- |
| P1 | 2 | 3 |
| P2 | 3 | 5 |
| P3 | 0 | 1 |
| P4 | 1 | 2 |

a. (4 points) Determine the Need Matrix for plates and bowls.

| Need | Plates | Bowls |
| :--- | :--- | :--- |
| P1 |  |  |
| P2 |  |  |
| P3 |  |  |
| P4 |  |  |

b. (7 points) Will the restaurant be able to feed all four parties successfully? Clearly explain your answer - specifically, why no or why/how there is a safe serving order.
4. (continued) Deadlock
c. (11 points) Assume a new dinner party, P5, comes to the restaurant at this time. Their maximum needs are 5 plates and 3 bowls. Initially, the waiter brings 2 plates to them. In order to be able to feed all five parties successfully, the restaurant needs more plates.
i. (2 points) Determine the new Need Matrix for plates and bowls.

| Need | Plates | Bowls |
| :--- | :--- | :--- |
| P1 |  |  |
| P2 |  |  |
| P3 |  |  |
| P4 |  |  |
| P5 |  |  |

ii. (6 points) At least how many plates would the restaurant need to add?
iii. (3 points) Show a safe serving sequence.
$\qquad$
$\qquad$

Question 2. Deadlock (15 points)
Consider a system with four processes P1, P2, P3, and P4, and two resources, R1, and R2, respectively. Each resource has two instances. Furthermore:

- P1 allocates an instance of R2, and requests an instance of R1;
- P2 allocates an instance of R1, and doesn't need any other resource;
- P3 allocates an instance of R1 and requires an instance of R2;
- P4 allocates an instance of R2, and doesn't need any other resource.
(5 points each question)
(a) Draw the resource allocation graph.
(b) Is there a cycle in the graph? If yes name it.
(c) Is the system in deadlock? If yes, explain why. If not, give a possible sequence of executions after which every process completes.
$\qquad$

Question 4. Scheduling (20 points)
Consider three threads that arrive at the same time and they are enqueued in the ready queue in the order T1, T2, T3.

Thread T1 runs a four-iteration loop, with each iteration taking one time unit. At the end of each iteration, T1 calls yield; as a result, T1 is placed at the end of the ready queue. Threads T2 and T3 both run a twoiteration loop, which each iteration taking three time units. At the end of first iteration, T2 synchronizes with T3, i.e., T2 cannot start the second iteration before T3 finishes the first iteration, and vice versa. While waiting, T 2 (T3) is placed in the waiting queue; once T 3 (T2) finishes its first iteration, T 2 (T3) is placed at the end of the ready queue. Each process exits after finishing its loop.

Assume the system has one CPU. On the timeline below, show how the threads are scheduled using two scheduling policies (FCFS and Round Robin). For each unit of time, indicate the state of the thread by writing " $R$ " if the thread is running, " $A$ " if the thread is in the ready queue, and " $W$ " if the thread is in the waiting queue (e.g., T 2 waits for T 3 to finish the first iteration, before T 2 can run its second iteration).
(a) (6 points) FCFS (No-preemption) FCFS always selects the thread at the head of the ready queue. A thread only stops running when it calls yield or waits to synchronize with another thread. What is the average completion time? (Each column corresponds to one time unit. The first column is already filled in.)

| T1 | R |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| T 2 | A |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| T 3 | A |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

(b) (6 points) Round Robin (time quantum $=\mathbf{2}$ units) When a thread is preempted it is moved at the end of the ready queue. What is average completion time?

| T1 | R |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| T2 | A |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| T3 | A |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

(c) (8 points) Assume there are two processors P1 and P2 in the system. The scheduler follows the policy of FCFS with no preemption. When the scheduler assigns tasks, always assign a task to P1 before assigning to P2. Instead of using "R" to mark running, use "P1" or "P2" to indicate where the task runs. What is the average completion time?

| T1 | P1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| T2 | P2 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| T3 | A |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

4. (24 points total) CPU scheduling. Consider the following single-threaded processes, arrival times, and CPU processing requirements:

| Process ID (PID) | Arrival Time | Processing Time |
| :---: | :---: | :---: |
| 1 | 0 | 6 |
| 2 | 2 | 4 |
| 3 | 3 | 5 |
| 4 | 6 | 2 |

a) (12 points): For each scheduling algorithm, fill in the table with the ID of the process that is running on the CPU. Each row corresponds to a time unit.

- For time slice-based algorithms, assume one unit time slice.
- When a process arrives it is immediately eligible for scheduling, e.g., process 2 that arrives at time 2 can be scheduled during time unit 2 .
- If a process is preempted, it is added at the tail of the ready queue.

| Time | FIFO | RR | SJF |
| :---: | :---: | :---: | :---: |
| $\mathbf{0}$ |  |  |  |
| $\mathbf{1}$ |  |  |  |
| $\mathbf{2}$ |  |  |  |
| 3 |  |  |  |
| $\mathbf{4}$ |  |  |  |
| $\mathbf{5}$ |  |  |  |
| $\mathbf{6}$ |  |  |  |
| 7 |  |  |  |
| $\mathbf{8}$ |  |  |  |
| $\mathbf{1 0}$ |  |  |  |
| 11 |  |  |  |
| 12 |  |  |  |
| 13 |  |  |  |
| 15 |  |  |  |
| 16 |  |  |  |

b) (6 points): Calculate the response times of individual processes for each of the scheduling algorithms. The response time is defined as the time a process takes to complete after it arrives.

|  | PID 1 | PID 2 | PID 3 | PID 4 |
| :---: | :---: | :---: | :---: | :---: |
| FIFO |  |  |  |  |
| RR |  |  |  |  |
| SJF |  |  |  |  |

c) (6 points) Consider same processes and arrival times, but assume now a processor with two CPUs. Assume CPU 0 is busy for the first two time units. For each scheduling algorithm, fill in the table with the ID of the process that is running on each CPU.

- For any non-time slice-based algorithm, assume that once a process starts running on a CPU, it keeps running on the same CPU till the end.
- If both CPUs are free, assume CPU 0 is allocated first.

| Time | CPU \# | FIFO | RR | SJF |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 0 |  |  |  |
|  | 1 |  |  |  |
| 1 | 0 |  |  |  |
|  | 1 |  |  |  |
| 2 | 0 |  |  |  |
|  | 1 |  |  |  |
| 3 | 0 |  |  |  |
|  | 1 |  |  |  |
| 4 | 0 |  |  |  |
|  | 1 |  |  |  |
| 5 | 0 |  |  |  |
|  | 1 |  |  |  |
| 6 | 0 |  |  |  |
|  | 1 |  |  |  |
| 7 | 0 |  |  |  |
|  | 1 |  |  |  |
| 8 | 0 |  |  |  |
|  | 1 |  |  |  |
| 9 | 0 |  |  |  |
|  | 1 |  |  |  |
| 10 | 0 |  |  |  |
|  | 1 |  |  |  |

5. (15 points total) Scheduling. Consider the following processes, arrival times, and CPU processing requirements:

| Process Name | Arrival Time | Processing Time |
| :---: | :---: | :---: |
| 1 | 0 | 4 |
| 2 | 2 | 3 |
| 3 | 5 | 3 |
| 4 | 6 | 2 |

For each scheduling algorithm, fill in the table with the process that is running on the CPU (for timeslice-based algorithms, assume a 1 unit timeslice). For RR and SRTF, assume that an arriving thread is run at the beginning of its arrival time, if the scheduling policy allows it. Also, assume that the currently running thread is not in the ready queue while it is running. The turnaround time is defined as the time a process takes to complete after it arrives.

| Time | FIFO | RR | SRTF |
| :--- | :--- | :--- | :--- |
| $\mathbf{0}$ | 1 | 1 | $\mathbf{1}$ |
| $\mathbf{1}$ |  |  |  |
| 2 |  |  |  |
| $\mathbf{3}$ |  |  |  |
| $\mathbf{4}$ |  |  |  |
| $\mathbf{5}$ |  |  |  |
| $\mathbf{6}$ |  |  |  |
| 7 |  |  |  |
| $\mathbf{8}$ |  |  |  |
| $\mathbf{9}$ |  |  |  |
| 10 |  |  |  |
| 11 |  |  |  |
| Average <br> Turnaround <br> Time |  |  |  |

5. (18 points) Paging:

Suppose you have a system with 32-bit pointers and 4 megabytes of physical memory that is partitioned into 8192-byte pages. The system uses an Inverted Page Table (IPT). Assume that there is no page sharing between processes.
a. (8 points) Describe what page table entries should look like. Specifically, how many bits should be in each page table entry, and what are they for? Also, how many page table entries should there be in the page table?
b. (5 points) Describe how an IPT is used to translate a virtual address into a physical address.
c. (3 points) How can you make an IPT more efficient? Explain your solution and how it works in detail.
d. (2 points) What effect, if any, does your solution in part (c) have on what happens on a context switch?

## Problem 4：Virtual Memory［20 pts］

Consider a multi－level memory management scheme with the following format for virtual addresses：

| Virtual Page \＃ <br> $(10$ bits $)$ | Virtual Page \＃ <br> $(10$ bits $)$ | Offset <br> $(12$ bits $)$ |
| :---: | :---: | :---: |

Virtual addresses are translated into physical addresses of the following form：

| Physical Page \＃ <br> $(20$ bits $)$ | Offset <br> （12 bits） |
| :---: | :---: |

Page table entries（PTE）are 32 bits in the following format，stored in big－endian form in memory （i．e．the MSB is first byte in memory）：

| Physical Page \＃ （20 bits） | OS Defined （3 bits） | $\bigcirc$ |  | 守 | 呙 |  |  | $\underset{\stackrel{\sim}{\oplus}}{\stackrel{G}{\oplus}}$ | 号 | 岂 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |

Here，＂Valid＂means that a translation is valid，＂Writeable＂means that the page is writeable，＂User＂ means that the page is accessible by the User（rather than only by the Kernel）．Note：the phrase ＂page table＂in the following questions means the multi－level data structure that maps virtual addresses to physical addresses．

Problem 4a［2pts］：How big is a page？Explain．

Problem 4b［2pts］：Suppose that we want an address space with one physical page at the top of the address space and one physical page at the bottom of the address space．How big would the page table be（in bytes）？Explain．

Problem $\mathbf{4 c}[\mathbf{2 p t s}]$ ：What is the maximum size of a page table（in bytes）for this scheme？Explain．

Problem 4d［2pts］：How big would each entry of a fully－associative TLB be for this management scheme？Explain．

Problem 4e[2pts]: Sketch the format of the page-table for the multi-level virtual memory management scheme of (4a). Illustrate the process of resolving an address as well as possible.

Problem 4f[10pts]: Assume the memory translation scheme from (4a). Use the Physical Memory table given on the next page to predict what will happen with the following load/store instructions. Assume that the base table pointer for the current user level process is $0 \times 00200000$.

Addresses are virtual. The return value for a load is an 8-bit data value or an error, while the return value for a store is either "ok" or an error. Possible errors are: invalid, read-only, kernel-only. Hint: Don't forget that Hexidecimal digits contain 4 bits!

| Instruction | Result |
| ---: | :---: |
| $\left[\begin{array}{r}\text { Load }\end{array}\right.$ | $0 \times 50$ |
| $[0 \times 00001047]$ | ok |
| Store |  |
| [0x00C005FF] | ERROR: <br> read-only |
| $[0 \times 00003012]$ |  |


| Instruction | Result |
| ---: | :--- |
| Store |  |
| $[0 \times 02001345]$ |  |
| Load |  |
| $[0 \times F F 80078 \mathrm{~F}]$ |  |
| Load |  |
| TexFFFFF005] |  |
| $[0 \times F F F F F 006]$ |  |

## Physical Memory [All Values are in Hexidecimal]

| Address | +0 | +1 | +2 | +3 | +4 | +5 | +6 | +7 | +8 | +9 | +A | +B | +C | +D | +E | +F |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 00000000 | 0E | 0 F | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 1A | 1B | 1C | 1D |
| 00000010 | 1E | 1F | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 2A | 2B | 2 C | 2D |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 00001010 | 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 4A | 4B | 4 C | 4D | 4E | 4 F |
| 00001020 | 40 | 03 | 41 | 01 | 30 | 01 | 31 | 03 | 00 | 03 | 00 | 00 | 00 | 00 | 00 | 00 |
| 00001030 | 00 | 11 | 22 | 33 | 44 | 55 | 66 | 77 | 88 | 99 | AA | BB | CC | DD | EE | FF |
| 00001040 | 10 | 01 | 11 | 03 | 31 | 03 | 13 | 00 | 14 | 01 | 15 | 03 | 16 | 01 | 17 | 00 |
| .... |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 00002030 | 10 | 01 | 11 | 00 | 12 | 03 | 67 | 03 | 11 | 03 | 00 | 00 | 00 | 00 | 00 | 00 |
| 00002040 | 02 | 20 | 03 | 30 | 04 | 40 | 05 | 50 | 01 | 60 | 03 | 70 | 08 | 80 | 09 | 90 |
| 00002050 | 10 | 00 | 31 | 01 | 10 | 03 | 31 | 01 | 12 | 03 | 30 | 00 | 10 | 00 | 10 | 01 |
| ... |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 00004000 | 30 | 00 | 31 | 01 | 11 | 01 | 33 | 03 | 34 | 01 | 35 | 00 | 43 | 38 | 32 | 79 |
| 00004010 | 50 | 28 | 84 | 19 | 71 | 69 | 39 | 93 | 75 | 10 | 58 | 20 | 97 | 49 | 44 | 59 |
| 00004020 | 23 | 03 | 20 | 03 | 00 | 01 | 62 | 08 | 99 | 86 | 28 | 03 | 48 | 25 | 34 | 21 |
| .... |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 00100000 | 00 | 00 | 10 | 65 | 00 | 00 | 20 | 67 | 00 | 00 | 30 | 00 | 00 | 00 | 40 | 07 |
| 00100010 | 00 | 00 | 50 | 03 | 00 | 00 | 00 | 00 | 00 | 00 | 00 | 00 | 00 | 00 | 00 | 00 |
| ... |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 00103000 | 11 | 22 | 00 | 05 | 55 | 66 | 77 | 88 | 99 | AA | BB | CC | DD | EE | FF | 00 |
| 00103010 | 22 | 33 | 44 | 55 | 66 | 77 | 88 | 99 | AA | BB | CC | DD | EE | FF | 00 | 67 |
| ... |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 001FE000 | 04 | 15 | 00 | 00 | 48 | 59 | 70 | 7B | 8C | 9D | AE | BF | D0 | E1 | F2 | 03 |
| 001 FE 010 | 10 | 15 | 00 | 67 | 10 | 15 | 10 | 67 | 10 | 15 | 20 | 67 | 10 | 15 | 30 | 67 |
| -.. |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 001FF000 | 00 | 00 | 00 | 00 | 00 | 00 | 00 | 65 | 00 | 00 | 10 | 67 | 00 | 00 | 00 | 00 |
| 001FF010 | 00 | 00 | 20 | 67 | 00 | 00 | 30 | 67 | 00 | 00 | 40 | 65 | 00 | 00 | 50 | 07 |
| ... |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 001 FFFF 0 | 00 | 00 | 00 | 00 | 00 | 00 | 00 | 00 | 10 | 00 | 00 | 67 | 00 | 10 | 30 | 65 |
| ... |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 00200000 | 00 | 10 | 00 | 07 | 00 | 10 | 10 | 07 | 00 | 10 | 20 | 07 | 00 | 10 | 30 | 07 |
| 00200010 | 00 | 10 | 40 | 07 | 00 | 10 | 50 | 07 | 00 | 10 | 60 | 07 | 00 | 10 | 70 | 07 |
| 00200020 | 00 | 10 | 00 | 07 | 00 | 00 | 00 | 00 | 00 | 00 | 00 | 00 | 00 | 00 | 00 | 00 |
| ... |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 00200FF0 | 00 | 00 | 00 | 00 | 00 | 00 | 00 | 00 | 00 | 1F | E0 | 07 | 00 | 1F | F0 | 07 |
| ... |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

3. (17 points total) Memory management:
a. (7 points) Consider a memory system with a cache access time of 10 ns and a memory access time of 200 ns , including the time to check the cache. What hit rate $H$ would we need in order to achieve an effective access time $10 \%$ greater than the cache access time? (Symbolic and/or fractional answers are OK)
b. (10 points) Suppose you have a 47 -bit virtual address space with a page size of 16 KB and that page table entry takes 8 bytes. How many levels of page tables would be required to map the virtual address space if every page table is required to fit into a single page? Be explicit in your explanation and show how a virtual address is structured.
$\qquad$

Question 6. Caches (20 points) A tiny system has 1-byte addresses and a 2-way associative cache with four entries. Each block in the cache holds two bytes. The cache controller uses the LRU policy for evicting from cache when both rows with the same "index" are full.
(a) (4 points) Use the figure below to indicate the number of bits in each field.

(b) ( 6 points) Assume the following access sequence to the memory: $0 \times \mathrm{ff}, 0 \times 22,0 \times 27,0 \times 24,0 \times 27,0 \times f f$, $0 \times f 0,0 \times 24,0 \times 27,0 \times 22$. Fill in the following table with the addresses whose content is in the cache. Initially assume the cache is empty. The first entry (i.e., the one corresponding to address 0 xff ) is filled for you.

|  |  | 0xff | 0x22 | 0x27 | 0x24 | 0x27 | 0xff | 0xf0 | 0x24 | 0x27 | 0x22 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Set$1$ | Index: 0 |  |  |  |  |  |  |  |  |  |  |
|  | Index: 1 | 0xfe, 0xff |  |  |  |  |  |  |  |  |  |
| $\begin{aligned} & \text { Set } \\ & 2 \end{aligned}$ | Index: 0 |  |  |  |  |  |  |  |  |  |  |
|  | Index: 1 |  |  |  |  |  |  |  |  |  |  |

(c) (4 points) How many cache misses did the access sequence at point (b) cause? What is the hit rate?
(d) (3 points) How many compulsory misses (i.e., misses which could never be avoided) did the access pattern at point (b) cause?
(e) (3 points) Assuming the cache access time is 10 ns , and that the miss time is 100 ns (this includes the time to check the cache), what is the average access time assuming the access pattern at point (b)?
6. (10 points total) Caching: Assume a computer system employing a cache, where the access time to the main memory is 100 ns , and the access time to the cache is 20 ns .
a. ( 2 points) Assume the cache hit rate is $95 \%$. What is the average access time?
b. (2 points) Assume the system implements virtual memory using a two-level page table with no TLB, and assume the CPU loads a word X from main memory. Assume the cache hit rate for the page entries as well as for the data in memory is $95 \%$. What is the average time it takes to load X?
c. (3 points) Assume the same setting as in point (b), but now assume that page translation is cached in the TLB (the TLB hit rate is $98 \%$ ), and the access time to the TLB is 16 ns . What is the average access time to X ?
d. (3 points) Assume we increase the cache size. Is it possible that this increase to lead to a decrease in the cache hit rate? Use no more than three sentences to explain your answer.

