Midterm I
SOLUTIONS
October 15th, 2008
CS162: Operating Systems and Systems Programming

General Information:
This is a closed book exam. You are allowed 1 page of hand-witten notes (both sides). You have 3 hours to complete as much of the exam as possible. Make sure to read all of the questions first, as some of the questions are substantially more time consuming.

Write all of your answers directly on this paper. Make your answers as concise as possible. On programming questions, we will be looking for performance as well as correctness, so think through your answers carefully. If there is something about the questions that you believe is open to interpretation, please ask us about it!

<table>
<thead>
<tr>
<th>Problem</th>
<th>Possible</th>
<th>Score</th>
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<tbody>
<tr>
<td>1</td>
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<td>Total</td>
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Problem 1: Short Answer [24pts]

Problem 1a[2pts]: Give at least two reasons why the following implementation of a condition variable is incorrect (assume that MySemi is a semaphore initialized to 0):

```c
Wait() { MySemi.P(); }
Signal() { MySemi.V(); }
```

Some answers:
- Semaphores are commutative, while condition variables are not. Practically speaking, if someone executes MySemi.V() followed by MySemi.P(), the latter will not wait. In contrast, execution of Signal() before Wait() should have no impact on Wait().
- The above implementation of Wait() will deadlock the system if it goes to sleep on MySemi.P() (since it will go to sleep while holding the monitor lock).

Problem 1b[4pts]: What is the difference between Mesa and Hoare scheduling for monitors? Include passing of locks between signaler and callee, scheduling of CPU resources, and impact on programmer.

For Mesa scheduling, the signaler keeps the lock and CPU, while the signaled thread is simply put on the ready queue and will run at a later time. Further, a programmer with Mesa scheduled monitors must recheck the condition after being awoken from a Wait() operation [i.e. they need a while loop around the execution of Wait()]. For Hoare scheduling, the signaler gives the lock and CPU to the signaled thread which begins running until it releases the lock, at which point the signaler regains the lock and CPU. A programmer with Hoare scheduled monitors does not need to recheck the condition after being awoken, since they know that the code after the Wait() is executed immediately after the Signal() [i.e. they do not need a while loop around the execution of Wait()].

Problem 1c[3pts]: The SRTF algorithm requires knowledge of the future. Why is that? Name two ways to approximate the information required to implement this algorithm.

SRTF stands for “Shortest Remaining Time First” and needs knowledge of the future to know which of the threads on the ready queue will run for the shortest time. Ways to approximate this include:
1. Using a prediction mechanism like a Kalman filter or exponential filter to predict the future given information about the past.
2. With a multi-level scheduler. Short tasks migrate to the top queue, while long ones migrate to the bottom queue.

Problem 1d[3pts]: What is priority donation? What sort of information must the OS track to allow it to perform priority donation?

Priority donation is the process of avoiding priority inversion by giving (“donating”) priority from a high-priority blocked thread to a lower-priority thread holding a lock needed by the high-priority thread. The OS must keep trace of lock acquisition and release operations and associate them with threads in order to perform this optimization.
Problem 1e[3pts]: Above, we show the Readers-Writers example given in class. It used two condition variables, one for waiting readers and one for waiting writers. Suppose that all of the following requests arrive in very short order (while R1 and R2 are still executing):

Incoming stream: R1 R2 W1 W2 R3 R4 R5 W3 R6 W4 W5 R7 R8 W6 R9

In what order would the above code process the above requests? If you have a group of requests that are equivalent (unordered), indicate this clearly by surrounding them with braces ‘{}’. You can assume that the wait queues for condition variables are FIFO in nature (i.e. signal() wakes up the oldest thread on the queue). Explain how you got your answer.

Assuming that the requests all come in during the processing of R1 and R2, then all of the Write requests will be queued on okToWrite and all of the read requests will be queued on okToRead. On exit of R1 and R2 the above code will proceed to execute each of the write requests, one at a time, in the order in which they arrived (see our assumption about the wait queues for the condition variables). We know this because both the Reader() and Writer() code on exit will signal the next waiting writer, if they exist. After all of the writers are gone, the Writer() code will broadcast to wakeup all of the Readers(), who will be able to run in parallel (see entry code of Reader()).

Answer: {R1, R3}, W1, W2, W3, W4, W5, W6, {R3, R4, R5, R6, R7, R8, R9}
Problem 1f[4pts]:
Suppose that you were to redesign the code in (1e). What is the minimum number of condition variables that we would need in order to handle the above requests in an order that guarantees that a read always returns the results of writes that have arrived before it but not after it? (Another way to say this is that the reads and writes occur in the order in which they arrive, while still allowing groups of reads that arrive together to occur simultaneously.) Provide a two or three sentence sketch of your scheme (do not try to write code!).

We accepted a couple of answers here:

1) The truly correct one is: 1! We only need a single condition variable on which to put threads to sleep. We put the scheduling logic into a state variables manipulated by the monitor lock. Here, we assume that each incoming request is assigned a sequentially increasing integer before it is put to sleep. This allows us to divide all of the requests into phases. Each phase is either a read phase or a write phase and includes a contiguous range of integers. When a new request arrives, we see whether it is compatible (same type of request) with the phase at the end of the queue, in which case we increase the last integer for that phase, or incompatible with the phase at the end of the queue, in which case we allocate a new entry to the queue and place the new request into the new bin.

This information is sufficient to allow us to schedule each phase sequentially. If the head phase is a read phase, we can wakeup all threads in that phase. If the head phase is a write phase, we can wake them up one at a time.

2) A less optimal solution (not the “minimum number”) would be to have a queue of condition variables, one for each phase of threads (readers or writers). For each incoming request, you would check the last phase – if it matches the request type, you would put the request to sleep on the condition variables at the end of the queue. If it doesn’t match the request type, you would allocate a new condition variable to place at the end of the queue (i.e. create a new phase).

Problem 1g[3pts]: What are exceptions? Name two different types of exceptions and give an example of each type:

Exceptions are interruptions in the flow of execution. There are two types of exceptions that we talked about in class synchronous (traps) and and asynchronous (interrupts). Examples of synchronous exceptions are: system calls, bad instruction exceptions, division by zero, page faults. Examples of asynchronous exceptions are device interrupts of all sorts.

Problem 1h[2pts]: What was the problem with the Therac-25? Your answer should involve one of the topics of the class.

The Therac-25 was a medical device for doing radiation therapy. Due to race conditions in its software, it ended up killing a number of patients through overdoses. Among other things, if the operator typed too fast while setting up a radiation session, it would exhibit the bug.
Problem 2: TRUE/FALSE [12 pts]

In the following, it is important that you EXPLAIN your answer in TWO SENTENCES OR LESS (Answers longer than this may not get credit!). Also, answers without an explanation GET NO CREDIT.

Problem 2a[2pts]: The kernel on a multiprocessor can use the local disabling of interrupts (within one CPU) to produce critical sections between the OSs on different CPUs.

True / False

Explain: Disabling interrupts on one CPU doesn’t affect other CPUs. Consequently, other CPUs keep executing when a single one has interrupts disabled.

Problem 2b[2pts]: When designing a multithreaded application, you must use synchronization primitives to make sure that the threads do not overwrite each other’s registers.

True / False

Explain: Registers are kept in the TCB and are private to each thread. Consequently, one thread will not overwrite another thread’s register under normal circumstances.

Problem 2c[2pts]: A system that provides segmentation without paging can fragment the physical address space, forcing the operating system to waste physical memory.

True / False

Explain: The physical chunks of memory are the same size as the virtual chunks of memory. Consequently, the physical address space can become fragmented as the OS attempts to fit segments into the physical address space.

Problem 2d[2pts]: A user-level library implements each system call by first executing a “transition to kernel mode” instruction. The library routine then calls an appropriate subroutine in the kernel.

True / False

Explain: If this were true, user-level code could execute anything it wanted – a serious security violation. Instead, syscalls use a trap instruction that simultaneously changes to kernel mode and jumps to well-defined places in the kernel.

Problem 2e[2pts]: The difference between processes and threads is purely historical.

True / False

Explain: A process is a combination of an address space and a set of threads. Thus, threads are the execution context portion of a process.

Problem 2f[2pts]: Round robin scheduling provides a latency improvement over FCFS scheduling for interactive jobs.

True / False

Explain: By timeslicing jobs, RR scheduling avoids the problem FCFS in which interactive jobs can get stuck behind long-running jobs.
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Problem 3: Atomic Synchronization Primitives [25 pts]

In class, we discussed a number of atomic hardware primitives that are available on modern architectures. In particular, we discussed “test and set” (TSET), SWAP, and “compare and swap” (CAS). They can be defined as follows (let “expr” be an expression, “&addr” be an address of a memory location, and “M[addr]” be the actual memory location at address addr):

<table>
<thead>
<tr>
<th>Test and Set (TSET)</th>
<th>Atomic Swap (SWAP)</th>
<th>Compare and Swap (CAS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TSET(&amp;addr) {</td>
<td>SWAP(&amp;addr, expr) {</td>
<td>CAS(&amp;addr, expr1, expr2) {</td>
</tr>
<tr>
<td>int result = M[addr];</td>
<td>int result = M[addr];</td>
<td>if (M[addr] == expr1) {</td>
</tr>
<tr>
<td>M[addr] = 1;</td>
<td>M[addr] = expr;</td>
<td>M[addr] = expr2;</td>
</tr>
<tr>
<td>return (result);</td>
<td>return (result);</td>
<td>return true;</td>
</tr>
<tr>
<td></td>
<td>}</td>
<td>}</td>
</tr>
<tr>
<td></td>
<td>}</td>
<td>}</td>
</tr>
</tbody>
</table>

Both TSET and SWAP return values (from memory), whereas CAS returns either true or false. Note that our &addr notation is similar to a reference in C++, and means that the &addr argument must be something that can be stored into (an “lvalue”). For instance, TSET could be used to implement a spin-lock acquire as follows:

```c
int lock = 0; // lock is free
// Later: acquire lock
while (TSET(lock));
```

CAS is general enough as an atomic operation that it can be used to implement both TSET and SWAP. For instance, consider the following implementation of TSET with CAS:

```c
TSET(&addr) {
    int temp;
    do {
        temp = M[addr];
    } while (!CAS(addr, temp, 1));
    return temp;
}
```

**Problem 3a[3pts]:**
Show how to implement a spinlock acquire with a single while loop using CAS instead of TSET. You must only fill in the arguments to CAS below:

```c
// Initialization
int lock = 0; // Lock is free

// acquire lock
while ( !CAS( lock , 0 , 1 ) );
```
**Problem 3b[2pts]:**
Show how SWAP can be implemented using CAS. Don’t forget the return value.

```
SWAP(addr, reg1) {

    Object return;
    do {
        return = M[addr];
    } while (!CAS(addr, return, reg1));
}
```

**Problem 3c[3pts]:**
With spinlocks, threads spin in a loop (busy waiting) until the lock is freed. In class we argued that spinlocks were a bad idea because they can waste a lot of processor cycles. The alternative is to put a waiting process to sleep while it is waiting for the lock (using a blocking lock). Contrary to what we implied in class, there are cases in which spinlocks would be more efficient than blocking locks. Give a circumstance in which this is true and explain why a spinlock is more efficient.

*If the expected wait time of the lock is very short (such as because the lock is rarely contested or the critical sections are very short), then it is possible that a spin lock will waste many fewer cycles than putting threads to sleep/waking them up. The important issue is that the expected wait time must be less than the time to put a thread to sleep and wake it up.*

*Short expected wait times are possible to capitalize on, for instance, in a multiprocessor because waiting threads can be stalled on other processors while the lock-holder makes progress. Spin-locks are much less useful in a uniprocessor because the lock-holder is sleeping while the waiter is spinning.*

*Some people mentioned I/O. However, you would have had to come up with a specific example of locks in use between a thread and an I/O operation as well as mentioned interrupts for releasing the lock.*

*We were looking for mention of (1) expected wait time being important, (2) the length of time for putting locks to sleep relative to the expected wait time of the lock, (3) a viable scenario such as a multiprocessor.*
An object such as a queue is considered “lock-free” if multiple processes can operate on this object simultaneously without requiring the use of locks, busy-waiting, or sleeping. In this problem, we are going to construct a lock-free FIFO queue using the atomic CAS operation. This queue needs both an Enqueue and Dequeue method.

We are going to do this in a slightly different way than normally. Rather than Head and Tail pointers, we are going to have “PrevHead” and Tail pointers. PrevHead will point at the last object returned from the queue. Thus, we can find the head of the queue (for dequeuing). If we don’t have to worry about simultaneous Enqueue or Dequeue operations, the code is straightforward (ignore the null-pointer exception for the Dequeue() operation for now):

```java
// Holding cell for an entry
class QueueEntry {
    QueueEntry next = null;
    Object stored;

    QueueEntry(Object newobject) {
        stored = newobject;
    }
}

// The actual Queue (not yet lock free!)
class Queue {
    QueueEntry prevHead = new QueueEntry(null);
    QueueEntry tail = prevHead;

    void Enqueue(Object newobject) {
        QueueEntry newEntry = new QueueEntry(newobject);
        QueueEntry oldtail = tail;
        tail = newEntry;
        oldtail.next = newEntry;
    }

    Object Dequeue() {
        QueueEntry oldprevHead = prevHead;
        QueueEntry nextEntry = oldprevHead.next;
        prevHead = nextEntry;
        return nextEntry.stored;
    }
}
```

Problem 3d[3pts]:
For this non-multithreaded code, draw the state of a queue with 2 queued items on it:
Problem 3e[3pts]:
For each of the following potential context switch points, state whether or not a context switch at that point could cause incorrect behavior of Enqueue(); Explain!

```java
void Enqueue(Object newobject) {
    QueueEntry newEntry = new QueueEntry(newobject);
    QueueEntry oldtail = tail;
    tail = newEntry;
    oldtail.next = newEntry;
}
```

Point 1: No. Construction of a QueueEntry is a purely local operation (and does not touch shared state in any way).

Point 2: Yes. An intervening Enqueue() operation will move the shared variable “tail” (and enqueue another object). As a result, the subsequent “tail=newEntry” will overwrite the other entry.

Point 3: No. At this point in the execution, only the local thread will ever touch “oldtail.next” (since we have moved the tail). Thus, we can reconnect at will. People who worried that the linked list is “broken” until this operation can relax. The worse that will happen is that the list appears to be shorter than it actually is until execution of “oldtail.next=newEntry,” at which point the new entry becomes available for subsequent dequeue.

Problem 3f[4pts]:
Rewrite code for Enqueue(), using the CAS() operation, such that it will work for any number of simultaneous Enqueue and Dequeue operations. You should never need to busy wait. Do not use locking (i.e. don’t use a test-and-set lock). The solution is tricky but can be done in a few lines. We will be grading on conciseness. Do not use more than one CAS() or more than 10 lines total (including the function declaration at the beginning). Hint: wrap a do-while around vulnerable parts of the code identified above.

```java
void Enqueue(Object newobject) {
    QueueEntry newEntry = new QueueEntry(newobject);
    // Insert code here

    // Here, 'tail' is the shared variable that needs to be protected by CAS. We must atomically swap in 'newEntry' to 'tail', giving us the old value so that we can link it to the new item.
    QueueEntry oldtail;
    do {
        oldtail = tail;
        // Tentative pointer to tail
    } while (!CAS(tail, oldtail, newEntry);
    oldtail.next = newEntry;
}
```
Problem 3g[3pts]:
For each of the following potential context switch points, state whether or not a context switch at that point could cause incorrect behavior of Dequeue(); Explain!

Object Dequeue() {
    QueueEntry oldprevHead = prevHead;
    QueueEntry nextEntry = oldprevHead.next;
    prevHead = nextEntry;
    return nextEntry.stored;
}

Point 1: Yes. The problem is that an intervening Dequeue() could end up getting the same entry ‘nextEntry’ that we are returning; consequently we end up dequeuing the same entry multiple times.

Point 2: Yes. The problem is that an intervening Dequeue() could end up getting the same entry ‘nextEntry’ that we are returning; consequently we end up dequeuing the same entry multiple times.

Point 3: No. The nextEntry has already been detached from the queue and is purely local. Thus, all that we are doing is removing the stored value from nextEntry for returning it.

Problem 3h[4pts]:
Rewrite code for Dequeue(), using the CAS() operation, such that it will work for any number of simultaneous Enqueue and Dequeue operations. You should never need to busy wait. Do not use locking (i.e. don’t use a test-and-set lock). The solution can be done in a few lines. We will be grading on conciseness. Do not use more than one CAS() or more than 10 lines total (including the function declaration at the beginning). Hint: wrap a do-while around vulnerable parts of the code identified above.

Object Dequeue() {
    // Insert code here
    // Here, ‘prevHead’ is the shared variable that needs to be
    // protected by CAS. We must atomically grab the value of
    // prevHead.next and swap it into prevHead. The CAS lets
    // us do this operation by making sure that prevHead is
    // still equal to oldprevHead at the time that we swap
    // in prevHead.next. Note that we have included a check to
    // handle empty queues (not required for your solution)

    QueueEntry oldprevHead, nextEntry;
    do {
        oldprevHead = prevHead;
        nextEntry = oldprevHead.next;
        if (nextEntry == null) // handle empty queue (not required
            return null; // for your solution)
    } while (!CAS(prevHead, oldprevHead, nextEntry));
    return oldprevHead.stored;
}
Problem 4: Deadlock[21 pts]

Problem 4a[5pts]:
The figure at the right illustrates a 2D mesh of network routers. Each router is connected to each of its neighbors by two network links (small arrows), one in each direction. Messages are routed from a source router to a destination router and can stretch through the network (i.e. consume links along the route from source to destination). Messages can cross inside routers.

Assume that no network link can service more than one message at a time, and that each message must consume a continuous set of channels (like a snake). Messages always make progress to the destination and never wrap back on themselves. The figure shows two messages (thick arrows).

Assume that each router or link has a very small amount of buffer space and that each message can be arbitrarily long. Show a situation (with a drawing) in which messages are deadlocked and can make no further progress. Explain how each of the four conditions of deadlock are satisfied by your example. *Hint: Links are the limited resources in this example.*

Answer: The simplest deadlock example is a set of four messages in a loop (as shown in the figure at the left). Each message is blocked attempting to make a counter-clockwise turn by a message utilizing the target channel. Four conditions:

1. **Mutual Exclusion**: Each channel held by one message at a time
2. **Hold and Wait**: messages hold channels while waiting to acquire other channels
3. **No preemption**: Channels can not be preempted from messages after they are acquired by them
4. **Circular wait**: We have a cycle of waiting here – four messages

Problem 4b[3pts]:
Define a routing policy that avoids deadlocks in the network of (4a). Name one of the four conditions that is no longer possible, given your routing policy. Explain.

*Several answers are possible here. The one given in class was to force messages to route in the X direction first, then Y. This removes the possibility of Circular Wait because it is no longer possible to have the North→East or South→West turns which would be required in any loop.*

Problem 4c[3pts]:
Suppose that each router node contains sufficient queue space to hold complete messages (assume infinite space, if you like). Why is it impossible for deadlocks such as in (4a) to occur? Name one of the four conditions that is no longer possible, given infinite queue space in the router. Explain.

*Deadlocks can no longer occur as in (4a) because blocked messages simply absorb into the queues at the blocking router – thereby freeing up the channels held by them. Clearly Hold and Wait is no longer possible (messages do not hold channels while waiting for others). You might be able to argue that Circular wait is no longer possible (any cycle would be transient).*
**Problem 4d[4pts]:**
Suppose that we have the following resources: A, B, C and threads T1, T2, T3, T4. The total number of each resource is:

<table>
<thead>
<tr>
<th>Total</th>
</tr>
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<tbody>
<tr>
<td>A</td>
</tr>
<tr>
<td>B</td>
</tr>
<tr>
<td>C</td>
</tr>
<tr>
<td>12</td>
</tr>
<tr>
<td>9</td>
</tr>
<tr>
<td>12</td>
</tr>
</tbody>
</table>

Further, assume that the processes have the following maximum requirements and current allocations:

<table>
<thead>
<tr>
<th>Thread ID</th>
<th>Current Allocation</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>T1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>T2</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>T3</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>T4</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

Is the system in a safe state? If “yes”, show a non-blocking sequence of thread executions. Otherwise, provide a proof that the system is unsafe. Show all steps, intermediate matrices, etc.

**Answer:** Yes, this system is in a safe state.

To prove this, we first compute the currently free allocations:

<table>
<thead>
<tr>
<th>Available</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
</tr>
<tr>
<td>2</td>
</tr>
</tbody>
</table>

Further, we compute the number needed by each thread (Maximum – Current Allocation):

<table>
<thead>
<tr>
<th>Thread ID</th>
<th>Needed Allocation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
</tr>
<tr>
<td>T1</td>
<td>2</td>
</tr>
<tr>
<td>T2</td>
<td>4</td>
</tr>
<tr>
<td>T3</td>
<td>1</td>
</tr>
<tr>
<td>T4</td>
<td>2</td>
</tr>
</tbody>
</table>

Thus, we can see that a possible sequence is: T3, T2, T4, T1:
Problem 4e[3pts]:
Assume that we start with a system in the state of (4d). Suppose that T1 asks for 2 more copies of resource A. Can the system grant this if it wants to avoid deadlock? Explain.

No. This cannot be granted. Assume that T1 gets 2 more of A.
Then, our available allocation is:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Then, looking at our needed allocations, we see:

<table>
<thead>
<tr>
<th>Thread ID</th>
<th>Needed Allocation</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>0 8 1</td>
</tr>
<tr>
<td>T2</td>
<td>4 4 0</td>
</tr>
<tr>
<td>T3</td>
<td>1 0 0</td>
</tr>
<tr>
<td>T4</td>
<td>2 7 0</td>
</tr>
</tbody>
</table>

At this point, the available allocation is insufficient to start any of the threads, much less find a safe sequence that finishes all of them.

Problem 4f[3pts]:
Assume that we start with a system in the state of (4d). What is the maximum number of additional copies of resources (A, B, and C) that T1 can be granted in a single request without risking deadlock? Explain.

We cannot ask for more than (A,B,C)=(2,1,1) since this is all that is available. However, the previous problem showed that A must be < 2. Further, note that the resources given to T1 are tied up until the very end of the execution (look at sequence in 4d).

Thus, looking at our safe sequence from 4d, we can see that it can still work if it is missing one A and one C. Just work through it with the first “Available Before” allocation of 1,1,0 instead of 2,1,1. However, it will not work if it is missing one more B (we would be unable to execute T4 in the sequence), i.e. setting “Available Before” to 1,0,0 prevents the execution of T4.

Thus the maximum number of additional resources that can be requested by T1 is (A,B,C)=(1,0,1)
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Problem 5: Address Translation [18 pts]

Problem 5a[3pts]:
Suppose we have a 32-bit processor (with 32-bit virtual addresses) and 8 KB pages. Assume that it can address up to 2 TB (terabytes) of DRAM; 1 TB = 1024 GB = (1024)^2 MB. Assume that we need 4 permissions bits in each page table entry (PTE), namely Valid (V), Writable (W), Accessed (A), and Dirty (D). Show the format of a PTE, assuming that each page should be able to hold an integer number of PTEs. If you have extra bits in the PTE, you can mark them as “unused”. Explain.

Since the PTE must have sufficient bits to address all of the physical pages, we must ask how many physical pages there are. \(2^{20} / 8 \text{KB} = 2^{20} / (2^{3} \cdot 2^{13}) = 2^{20} / 2^{21} = 2^{-1}\). Consequently, we need 28 bits in the PTE for the physical page number. With the 4 other bits, this leads us to a 32 bit PTE (which fits an integral number of times in an 8KB page (with no wasted bits)). Our PTE looks like (bits in any order):

```
<table>
<thead>
<tr>
<th>Physical Page Number (28 bits)</th>
<th>D</th>
<th>A</th>
<th>W</th>
<th>V</th>
</tr>
</thead>
</table>
```

32 Bits

Problem 5b[5pts]:
Assume that we wish to build a two-level page table for the processor from (5a) in which each piece of the page table consumes exactly a page (no more, no less). We may end up wasting space as a result. Draw and label a figure showing how a virtual address gets mapped into a physical address. Show the format of the page table (complete with access checks), the virtual address, and physical address. Minimize pieces of the page table that consume less than a page (and thus waste space).

Looking at the virtual address, we see that there are 32 – 13 bits = 19 bits of virtual page number that we need to translate into a physical page number. Each page can hold \(8 \text{KB} / 4 = 2048\) PTEs. Thus, one level of the page table can translate 11 bits. Thus, our 19 bit address can be divided into an 11 bit piece and an 8 bit piece. Thus, one level of our page table will only have 256 entries in it, even though we could fit 2048. To avoid wasting too much space, we will put the 256 entry chunk into the top-level of the page table (this way we waste a single chunk of size \(8 \text{KB} - 256 \cdot 4 = 7168\) bytes). If we put the small chunks at the lower-levels of the page table, we would waste 2048 * 7168 bytes.
Problem 5c[3pts]:
Consider a multi-level memory management scheme using the following format for virtual addresses:

<table>
<thead>
<tr>
<th>Virtual seg # (2 bits)</th>
<th>Virtual Page # (6 bits)</th>
<th>Offset (12 bits)</th>
</tr>
</thead>
</table>

Virtual addresses are translated into physical addresses of the following form:

<table>
<thead>
<tr>
<th>Physical Page # (8 bits)</th>
<th>Offset (12 bits)</th>
</tr>
</thead>
</table>

Page table entries (PTE) are 16 bits in the following format, stored in big-endian form in memory (i.e. the MSB is first byte in memory):

<table>
<thead>
<tr>
<th>Physical Page # (8 bits)</th>
<th>Kernel</th>
<th>Nocache</th>
<th>0</th>
<th>0</th>
<th>Dirty</th>
<th>Use</th>
<th>Writeable</th>
<th>Valid</th>
</tr>
</thead>
</table>

1) How big is a page? Explain.

_We just have to look at the offset of 12 bits. Since 2^{12} = 4096, thus pages are 4096 bytes in size._

2) What is the maximum amount of virtual memory supported by this scheme? Explain

_Since virtual addresses are 20 bits, the maximum amount of virtual memory is 2^{20} = 1MB_

3) What is the maximum amount of physical memory supported by this scheme? Explain

_Since physical addresses are 20 bits, the maximum amount of physical memory is 2^{20} = 1MB_

Problem 5d[7pts]: Assume the memory translation scheme from (5c). Use the Segment Table and Physical Memory table given on the next page to predict what will happen with the following load/store instructions. Addresses are virtual. The return value for a load is an 8-bit data value or an error, while the return value for a store is either “ok” or an error. If there is an error, make sure to say which error. Possibilities are: “bad segment” (invalid segment), “segment overflow” (address outside range of segment), or “access violation” (page invalid, or attempt to write a read only page). A few answers are given:

<table>
<thead>
<tr>
<th>Instruction</th>
<th>Result</th>
<th>Instruction</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load [0xC1015]</td>
<td>0x57</td>
<td>Store [0x52002]</td>
<td>Segment Overflow</td>
</tr>
<tr>
<td>Store [0x43045]</td>
<td>ok</td>
<td>Load [0x04013]</td>
<td>0x44</td>
</tr>
<tr>
<td>Store [0xC1016]</td>
<td>Access violation</td>
<td>Store [0x81015]</td>
<td>Bad Segment</td>
</tr>
<tr>
<td>Load [0xD2002]</td>
<td>0x10</td>
<td>Store [0x03010]</td>
<td>Ok</td>
</tr>
<tr>
<td>Store [0xD2031]</td>
<td>Access violation</td>
<td>Load [0x13035]</td>
<td>0x59</td>
</tr>
</tbody>
</table>
### Segment Table (Max Segment=3)

<table>
<thead>
<tr>
<th>Seg #</th>
<th>Page Table Base</th>
<th>Max Page Entries</th>
<th>Segment State</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0x02030</td>
<td>0x20</td>
<td>Valid</td>
</tr>
<tr>
<td>1</td>
<td>0x01020</td>
<td>0x10</td>
<td>Valid</td>
</tr>
<tr>
<td>2</td>
<td>0x01040</td>
<td>0x40</td>
<td>Invalid</td>
</tr>
<tr>
<td>3</td>
<td>0x04000</td>
<td>0x20</td>
<td>Valid</td>
</tr>
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</table>

### Physical Memory

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<th>+2</th>
<th>+3</th>
<th>+4</th>
<th>+5</th>
<th>+6</th>
<th>+7</th>
<th>+8</th>
<th>+9</th>
<th>+A</th>
<th>+B</th>
<th>+C</th>
<th>+D</th>
<th>+E</th>
<th>+F</th>
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</thead>
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</tr>
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</table>
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