Lecture 3: Finite Automata

Administrivia

- Everyone should now be registered electronically using the link on our webpage. If you haven’t, do so today!
- I’d like to have teams formed by next Monday at the latest.
- Please fill out the background survey linked to on the homework page.
An Alternative Style for Describing Languages

- Rather than giving a single pattern, we can give a set of rules.
- Each rule has the form

\[ A : \alpha_1 \alpha_2 \cdots \alpha_n, \ n \geq 0, \]

where

- \( A \) is a symbol that is intended to stand for a language (set of strings)—a metavariable or nonterminal symbol.
- Each \( \alpha_i \) is either a literal character (like "a") or a nonterminal symbol.
- The interpretation of this rule is

One way to form a string in \( L(A) \) (the language denoted by \( A \)) is to concatenate one string each from \( L(\alpha_1), L(\alpha_2), \ldots \).

(where \( L("c") \) is just the language \( \{"c"\} \)).

- This is Backus-Naur Form (BNF). A set of rules is a grammar.
- Aside: You’ll see that ‘:\=' written many different ways, such as ‘::=’, ‘\rightarrow’, etc. We’ll just use the same notation our tools use.
Some Abbreviations

- The basic form from the last slide is good for formal analysis, but not for writing.
- So, we can allow some abbreviations that are obviously exandable into the basic forms:

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A : \mathcal{R}_1</td>
<td>\cdots</td>
</tr>
<tr>
<td>$A : \cdots (\mathcal{R}) \cdots$</td>
<td>$B : \mathcal{R}$</td>
</tr>
<tr>
<td>$A : &quot;c_1&quot;</td>
<td>\cdots</td>
</tr>
</tbody>
</table>

(likewise other character classes)
Some Technicalities

• From the definition, each nonterminal in a grammar defines a language. Often, we are interested in just one of them (the start symbol), and the others are auxiliary definitions.

• The definition of what a rule means (“One way to form a string in $L(A)$ is...”) leaves open the possibility that there are other ways to form items in $L(A)$ than covered in the rule.

• We need that freedom in order to allow multiple rules for $A$, but we don’t really want to include strings that aren’t covered by some rule.

• So precise mathematical definitions throw in sentences like:

  A grammar defines the minimal languages that contain all strings that satisfy the rules.
A Big Restriction (for now)

- For the time being, we’ll also add a restriction. In each rule:
  \[ A : \alpha_1\alpha_2\cdots\alpha_n, \quad n \geq 0, \]
  we’ll require that if \( \alpha_i \) is a nonterminal symbol, then either
  - All the rules for that symbol have to occurred before all the rules
    for \( A \), or
  - \( i = n \) (i.e., is the last item) and \( \alpha_n \) is \( A \).

- We call such a restricted grammar a \textbf{Type 3} or \textit{regular} grammar. The languages definable by regular grammars are called \textit{regular languages}.

\textbf{Claim:} Regular languages are exactly the ones that can be defined by regular expressions.
Proof of Claim (I)

- Start with a regular expression, $R$, and make a (possibly not yet valid) rule,
  \[ R: \quad R \]

- Create a new (preceding) rule for each parenthesized expression.

- This will leave just the constructs `$X^*$`, `$X+$`, and `$X?$`. What do we do with them?
Proof of Claim (II)

Replace construct... | ... with \( Q \), where

\( R^* \)
Proof of Claim (II)

<table>
<thead>
<tr>
<th>Replace construct...</th>
<th>...with $Q$, where</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R^*$</td>
<td>$Q :$</td>
</tr>
<tr>
<td></td>
<td>$Q : R , Q$</td>
</tr>
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</table>

$R^+$
**Proof of Claim (II)**

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</tr>
<tr>
<td>$R^+$</td>
<td>$Q : R$</td>
</tr>
<tr>
<td></td>
<td>$Q : R \ Q$</td>
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</tbody>
</table>

$R^?$
# Proof of Claim (II)

Replace construct... with $Q$, where

<table>
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<th>$R^*$</th>
<th>$Q : R Q$</th>
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</thead>
<tbody>
<tr>
<td>$R^+$</td>
<td>$Q : R$</td>
</tr>
<tr>
<td>$R^?$</td>
<td>$Q : R$</td>
</tr>
</tbody>
</table>
Example

- Consider the regular expression \((\text{"+"|\text{"-"})?\text{"0"|\text{"1"})+\)

1. \(R: (\text{"+"|\text{"-"})?\text{"0"|\text{"1"})+ \quad \text{replace with} \ldots\)

2. \(Q_1: \text{"+"} \mid \text{"-"}\)
   \(Q_2: \text{"0"} \mid \text{"1"}\)
   \(R: Q_1? Q_2+ \quad \text{replace with} \ldots\)

3. \(Q_3: \epsilon \mid Q_1\)
   \(Q_4: Q_2 \mid Q_2 Q_4\)
   \(R: Q_3 Q_4\)
Classical Pattern-Matching Implementation

- For compilers, can generally make do with “classical” regular expressions.

- Implementable using finite(-state) automata or FAs. (“Finite state” = “finite memory”).

- Classical construction:

  \[
  \text{regular expression} \Rightarrow \text{nondeterministic FA (NFA)} \Rightarrow \text{deterministic FA (DFA)} \Rightarrow \text{table-driven program.}
  \]
Review: FA operation

• A FA is a graph whose nodes are states (of memory) and whose edges are state transitions. There are a finite number of nodes.

• One state is the designated start state.

• Some subset of the nodes are final states.

• Each transition is labeled with a set of symbols (characters, etc.) or $\epsilon$.

• A FA recognizes a string $c_1c_2\cdots c_n$ if there is a path (sequence of edges) from the start state to a final state such that the labels of the edges in sequence, aside from $\epsilon$ edges, respectively contain $c_1, c_2, \ldots, c_n$.

• If the edges leaving any node have disjoint sets of characters and if there are no $\epsilon$ nodes, FA is a DFA, else an NFA.
Example: What does this DFA recognize?

What is the simplest equivalent NFA you can think of?
Example: What does this DFA recognize?

Bit strings with # of 1’s divisible by 2 or 3.

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Strings of capitals ending in ABCDABD.

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Strings of capitals ending in ABCDABD.

What is the simplest equivalent DFA you can think of?

(Edges without labels mean “any character not covered by another edge.”)
Example: What does this NFA recognize?

What is the simplest equivalent DFA you can think of?
Review: Classical Regular Expressions to NFAs (I)

\[ \epsilon \]

\[ a \]

\[ R_1 \quad R_2 \quad R_1 \quad R_2 \]

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Review: Classical Regular Expressions to NFAs (II)

\[ R_1 \mid R_2 \]

\[ R_1 \]

\[ R_2 \]

\[ R^* \]
Extensions?

- How would you translate $\emptyset$ (the empty language, containing no strings) into an FA?
- How could you translate 'R?' into an NFA?
- How could you translate 'R+' into an NFA?
- How could you translate $R_1 | R_2 | \cdots | R_n$ into an NFA?
Example of Conversion

How would you translate \(((ab)^*|c)^*\) into an NFA (using the construction above)?
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How would you translate \(((ab)^* | c)^*\) into an NFA (using the construction above)?
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Abstract Implementation of NFAs

String: XYYZ
Review: Converting to DFAs

- **OBSERVATION:** The set of states that are marked (colored red) changes with each character in a way that depends only on the set and the character.

- In other words, machine on previous slide acted like this DFA:
DFAs as Programs

- Can realize DFA in program with control structure:

```c
state = INITIAL;
for (s = input; *s != '\0'; s += 1) {
    switch (state):
    case INITIAL:
        if (*s == 'a') state = A_STATE; break;
    case A_STATE:
        if (*s == 'b') state = B_STATE; else state = INITIAL; break;
        ...
    }
return state == FINAL1 || state == FINAL2;
```

- Or with data structure (table driven):

```c
state = INITIAL;
for (s = input; *s != '\0'; s += 1)
    state = transition[state][s];
return isfinal[state];
```
What Flex Does

• Flex program specification is giant regular expression of the form $R_1|R_2|\cdots|R_n$, where none of the $R_i$ match $\epsilon$.

• Each final state labeled with some action.

• Converted, by previous methods, into a table-driven DFA.

• But, this particular DFA is used to recognize prefixes of the (remaining) input: initial portions that put machine in a final state.

• Which final state(s) we end up in determine action. To deal with multiple actions:
  - Match longest prefix ("maximum munch").
  - If there are multiple matches, apply first rule in order.
How Do They Do It?

• How can we use a DFA to recognize longest match?

• How can we use DFA to act on first of equal-length matches?

• How can we use a DFA to handle the $R_1/R_2$ pattern (matches just $R_1$ but only if followed by $R_2$, like $R_1(?=R_2)$ in Python)?