Lecture 4: Parsing

Administrivia

• If you do not have a group, please post a request on Piazza (see the “Form project teams…” item. Be sure to update your post if you find one.

• We will assign orphans to groups randomly in a few days.

• Programming Contest coming up.
A Glance at the Map

Source code → Lexical Analysis → Tokens → Parsing → AST → Semantic Analysis → Decorated AST

We are here
Review: BNF

- BNF is another pattern-matching language;
- Alphabet typically set of *tokens*, such as from lexical analysis, referred to as *terminal symbols* or *terminals*.
- Matching rules have form:

  \[ X : \alpha_1\alpha_2 \cdots \alpha_n, \]

  where \( X \) is from a set of *nonterminal symbols* (or *nonterminals* or *meta-variables*), \( n \geq 0 \), and each \( \alpha_i \) is a terminal or nonterminal symbol.
- For emphasis, may write \( X : \varepsilon \) when \( n = 0 \).
- Read \( X : \alpha_1\alpha_2 \cdots \alpha_n \), as

  “An \( X \) may be formed from the concatenation of an \( \alpha_1, \alpha_2, \ldots, \alpha_n \).”
- Designate one nonterminal as the *start symbol*.
- Set of all matching rules is a *context-free grammar*. 
Review: Derivations

• String (of terminals) $T$ is in the language described by grammar $G$, $(T \in L(G))$ if there is a derivation of $T$ from the start symbol of $G$.

• Derivation of $T = \tau_1 \cdots \tau_k$ from nonterminal $A$ is sequence of sentential forms:

$$A \Rightarrow \alpha_{11} \alpha_{12} \cdots \Rightarrow \alpha_{21} \alpha_{22} \cdots \Rightarrow \cdots \Rightarrow \tau_1 \cdots \tau_k$$

where each $\alpha_{ij}$ is a terminal or nonterminal symbol.

• We say that

$$\alpha_1 \cdots \alpha_{m-1} B \alpha_{m+1} \cdots \alpha_n \Rightarrow \alpha_1 \cdots \alpha_{m-1} \beta_1 \cdots \beta_p \alpha_{m+1} \cdots \alpha_n$$

if $B : \beta_1 \cdots \beta_p$ is a production. ($1 \leq m \leq n$).

• If $\Phi$ and $\Phi'$ are sentential forms, then $\Phi_1 \xrightarrow{*} \Phi_2$ means that 0 or more $\Rightarrow$ steps turns $\Phi_1$ into $\Phi_2$. $\Phi_1 \xrightarrow{+} \Phi_2$ means 1 or more $\Rightarrow$ steps does it.

• So if $S$ is start symbol of $G$, then $T \in L(G)$ iff $S \xrightarrow{+} T$. 
Example of Derivation

1. $e : s\ ID$
2. $e : s\ '(\ e\ ')'$
3. $e : e\ '/'\ e$
4. $s :$
5. $s : '+$
6. $s : '-'$

Alternative Notation

$$
\begin{align*}
 & e : s\ ID \\
 & e : s\ '(\ e\ ')' \\
 & e : e\ '/'\ e \\
 & s : \epsilon \mid '+' \mid '-'
\end{align*}
$$

Problem: Derive $\ -\ ID\ /\ (\ ID\ /\ ID\ )$

$$
\begin{align*}
 e & \xrightarrow{3} e\ /\ e \xrightarrow{1} s\ ID\ /\ e \xrightarrow{6} -\ ID\ /\ e \xrightarrow{2} -\ ID\ /\ s\ (\ e ) \\
 & \xrightarrow{4} -\ ID\ /\ (\ e ) \xrightarrow{3} -\ ID\ /\ (\ e\ /\ e ) \xrightarrow{1} -\ ID\ /\ (\ s\ ID\ /\ e ) \\
 & \xrightarrow{4} -\ ID\ /\ (\ ID\ /\ e ) \xrightarrow{1} -\ ID\ /\ (\ ID\ /\ s\ ID ) \\
 & \xrightarrow{4} -\ ID\ /\ (\ ID\ /\ ID )
\end{align*}
$$

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Types of Derivation

- **Context free** means can replace nonterminals in any order (i.e., regardless of context) to get same result (as long as you use same productions).

- So, if we use a particular rule for selecting nonterminal to “produce” from, can characterize derivation by just listing productions.

- Previous example was leftmost derivation: always choose leftmost nonterminals. Completely characterized by list of productions: 3, 1, 6, 2, 4, 3, 1, 4, 1, 4.
Derivations and Parse Trees

- A leftmost derivation also completely characterized by parse tree:

```
  s
 /|
- ID /
```

- What is the rightmost derivation for this?
Derivations and Parse Trees

- A leftmost derivation also completely characterized by parse tree:

```
  e
 / \  
 e   e
 / \  /
 s   s
   /   /   /
  ID  ID (   ID / ID )
```

- What is the rightmost derivation for this?

```
e \rightarrow^3 e / e \rightarrow^2 e / s ( e ) \rightarrow^3 e / s ( e / e )
\rightarrow^1 e / s ( e / s ID ) \rightarrow^4 e / s ( e / ID )
\rightarrow^1 e / s ( s ID / ID ) \rightarrow^4 e / s ( ID / ID )
\rightarrow^4 e / ( ID / ID ) \rightarrow^1 s ID / ( ID / ID ) \rightarrow^6 - ID / ( ID / ID )
```
Ambiguity

• Only one derivation for previous example.
• What about 'ID / ID / ID'?
• Claim there are two parse trees, corresponding to two leftmost derivations. What are they?

• If there exists even one string like ID / ID / ID in $L(G)$, we say $G$ is ambiguous (even if other strings only have one parse tree).
Ambiguity

- Only one derivation for previous example.
- What about ‘ID / ID / ID’?
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![Parse Trees](image)

- If there exists even one string like ID / ID / ID in $L(G)$, we say $G$ is ambiguous (even if other strings only have one parse tree).
Review: Syntax-Directed Translation

- Want the structure of sentences, not just whether they are in the language, because this drives translation.

- Associate translation rules to each production, just as Flex associated actions with matching patterns.

- Bison notation:
  
  ```
  e : e '/' e { $$ = doDivide($1, $3); }
  ```

  provides way to refer to and set *semantic values* on each node of a parse tree.

- Compute these semantic values from leaves up the parse tree.

- Same as the order of a *rightmost derivation in reverse* (a.k.a a *canonical derivation*).

- Alternatively, just perform arbitrary actions in the same order.
Example: Conditional statement

Problem:  
   * if-else  
   * if-elif-else  

statements in Python (else optional). Assume that only (indented) suites may be used for then and else clauses, that nonterminal stmt defines an individual statement (one per line), and that nonterminal expr defines an expression. Lexer supplies INDENTS and DEDENTs. A cond is a kind of stmt.
Example: Conditional statement

Problem: if-else or if-elif-else statements in Python (else optional). Assume that only (indented) suites may be used for then and else clauses, that nonterminal stmt defines an individual statement (one per line), and that nonterminal expr defines an expression. Lexer supplies INDENTS and DEDENTS. A cond is a kind of stmt.

```
expr : ...
stmt : ... | cond | ...
cond : "if" expr ':' suite elifs else
suite: INDENT stmts DEDENT
stmts: stmt | stmts stmt
elifs: ε | "elif" expr ':' suite elifs
else : ε | "else" ':' suite
```
Example: Conditional statement in Java

Problem: if-else in Java. Assume that nonterminal stmt defines an individual statement (including a block in {}).
Example: Conditional statement in Java

Problem: if-else in Java. Assume that nonterminal stmt defines an individual statement (including a block in {}).

expr : ...
stmt : ... | cond | ...
cond : "if" '(' expr ')' stmt else
else : $\epsilon$ | "else" stmt

But this doesn't quite work: recognizes correct statements and rejects incorrect ones, but is ambiguous. E.g.,

if (foo) if (bar) walk(); else chewGum();

Do we chew gum if foo is false? That is, is this equivalent to

if (foo) { if (bar) walk(); } else chewGum();
/*or*/ if (foo) { if (bar) walk(); else chewGum(); } ?
Example resolved: Conditional statement in Java

The rule is supposed to be “each ‘else’ attaches to the nearest open ‘if’ on the left,” which is captured by:

\[
\begin{align*}
\text{expr} & : \ldots \\
\text{stmt} & : \ldots \mid \text{cond} \mid \ldots \\
\text{stmt\_closed} & : \ldots \mid \text{cond\_closed} \mid \ldots \\
\text{cond\_closed} & : "if\" (\text{expr})\" \text{stmt\_closed} \"else\" \text{stmt\_closed} \\
\text{cond} & : "if\" (\text{expr})\" \text{stmt} \\
& \quad \mid "if\" (\text{expr})\" \text{stmt\_closed} \"else\" \text{stmt}
\end{align*}
\]

This does not allow us to interpret

\[
\text{if (foo)} \text{if (bar)} \text{walk(); else chewGum();}
\]

as

\[
\text{if (foo)} \{ \text{if (bar) walk(); } \} \text{else chewGum();}
\]

But it’s not exactly clear, is it?
Puzzle: NFA to BNF

Problem: What BNF grammar accepts the same string as this NFA?
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Problem: What BNF grammar accepts the same string as this NFA?

A conventional answer (from class):

\[
\begin{align*}
S &: S2s \ Z \mid S3s \ Z \\
S2s &: S2 \mid S2 \ S2s \\
S3s &: S3 \mid S3 \ S3s \\

S2 &: Z \ '1' \ Z \ '1' \\
S3 &: Z \ '1' \ Z \ '1' \ Z \ '1' \\
Z &: \ '0' \ Z \mid \epsilon
\end{align*}
\]
Puzzle: NFA to BNF

Problem: What BNF grammar accepts the same string as this NFA?

General answer (adaptable to any NFA), with one nonterminal per state:

- **S0**: S1 | S4
- **S1**: ’1’ S2 | ’0’ S1
- **S2**: ’1’ S3 | ’0’ S2
- **S3**: S1 | ’0’ S3 | ε
- **S4**: ’1’ S5 | ’0’ S4
- **S5**: ’1’ S6 | ’0’ S5
- **S6**: ’1’ S7 | ’0’ S6
- **S7**: S4 | ’0’ S7 | ε

Nonterminal Sk is “the set of strings that will get me from Sk in the NFA to a final state in the NFA.”