Solutions to Written Assignment 2

1. Let L be the language consisting of all palindromes over the alphabet $\Sigma = \{a, b\}$. That is, L consists of all sequences of a's and b's that read the same forward or backward. For example, $aba \in L$ and $aabbbaa \in L$, but $abb \notin L$.

   Write a context-free grammar for the language L.

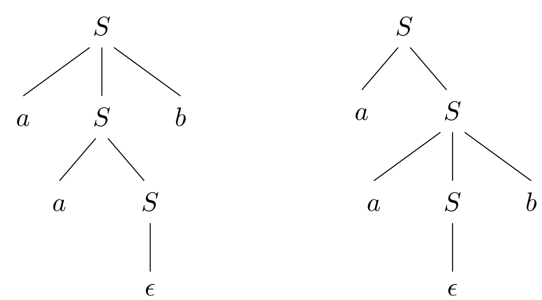
$$\begin{array}{lcl} S & \rightarrow & aSa \\ S & \rightarrow & bSb \\ S & \rightarrow & a \\ S & \rightarrow & b \\ S & \rightarrow & \epsilon \end{array}$$

2. Consider the following grammar:

$$\begin{array}{lcl} S & \rightarrow & aSb \\ S & \rightarrow & aS \\ S & \rightarrow & \epsilon \end{array}$$

   (a) Give a one-sentence description of the language generated by this grammar.
   The strings in this language consist of a sequence of n a's followed by m b's, for any n and m such that $n \geq m \geq 0$.

   (b) Show that this grammar is ambiguous by giving a string that can be parsed in two different ways. Draw both parse trees.
   The string aab can be parsed in two ways:



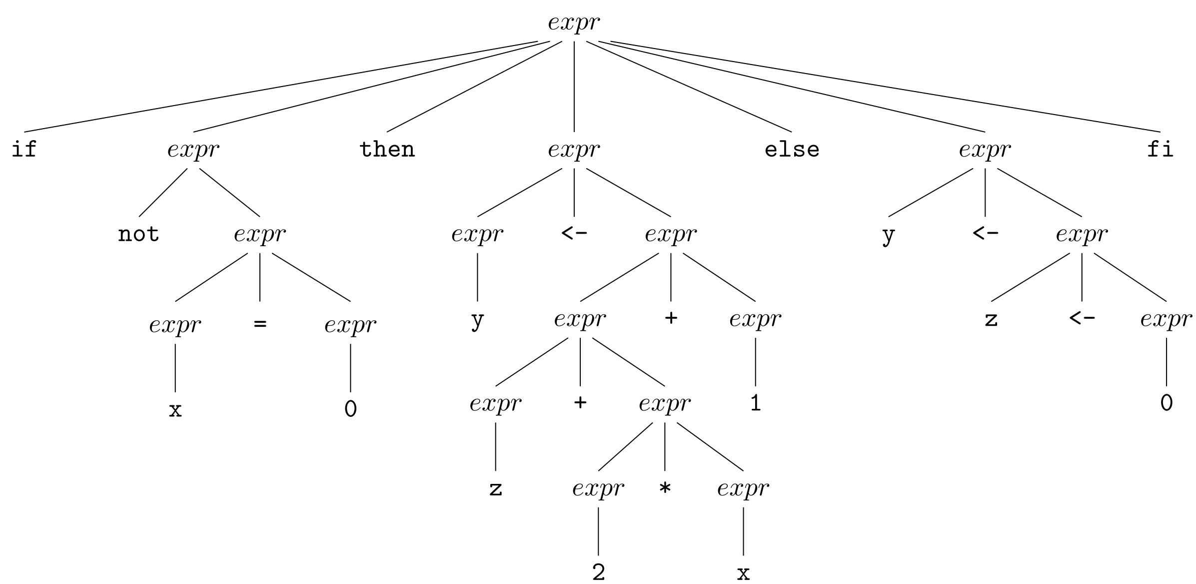
   (c) Give an unambiguous grammar that accepts the same language as the grammar above.

$$\begin{array}{lcl} S & \rightarrow & aSb \\ S & \rightarrow & T \\ T & \rightarrow & aT \\ T & \rightarrow & \epsilon \end{array}$$

3. Using the context-free grammar for Cool given in Section 11 of the Cool manual, draw a parse tree for the following expression.

```
if not x = 0 then
    y <- z + 2 * x + 1
else
    y <- z <- 0
fi
```

Note that the context-free grammar by itself is ambiguous, so you will need to use the precedence rules in Section 11.1 to get the correct tree.



4. Give an example of a grammar that is $LL(2)$ but not $LL(1)$.

$$S \to ab \mid ac$$