1. The Algol 68 language introduced an expression called the case conformity clause. Here’s one version of it:

\[
\text{case } I := E_0 \text{ in } T_1: E_1; T_2: E_2; \ldots; T_n: E_n; \text{ esac}
\]

where the \(E_i\) are expressions (i.e., having values), \(I\) is an identifier, and the \(T_i\) are types. The idea here is that the program first evaluates \(E_0\), and assigns \(I\) its value. If the dynamic type of \(I\) is \(T_i\) for some \(i\) (or a subtype of \(T_i\)), the program evaluates \(E_i\) and yields its value as the value of the entire clause (it will be a run-time error if no clauses match). If more than one \(T_i\) fits, the program chooses one arbitrarily and evaluates it (the expression must type properly regardless of which choice is made). The problem is to come up with a static typing rule for this expression. Assume that the AST for the case conformity clause above is represented in Prolog notation as

\[
\text{case_conform}(\hat{I}, \hat{E}_0, [\text{case}(\hat{T}_1, \hat{E}_1), \ldots, \text{case}(\hat{T}_n, \hat{E}_n)])
\]

where \(\hat{x}\) is the AST for \(x\). So the problem is to find an appropriate replacement for ‘??’ in

\[
\text{typeof}(\text{case_conform}(I,E0,Clausess), T, \text{Env}) :- ??
\]

The implication here is that all the clauses have to produce values of some common type, \(T\). There is no need to know the rest of this language to do this.

Produce two Prolog versions of this rule under the following alternative assumptions:

**Version 1:** Require that all the \(T_i\) be subtypes of the static type of \(E_0\).

**Version 2:** Require that at least one clause have a \(T_i\) that is a subtype of \(E_0\)’s type, but allow \(E_i\) in clauses for which \(T_i\) is not a subtype of \(E_0\)’s type to have any type at all.

So, under version 2, it’s OK to write

\[
\text{N + case } x := \text{head(shapeList) in}
\]
\[
\text{Rectangle : width(x);}
\]
\[
\text{Circle : radius(x);}
\]
\[
\text{Elephant : } "\text{Hi, there!}" \text{ esac}
\]

assuming that \(\text{shapeList} \) is a list of \(\text{Shapes}\), types \(\text{Rectangle}\) and \(\text{Circle}\) are subtypes of \(\text{Shape}\), and \(\text{Elephant}\) is not. Even though the type of the \(\text{Elephant}\) case is presumably incompatible with that of the other two (presumably numeric), we can ignore it, since the last case can never be taken. Under version 1, however, this case expression is illegal.

See the skeletons in \(\text{hw5/case_conform1.pl}\) and \(\text{hw5/case_conform2.pl}\). To do version 2, you’ll need some “impure” Prolog. The following pair of rules (in the order given) define \(\text{notok}(X)\) to succeed if \(\text{ok}(X)\) cannot be satisfied:
notok(X) :- ok(X), !, fail.
notok(_).

The cut symbol, ‘!’, basically says, “always succeed, but if you ever have to backtrack past this point, give up on the goal notok immediately and don’t try any other rules for notok.” So once you find that ok(foo) is satisfiable, you hit the cut symbol and then immediately fail (the goal fail has no rules, so that it always fails).

2. In Java, the following is legal:

   `String[] Y;
   Object[] X;
   ...
   X = Y;`

That is, an array of $T_1$ may be assigned to a variable of type array-of-$T_2$ as long as $T_1$ is a subtype of $T_2$. As it turns out, this rule is unsound in the sense that because of it, certain type errors can only be discovered at execution time, requiring a (somewhat) expensive check that slows down some operations. Give an example of how this can happen (by which I mean an actual Java program).

3. Write a legal Python program that simply prints “static” and that would also be legal if Python used dynamic scoping, but would print “dynamic” instead.

4. Show how the type rules from slide 15 of Lecture 23 work to determine the types of $Y$, $g$, and $\text{fact}$ in

   ```python
   def Y f = f (Y f)
   def g h x = if x = 0 then 1 else h(x-1) * x fi
   def fact x = Y g x
   ```

Assume that ‘-’ and ‘*’ obey the same rules as ‘+’. (Aside: for obvious reasons, this particular definition of $Y$, the “paradoxical combinator,” won’t actually work unless this language uses normal-order evaluation, in which expressions are not evaluated until their value is actually used in a primitive operation. However, evaluation is not the point here.)

5. Overloading in the project is a work in progress, so let’s take a shot in the homework. The Python file `overload.py` contains a skeleton that defines a simple AST with two types of node:

   **Leaf nodes**, labeled with an identifier string that names a type.

   **Call nodes**, containing labeled with a function identifier and having 0 or more children (each an AST).
It also defines a type `Signature`, which stands for the type of a function (that is, its argument types and its return type). We'll define an environment as a dictionary mapping function names to lists of function signatures (thus representing sets of overloads of a given function name).

The idea is to figure out the particular signature to choose for each of the function names in call nodes so as to make all signatures match the argument types.

Fill in the two functions:

**resolve1(T, Env):** As in Java or C++, require that each signature be selected unambiguously using only the types of the arguments. In other words, given a call such as \( f(g(\text{Int})) \), the type of \( g(\text{Int}) \) must be determined unambiguously without reference to the fact that its result will be an argument to \( f \).

**resolve2(T, Env):** As in the Ada language, find signatures for all functions so that the entire AST matches. In other words, given a call such as \( f(g(\text{Int})) \), it's OK to have two possible overloads of \( g \) that take \text{Int} arguments, as long as they have different return types and only one return type fits an overloading of \( f \).

6. Let's look at a very simplified sketch of scope analysis to give you a chance to work out the logic of scope analysis in our project on a simplified language. This language has the following syntax:

```plaintext
program: /* empty */ | program outer_stmt ;
outer_stmt: stmt | def | class ;
stmt: ID "::" type "=" expr ";" |
ID "=" expr ";" ;
stmts: /* empty */ |
| stmts stmt
| stmts def ;
def: "def" ID "{" stmts "}" ;
class: "class" ID "{" stmts "}"
type: ID
eexpr: /* empty */ | expr ID ;
```

The skeleton file `scoper.py` reads this syntax and produces an AST for it (see the skeleton). Fill in the skeleton to

1. Find all the distinct definitions, according to the rules. The definitions are names defined by `def`, names defined by `class`, and names that are assigned to.
2. Check that definitions are consistent: identifiers may not be multiply defined (no overloading here); multiple assignments to an identifier in the same declarative region result in only one local variable; a name may be defined to be only one kind of thing (local, method, or class) in the same declarative region.

3. Check that each name defined by an ‘outer_stmt’ (via def, class, or assignment) is so defined before any uses of it.

4. Make sure that all names appearing in ‘exprs’ are defined somewhere in an enclosing declarative region (for inner functions and locals, this can be before or after the use, as in Python.)

5. If there are multiple assignments to the same variable (i.e., in the same declarative region) using the :: syntax, make sure the type name is the same in each case (no check is needed for other assignments).

6. Make sure that all ‘type’ identifiers refer to classes (and are defined prior to the use of the type).

7. As in Python, members of a class are not directly visible inside a method of the class (i.e., without '.', which this problem does not address.)

8. Number each distinct defined local, function, or class, and decorate the identifiers with the appropriate numbers. To make things deterministic, the skeleton has you do this by decorating identifier nodes in the AST with objects (type Decl), so that identifiers that refer to the same entity point to the same Decl. There is machinery in the skeleton to number these decorations.

The skeleton will print out the resulting program and annotations. The files eg.scp and eg.out provide a sample input and output.