1. **Dijkstra with negative edges!**
Consider a directed graph in which the only negative edges are those that leave s; all other edges are positive. Can Dijkstra’s algorithm, started at s, fail on such a graph? Prove your answer.

2. **Dijkstra Jr.**
Suppose we want to run Dijkstra’s algorithm on a graph whose edge weights are integers in the range 0, · · · , W where W is a relatively small number.

   - (a) Show how Dijkstra’s algorithm can be made to run in time $O(W|V| + |E|)$.
   - (b) Show an alternative implementation that takes time just $O(|V| + |E|) \log W)$

3. **No one like you**
Let $G = (V, E)$ be an undirected graph. Prove that if all its edge weights are distinct, then it has a unique minimum spanning tree.

4. **To tell you the truth...**
The following statements may or may not be correct. In each case, either prove it (if it is correct) or give a counter example (if it isn’t correct). Always assume that the graph $G = (V, E)$ is undirected and connected. Do not assume that edge weights are distinct unless this is specifically stated.

   1. If graph $G$ has more than $|V| − 1$ edges, and there is a unique heaviest edge, then this edge cannot be part of a minimum spanning tree.

   2. Let $e$ be an edge of minimum weight in $G$. Then $e$ must be part of some MST.

   3. Kruskal’s algorithm fails to find the minimum spanning tree if the graph has negative edges.

   4. If $e$ is the heaviest edge in some cycle of the graph $G$, then there is a minimum spanning tree not containing $e$. 