Problem Set 1

CS172 Spring 2005

Out: January 26, 2005
Due: February 2, 2005 by noon to 327 Soda

1. (a) Show that the language of all strings (over $\Sigma = \{\texttt{0}, \texttt{1}\}$) which do not contain the substring `011` is a regular language by exhibiting a DFA/NFA that recognizes it.

   (b) Suppose A is a regular language over Σ. Let Σ' be some alphabet such that $\Sigma \subset \Sigma'$. Using only DFAs, show that A is a regular language over Σ'.

2. (*Sipser 1.24*) For any string $w = w_1 w_2 \ldots w_n$, the **reverse** of w, written $w^{\mathcal{R}}$, is the string w in reverse order, $w_n \ldots w_2 w_1$. For any language A, let $A^{\mathcal{R}} = \{w^{\mathcal{R}} \mid w \in A\}$. Show that if A is regular, so is $A^{\mathcal{R}}$.

3. (*Sipser 1.25*) Let

$$\Sigma_3 = \left\{ \begin{bmatrix} \texttt{0} \\ \texttt{0} \\ \texttt{0} \end{bmatrix}, \begin{bmatrix} \texttt{0} \\ \texttt{0} \\ \texttt{1} \end{bmatrix}, \begin{bmatrix} \texttt{0} \\ \texttt{1} \\ \texttt{0} \end{bmatrix}, \ldots, \begin{bmatrix} \texttt{1} \\ \texttt{1} \\ \texttt{1} \end{bmatrix} \right\}$$

   Σ_3 contains all size 3 columns of `0`s and `1`s. A string of symbols in Σ_3 gives three rows of `0`s and `1`s. Consider each row to be a binary number and let

$$B = \{w \in \Sigma_3^* \mid \text{the bottom row of } w \text{ is the sum of the top two rows}\}$$

   For example,

$$\begin{bmatrix} \texttt{0} \\ \texttt{0} \\ \texttt{1} \end{bmatrix} \begin{bmatrix} \texttt{1} \\ \texttt{0} \\ \texttt{0} \end{bmatrix} \begin{bmatrix} \texttt{1} \\ \texttt{1} \\ \texttt{0} \end{bmatrix} \in B, \text{ but } \begin{bmatrix} \texttt{0} \\ \texttt{0} \\ \texttt{1} \end{bmatrix} \begin{bmatrix} \texttt{1} \\ \texttt{0} \\ \texttt{1} \end{bmatrix} \notin B.$$

   Show that B is regular. (Hint: Working with $B^{\mathcal{R}}$ is easier. You may assume the result claimed in the last problem.)

4. (*Sipser 1.28*) Let

$$\Sigma_2 = \left\{ \begin{bmatrix} \texttt{0} \\ \texttt{0} \end{bmatrix}, \begin{bmatrix} \texttt{0} \\ \texttt{1} \end{bmatrix}, \begin{bmatrix} \texttt{1} \\ \texttt{0} \end{bmatrix}, \begin{bmatrix} \texttt{1} \\ \texttt{1} \end{bmatrix} \right\}.$$

Here, Σ_2 contains all columns of 0s and 1s of height two. Consider the top and bottom rows to be strings of 0s and 1s and let

$$E = \{w \in \Sigma_2^* \mid \text{the bottom row of } w \text{ is the reverse of the top row of } w\}.$$

Show that E is not regular.

5. Let x and y be strings and let L be any language. We say that x and y are *distinguishable by* L if some string z exists whereby exactly one of the strings xz and yz is a member of L. Otherwise, if for every string z, $xz \in L$ if and only if $yz \in L$, we say that x and y are *indistinguishable by* L. If x and y are indistiguishable by L we write $x \equiv_L y$.

   (a) (*Sipser 1.34*) Show that $\equiv_L$ is an equivalence relation.

   (b) Suppose we have a set of strings $\{x_1, x_2, \ldots, x_k\}$ such that x_i and x_j are distinguishable by L for $i \neq j$. Show that any DFA which recognizes L must have at least k states.