Problem Set 2

CS172 Spring 2005

Out: February 2, 2005
Due: February 9, 2005 by noon to 327 Soda

1. (*Sipser 1.42*) If A is any language, let $A_{\frac{1}{2}-}$ be the set of all first halves of strings in A so that

$$A_{\frac{1}{2}-} = \{x \mid \text{for some } y, |x| = |y| \text{ and } xy \in A\}$$

   Show that if A is regular, than so is $A_{\frac{1}{2}-}$.

2. (a) Give a context-free grammar that generates the language

$$L = \{0^i 1 2^j \mid i > j\}$$

   (b) Let L be the language: $L = \{0^n 1^n \mid n \geq 0\}$ Give a context-free grammar which generates the *complement* of L.

   (c) Characterize the language generated by the following grammar over the alphabet $\Sigma = \{0, 1, 2\}^*$

$$S \to 0S0A \mid 1S1A \mid 2$$
$$A \to 0 \mid 1$$

3. (*Sipser 2.18*) Use the pumping lemma to show that the following languages are not context-free.

   (a) $\{0^n 1^n 0^n 1^n \mid n \geq 0\}$

   (b) $\{0^n \# 0^{2n} \# 0^{3n} \mid n \geq 0\}$

   (c) $\{w \# x \mid \ w \text{ is a substring of } x, \text{ where } w, x \in \{a, b\}^*\}$

   (d) $\{x_1 \# x_2 \# \cdots \# x_k \mid k \geq 2, \text{each } x_i \in \{a, b\}^*, \text{ and for some } i \neq j, x_i = x_j\}$

4. (*Sipser 2.23*) Give an example of a language that is not context free but that does satisfy the three conditions of the pumping lemma. Prove that your example works.

5. (*Sipser 2.26*) Let $C = \{x \# y \mid \ x, y \in \{0, 1\}^* \text{ and } x \neq y\}$. Show that C is a context free language.