Problem Set 4

CS172 Spring 2005

Out: February 16, 2005
Due: February 23, 2005 by 5 PM to 327 Soda

1. (*Sipser 4.10*) Let

   $A = \{\langle M\rangle | M$ is a DFA which doesn't accept any string containing an odd number of 1s$\}$.

   Show that A is decidable.

2. (*Sipser 4.18*) Let A and B be two disjoint languages. Say that language C *separates* A and B if $A \subseteq C$ and $B \subseteq \overline{C}$. Show that any two disjoint co-Turing-recognizable languages are separable by some decidable language.

3. (*Sipser 4.21*) Let A be a Turing-recognizable language consisting of descriptions of Turing machines, $\{\langle M_1\rangle, \langle M_2\rangle, \ldots\}$, where every M_i is a decider. Prove that some decidable language is not decided by any decider M_i whose description appears in A. (Hint: You may find it helpful to consider an enumerator for A.)

4. (*Sipser 4.13*) Show that the problem of testing whether a CFG generates all strings in 1^* is decidable. In other words, show that

   $$\{\langle G\rangle | G \text{ is a CFG over } \{0,1\}^* \text{ and } 1^* \subseteq L(G)\}$$

   is a decidable language.