Useful Sums:
\[ \sum_{k=1}^{n} k^m = \frac{n^{m+1}}{m+1} + O(n^m) \]
\[ \sum_{k=0}^{\infty} a^k = \frac{1}{1-a} \]
\[ \sum_{k=1}^{n} \frac{1}{k} = H_n \approx \ln n \]
\[ \sum_{k=1}^{\infty} \frac{1}{k^2} = \frac{\pi^2}{6} \]

Exponentials and Stirling:
\[ (1 + \frac{1}{n})^n \approx e \]
\[ (1 - \frac{1}{n})^n \approx e^{-1} \]
\[ n! \approx \sqrt{2\pi n} \left( \frac{n}{e} \right)^n \left( 1 + \frac{1}{12n} + O\left( \frac{1}{n^2} \right) \right) \]

Binomial Distribution: with parameters \( n \) and \( p \):
\[ \Pr[X = k] = \binom{n}{k} p^k (1-p)^{n-k} \]
and \( \binom{n}{k} \) is also the coefficient of \( x^k \) in \( (1 + x)^n \)
with \( \mathbb{E}[X] = np \) and \( \text{Var}(X) = np(1-p) \).

Geometric Distribution: with parameter \( p \):
\[ \Pr[X = k] = (1-p)^{k-1} p \]
where \( \mathbb{E}[X] = 1/p \) and \( \text{Var}(X) = (1-p)/p^2 \).

Markov Bound: For \( X \) a non-negative random variable:
\[ \Pr[X \geq t] \leq \frac{\mathbb{E}[X]}{t} \]

Chebyshev Bound: For \( X \) any random variable:
\[ \Pr[|X - \mu| \geq t \sigma_X] \leq \frac{1}{t^2} \quad \text{or} \quad \Pr[|X - \mu| \geq s] \leq \frac{\text{Var}(X)}{s^2} \]

Chernoff lower tail bound: For \( X \) any random variable which is a sum of independent Poisson trials with \( \mathbb{E}[X] = \mu \) and \( \delta \in (0, 1] \):
\[ \Pr[X \leq (1 - \delta)\mu] \leq \left( \frac{e^{-\delta}}{(1 - \delta)^{(1-\delta)}} \right)^\mu \leq \exp(-\mu\delta^2/2) \]

Chernoff upper tail bound: For \( X \) any random variable which is a sum of independent Poisson trials with \( \mathbb{E}[X] = \mu \) and \( \delta > 0 \):
\[ \Pr[X \geq (1 + \delta)\mu] \leq \left( \frac{e^\delta}{(1 + \delta)^{(1+\delta)}} \right)^\mu \leq \begin{cases} \exp(-\mu\delta^2/3) & \text{for } \delta \leq 1 \\ 2^{-(1+\delta)\mu} & \text{for } \delta \geq 5 \end{cases} \]
**Poisson Distribution:** with parameter $\lambda$:

$$\Pr[X = k] = \frac{\lambda^k \exp(-\lambda)}{k!}$$

mean is $\lambda$ and variance equals $\lambda$.

**Lipschitz Condition** The function $f$ is said to be *Lipschitz with bound* $c$ if changing any single argument changes the value of the function by at most $c$, that is there exists a $c$ such that for all $i$, $x_1, \ldots, x_n$ and $y$:

$$|f(x_1, \ldots, x_{i-1}, x_i, x_{i+1}, \ldots, x_n) - f(x_1, \ldots, x_{i-1}, y, x_{i+1}, \ldots, x_n)| \leq c$$

**Azuma-Hoeffding Inequality:** Let $F = f(X_1, \ldots, X_n)$ be a function of random variables $X_1, \ldots, X_n$, so that

$$Z_i = \mathbb{E}[F|X_1, \ldots, X_i] \text{ for } i = 1, \ldots, n \text{ and } Z_0 = \mathbb{E}[F]$$

is a Doob Martingale. If $f$ is Lipschitz with bound $c$, then

$$\Pr(|Z_n - Z_0| \geq \lambda) \leq 2 \exp(-\lambda^2/(nc^2))$$