This homework is due by 5pm on Thursday Feb 21st. Please hand it to the CS174 homework box on the second floor of Soda Hall.

1. Suppose that we roll a fair dice 200 times. Let $X$ be the sum of the numbers that appear. Use Chebyshev to bound the probability that $\Pr(X > 800)$.

2. Let $X$ be a number chosen uniformly at random from $[1, \ldots, n]$.
   (a) What is $\text{Var}[X]$?
   (b) Let $Y$ be a sum of $k$ numbers chosen independently and uniformly at random from $[1, \ldots, n]$. What is $\text{Var}[Y]$?

3. Let $X$ be a random variable which is non-negative, and with expected value 1.
   (a) Give an upper bound on the probability that $X \geq 10$.
   (b) Can you give an upper bound on $\text{Var}[X]$?
   (c) Now suppose you are told in addition that $X \leq 3$. Can you give an upper bound on $\text{Var}[X]$? Is your bound tight?

4. Give a random variable $X$ with probability distribution such that $E[X]$ is finite, but $\text{Var}[X]$ is unbounded.
   (a) Do this first with no restriction on the values of $X$ (easier).
   (b) Now do it for an $X$ which is non-negative.

5. Let $X$ be a binomial random variable with parameters $n = 100$ and $p = 1/2$.
   (a) Use Chebyshev to bound the probability that $X \geq 70$.
   (b) Give a direct upper bound on the probability that $X \geq 70$ using the binomial distribution. Its fine to use a bound based on a geometric series (which will be very close).

6. Using generating functions, show that

$$1 = \sum_{k=0}^{n} \frac{\binom{m}{k} \binom{N-m}{n-k}}{\binom{N}{n}}$$

where $n, m \leq N$ are any values. This is called a hypergeometric distribution. It models the probability of drawing $k$ defective objects in a sample of $n$ objects. The $n$ objects are randomly drawn from a larger set of $N$ objects of which $m$ are defective.