CS-184: Computer Graphics

Lecture #15: Modeling w/ Points

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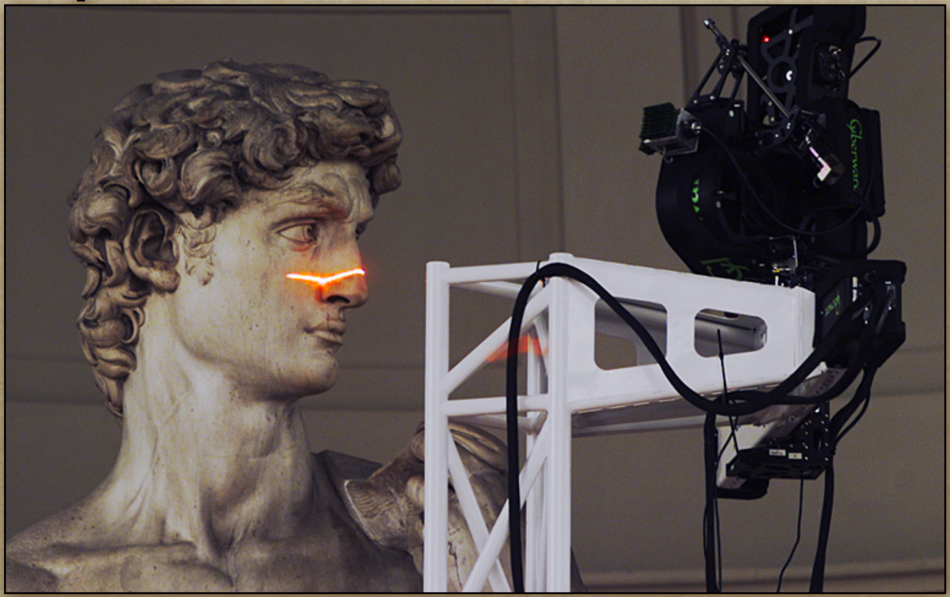
V2007F-15-1.0

Today

- Points as a graphics primitive

A Thought Experiment

- Laser scanners
  - Millions to *billions* of points
- Typical image
  - At most a few million pixels
- More points than pixels...



“Point-Based Graphics”

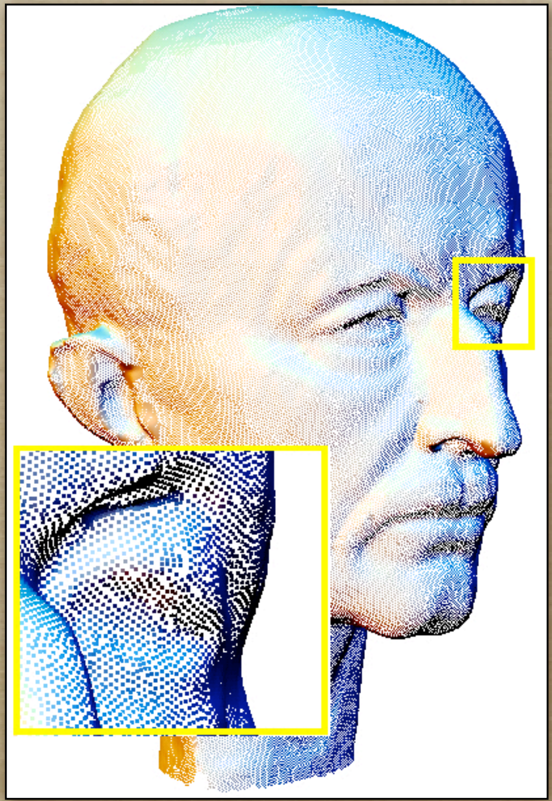
- Surfaces represented only by points
  - Maybe normals also
  - No topology
- How can we do
  - Rendering
  - Modeling opperations
  - Simulation

Rendering

- For each point draw a little "splat"
  - Use associated normal for shading
  - Possibly apply texture

If "splats" are small compared to spacing then gaps result

Splatting too many points would waste time

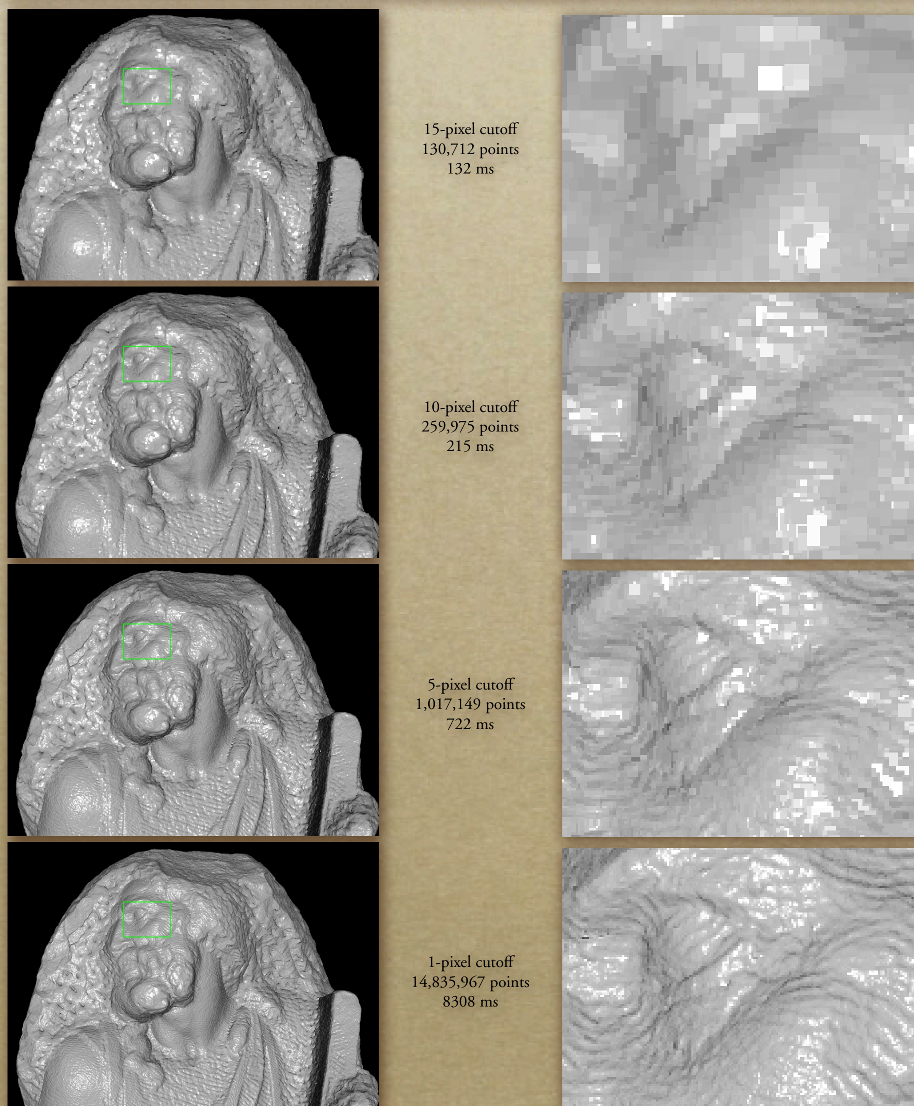


Ohtake, *et al.*, SIGGRAPH 2003

Rendering

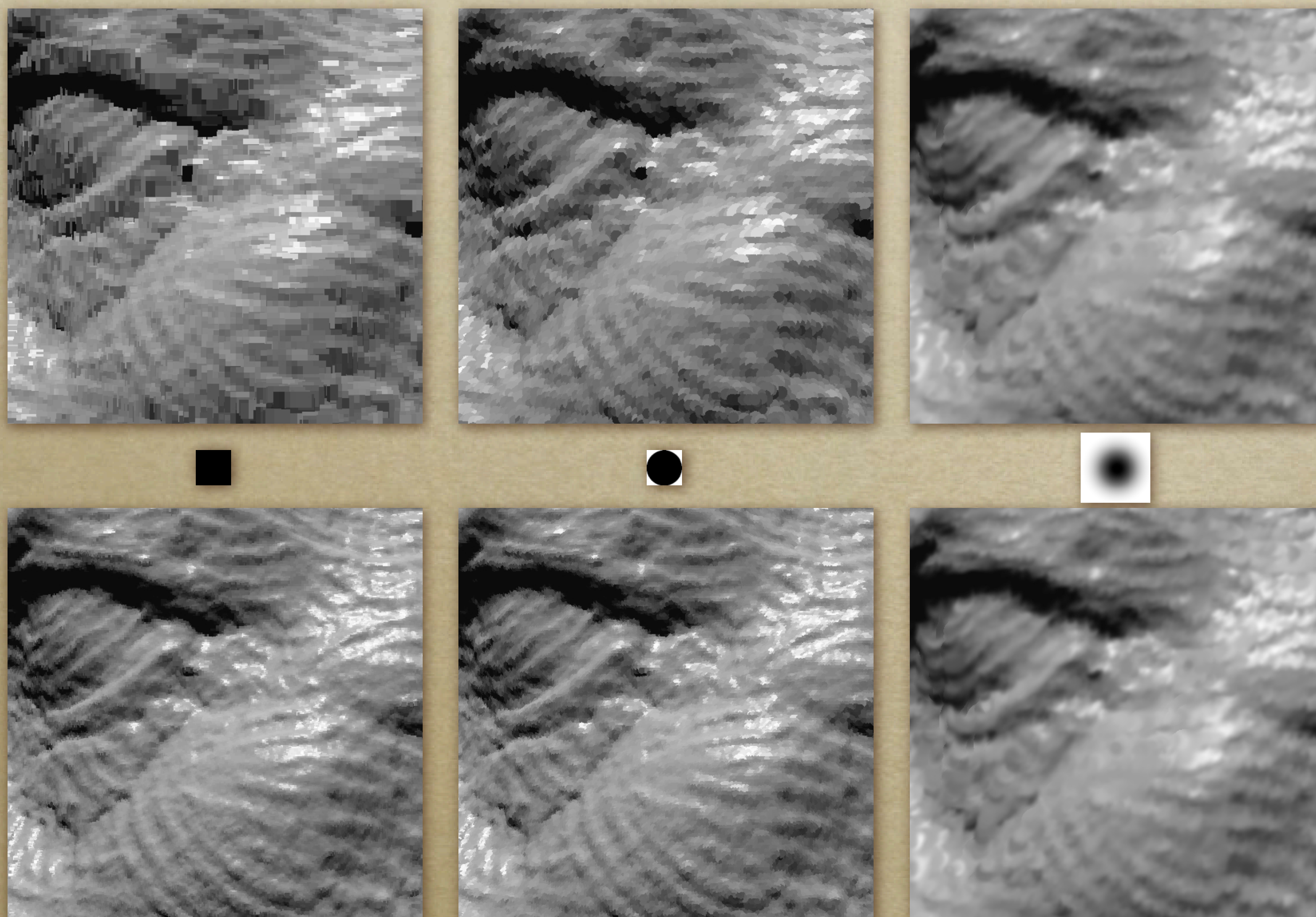
- “QSplat” algorithm
  - Build hierarchical tree of the points
  - Use bounding spheres to estimate size of clusters
  - Render clusters based on screen size
  - Use cluster-normals for internal nodes

Rendering



From Rusinkiewicz and Levoy, SIGGRAPH 2000.

Rendering



From Rusinkiewicz and Levoy, SIGGRAPH 2000.

Rendering

From Rusinkiewicz and Levoy, SIGGRAPH 2000.

Rendering


(a)
Points


(b)
Polygons – same number of primitives as (a)
Same rendering time as (a)


(c)
Polygons – same number of vertices as (a)
Twice the rendering time of (a)

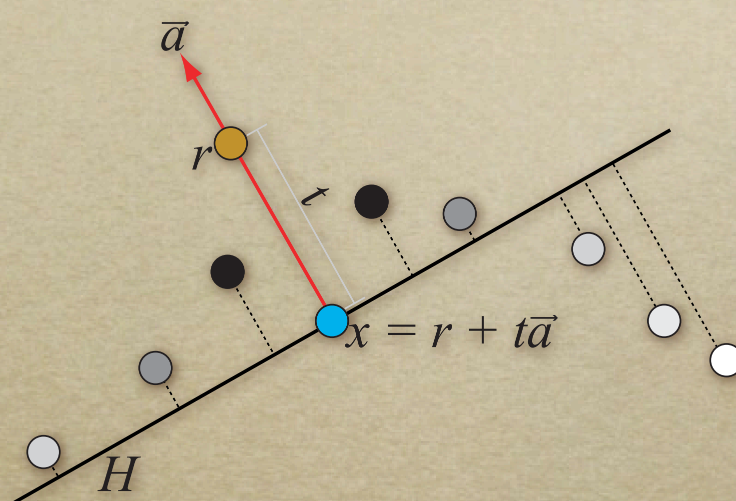
From Rusinkiewicz and Levoy, SIGGRAPH 2000.

Defining a Surface

- Two related methods
  - Surface is a point attractor
    - Point-set surfaces
  - Implicit surface
    - Multi-level Partition of Unity Implicits
    - Implicit Moving Least-Squares

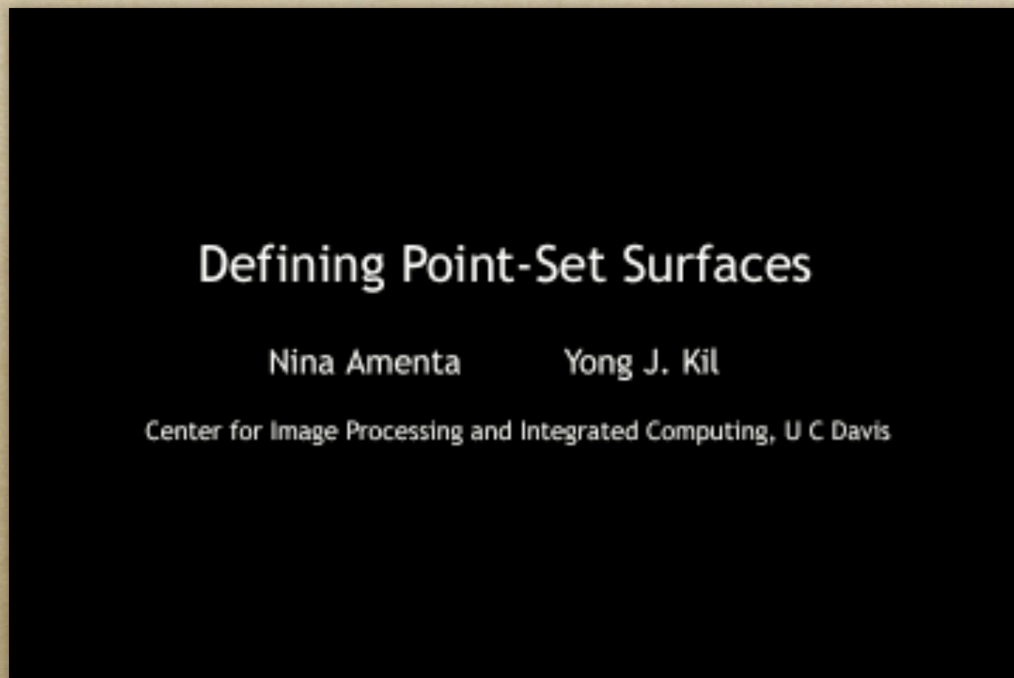
Point-Set Surfaces

- Surface is the attractor of a repeated projection process
  - Find nearby points
  - Fit plane (weighted)
  - Project into plane
  - Repeat
- Does it converge?
- How to weight points?

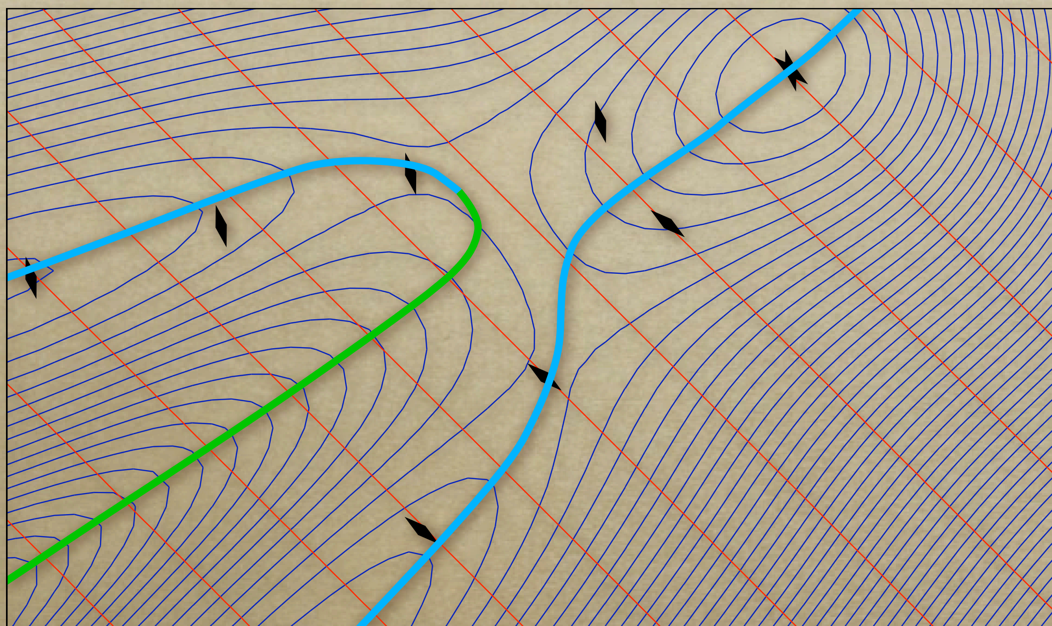


From Amenta and Kil, SIGGRAPH 2004.

Point-Set Surfaces



Point-Set Surfaces



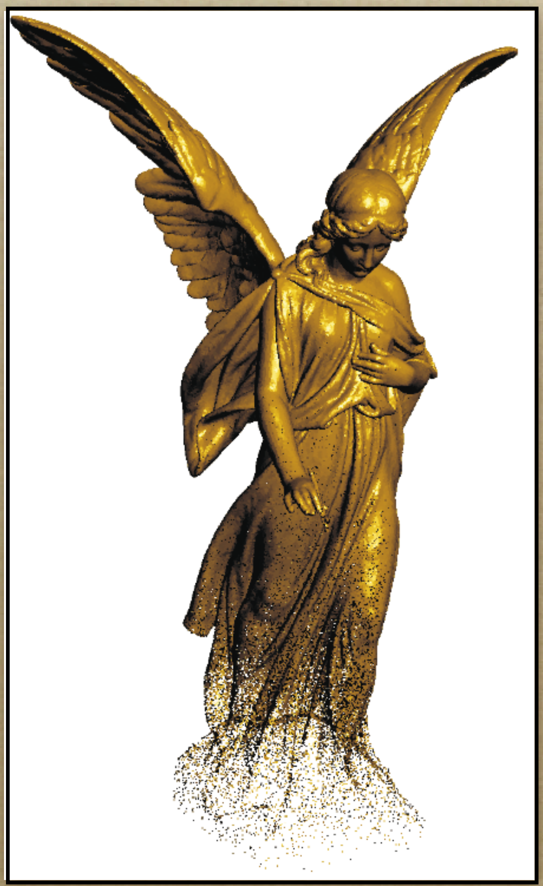
From Amenta and Kil, SIGGRAPH 2004.

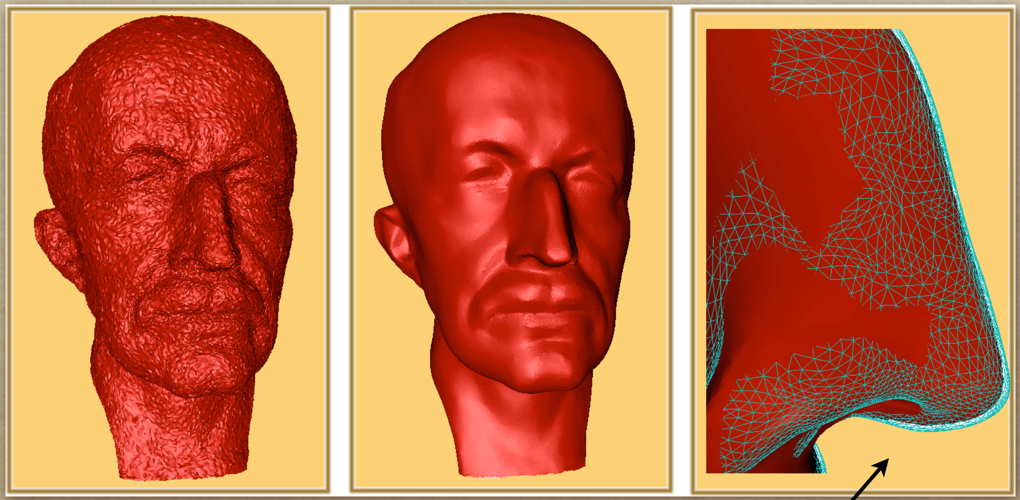
Does this give us a good surface?

New “robust” methods exist for sharp features

Point-Set Surfaces

- Some examples



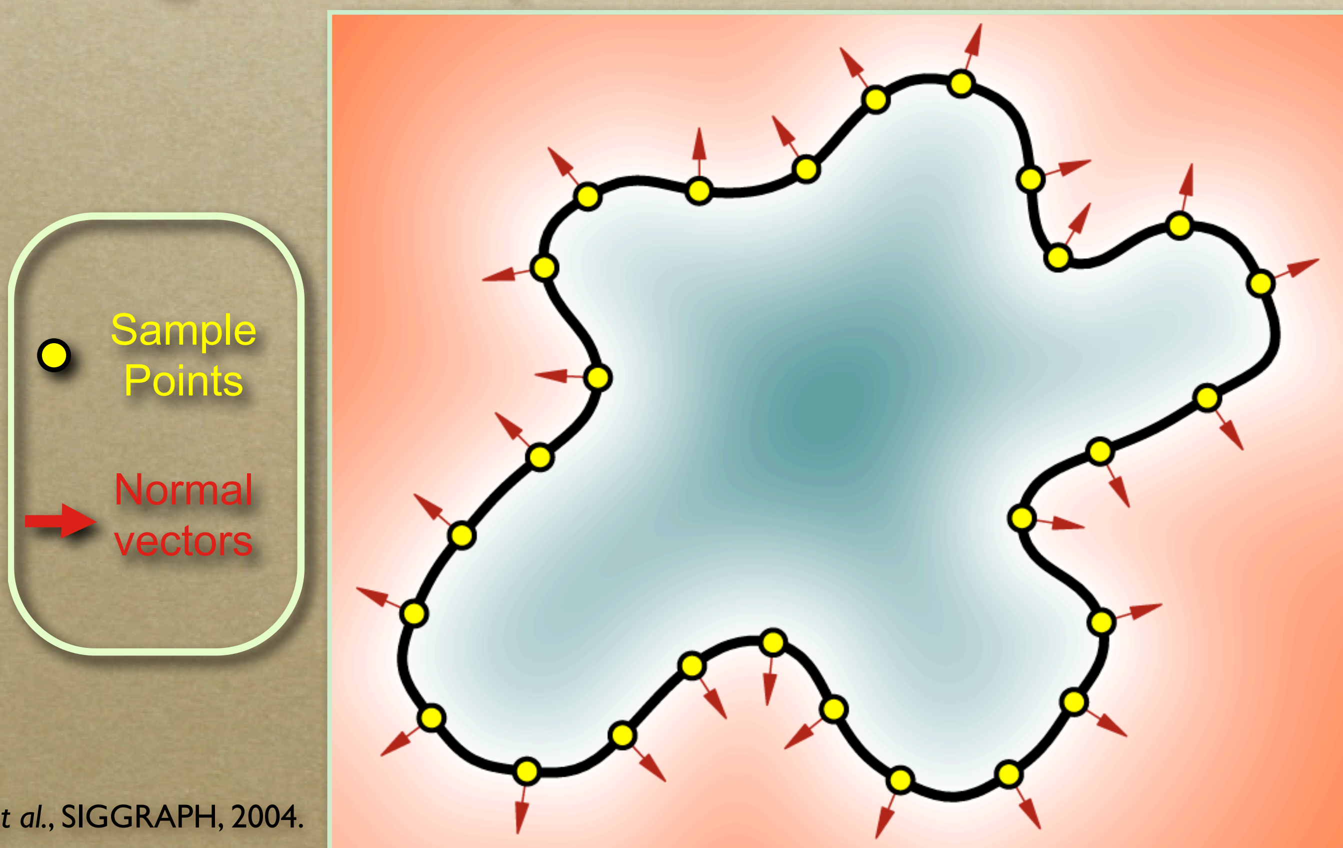


Note shrinkage

From Fleichman, Thesis, 2003.

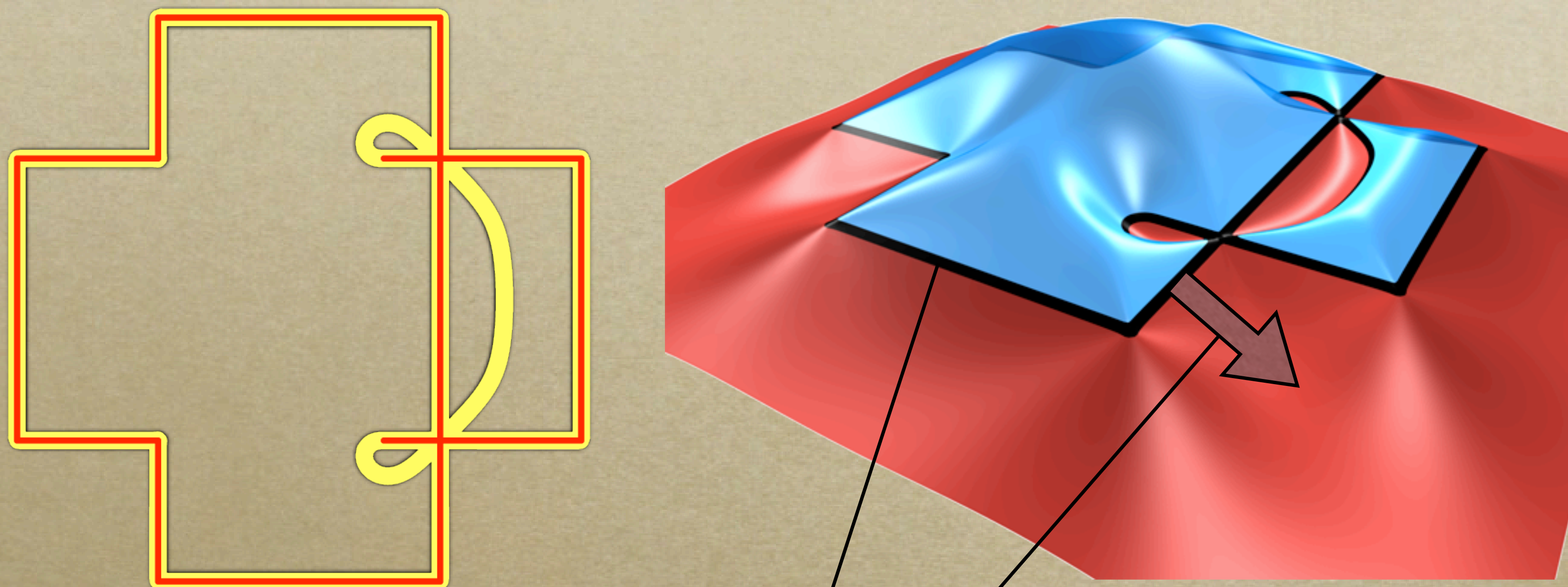
Implicit Moving Lest-Squares

- Define a scalar function that is zero passing through all the points



From Shen, *et al.*, SIGGRAPH, 2004.

Implicit Moving Lest-Squares



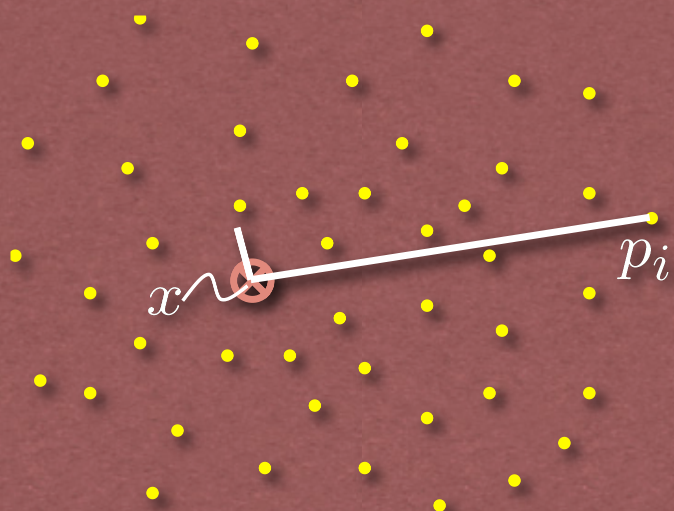
Function is zero on boundary
Decreases in outward direction

From Shen, *et al.*, SIGGRAPH, 2004.

Moving Least-Square Interpolation

Standard Least Square

$$\begin{bmatrix} b^{\top}(p_1) \\ \vdots \\ b^{\top}(p_N) \end{bmatrix} c = \begin{bmatrix} \phi_1 \\ \vdots \\ \phi_N \end{bmatrix}$$

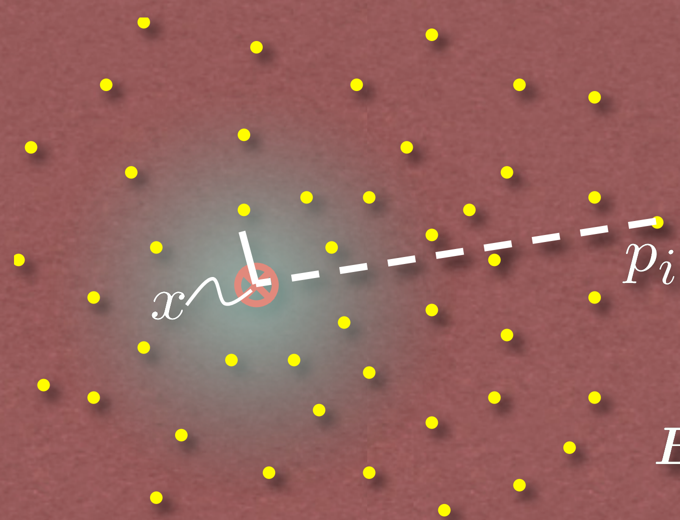


$$B^{\top} B\, c = B^{\top} \phi$$

Moving Least-Square Interpolation

Moving Least Square

$$\begin{bmatrix} w(\boldsymbol{x},\boldsymbol{p}_1) & & \\ & \ddots & \\ & & w(\boldsymbol{x},\boldsymbol{p}_N) \end{bmatrix} \begin{bmatrix} \boldsymbol{b}^\mathsf{T}(\boldsymbol{p}_1) \\ \vdots \\ \boldsymbol{b}^\mathsf{T}(\boldsymbol{p}_N) \end{bmatrix} \boldsymbol{c} = \begin{bmatrix} w(\boldsymbol{x},\boldsymbol{p}_1) & & \\ & \ddots & \\ & & w(\boldsymbol{x},\boldsymbol{p}_N) \end{bmatrix} \begin{bmatrix} \phi_1 \\ \vdots \\ \phi_N \end{bmatrix}$$

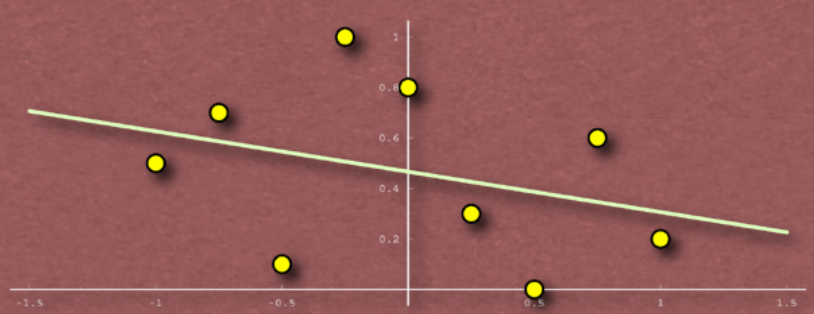


$$w(r) = \frac{1}{\left(r^2 + \epsilon^2\right)}$$

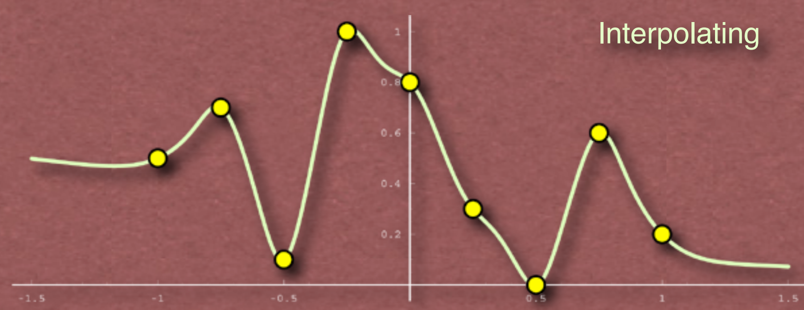
$$\boldsymbol{B}^\mathsf{T}\ (\boldsymbol{W}(x))^2\ \boldsymbol{B}\ \boldsymbol{c}(x) = \boldsymbol{B}^\mathsf{T}\ (\boldsymbol{W}(x))^2\ \phi$$

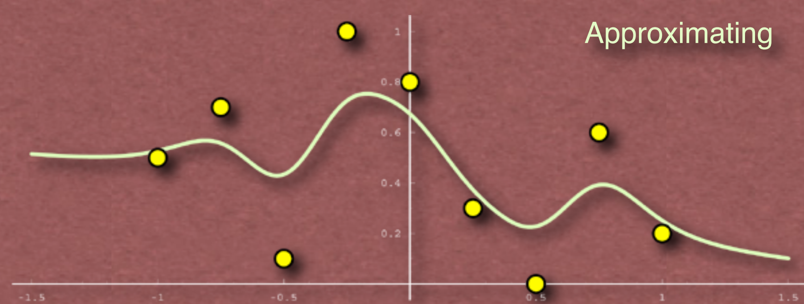
Moving Least-Square Interpolation

Least Square

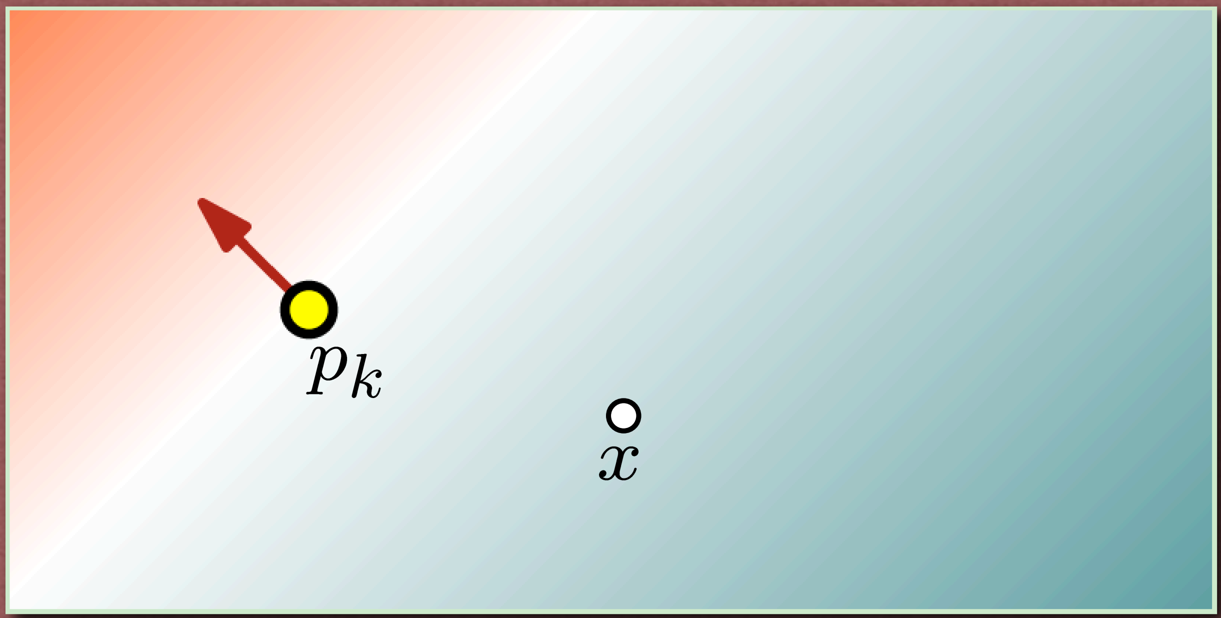


Moving Least Square



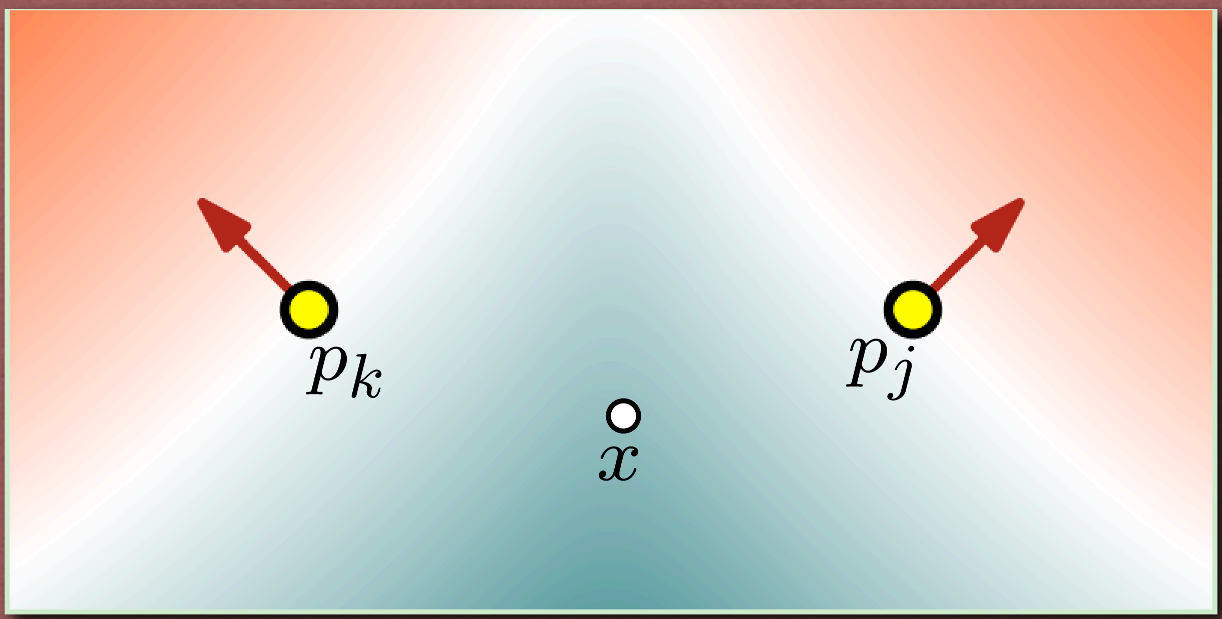


Interpolating Functions



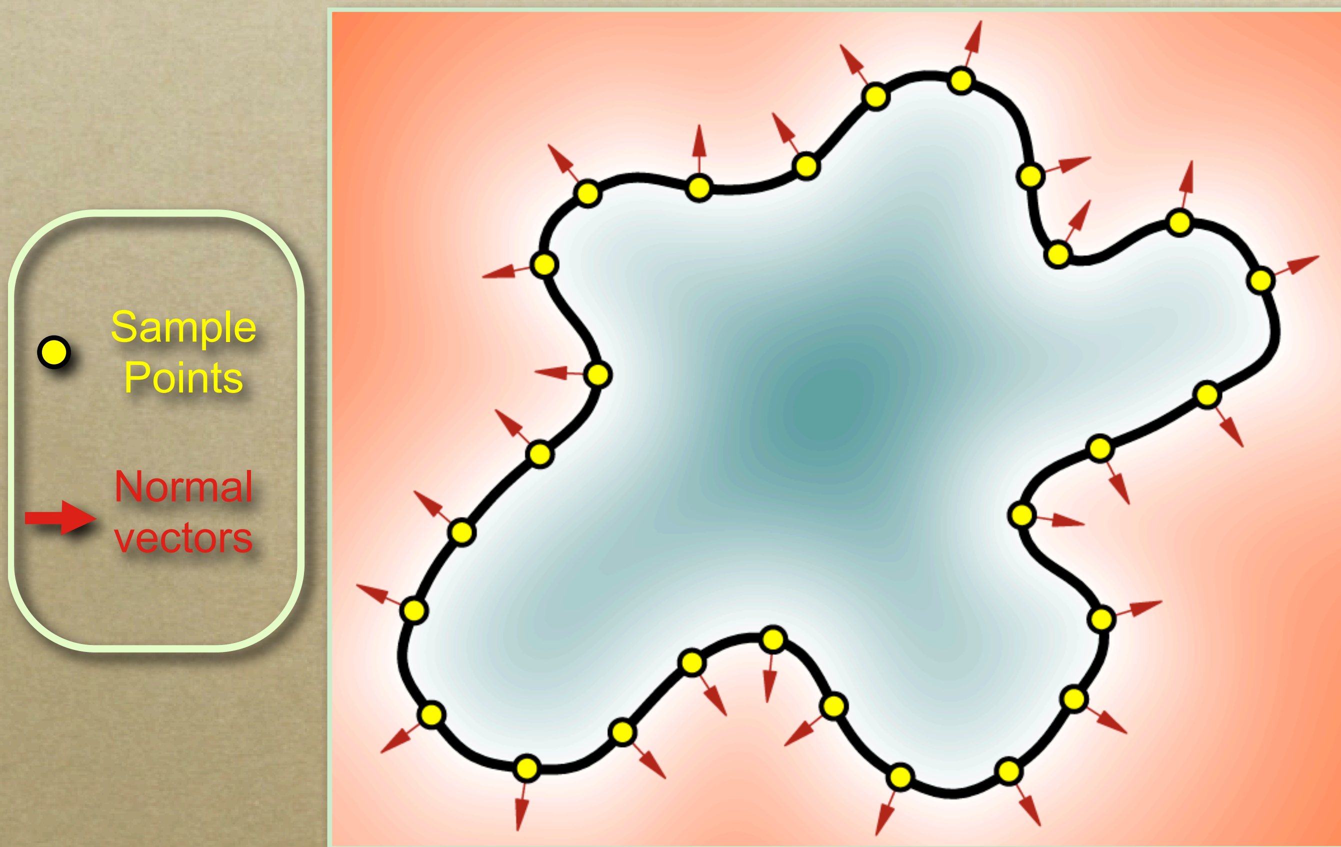
$$
\begin{aligned}
S_k(\boldsymbol{x}) &= \phi_k + (\boldsymbol{x} - \boldsymbol{p}_k)^{\mathsf{T}}\, \hat{\boldsymbol{n}}_k \\
&= \psi_{0k} + \psi_{xk}\, x + \psi_{yk}\, y + \psi_{zk}\, z
\end{aligned}
$$

Interpolating Functions



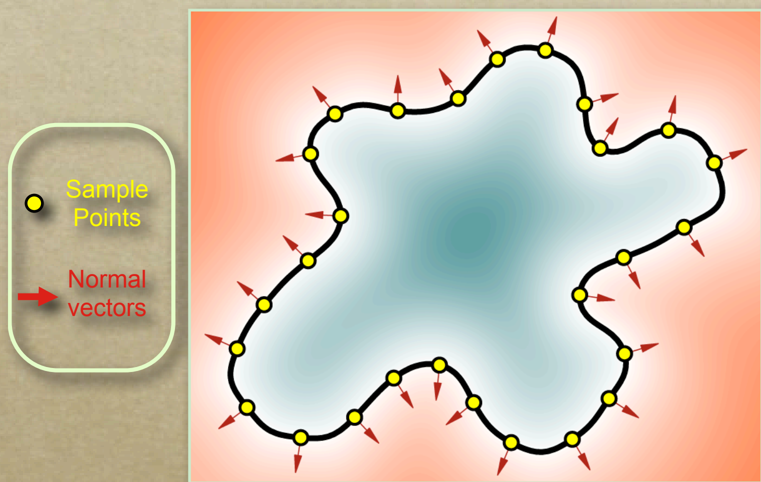
$$\begin{bmatrix} w(\boldsymbol{x}, \boldsymbol{p}_1) \\ \vdots \\ w(\boldsymbol{x}, \boldsymbol{p}_i) \end{bmatrix} c_1 = \begin{bmatrix} w(\boldsymbol{x}, \boldsymbol{p}_1) & & \\ & \ddots & \\ & & w(\boldsymbol{x}, \boldsymbol{p}_N) \end{bmatrix} \begin{bmatrix} S_1(\boldsymbol{x}) \\ \vdots \\ S_N(\boldsymbol{x}) \end{bmatrix}$$

Implicit Moving Lest-Squares

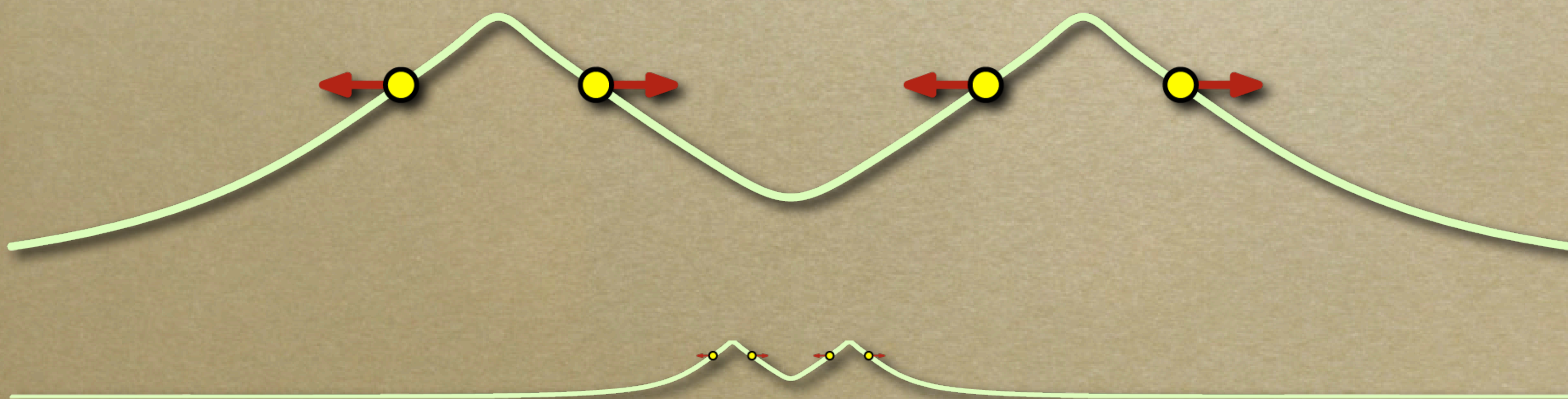


From Shen, *et al.*, SIGGRAPH, 2004.

Implicit Moving Lest-Squares



Proof of good behavior in Kolluri SODA 2005



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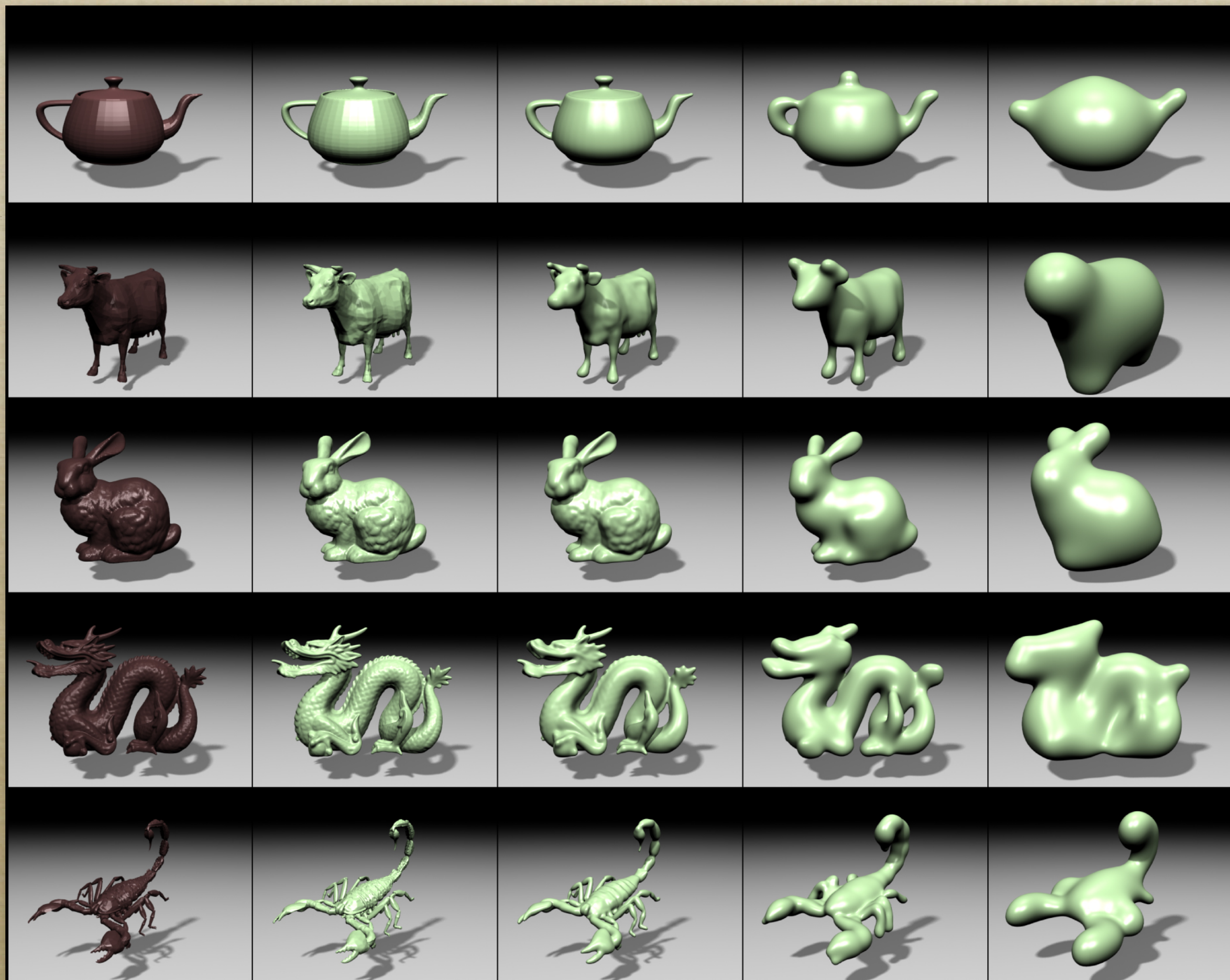
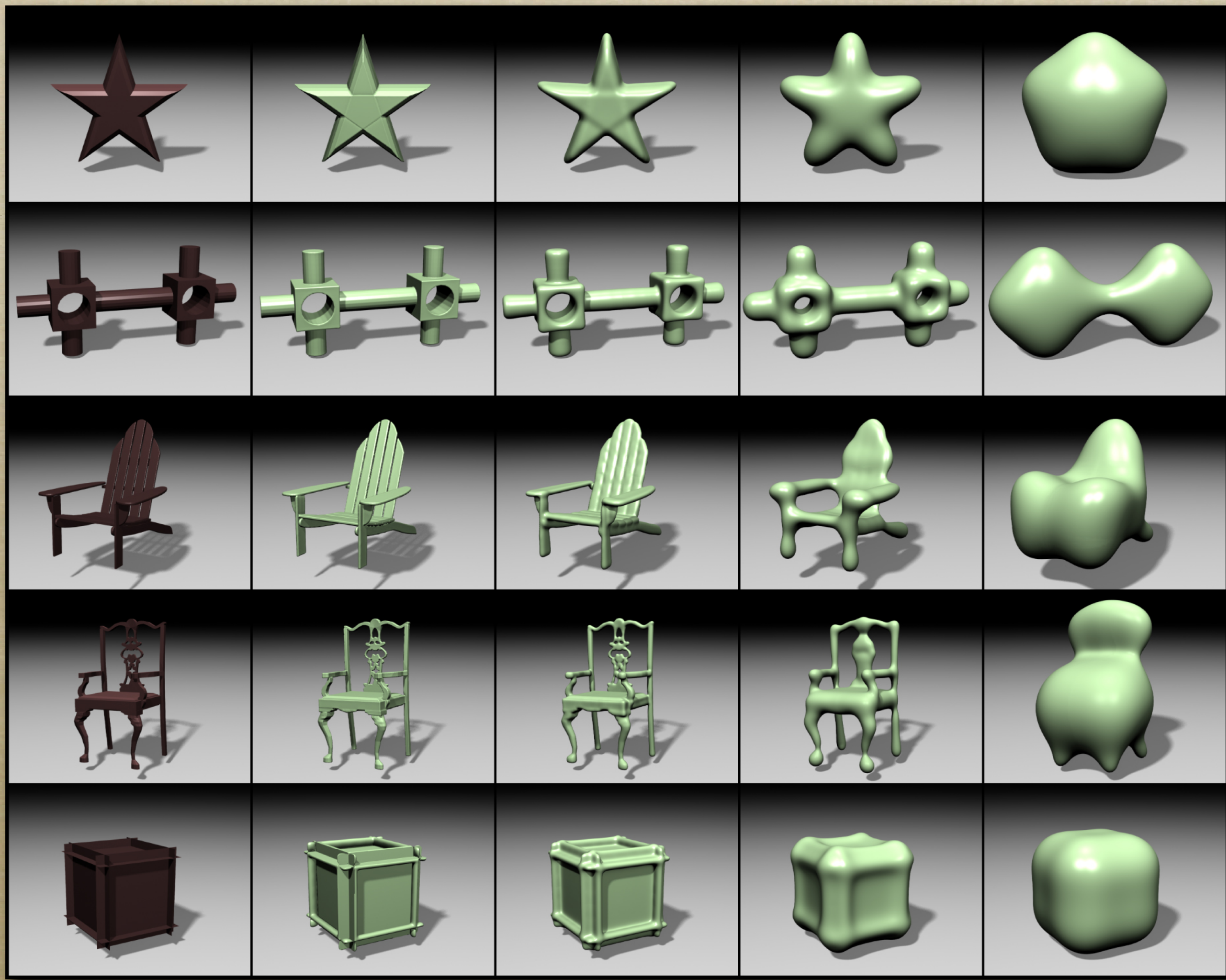


From Shen, *et al.*, SIGGRAPH, 2004.
(Actually based on polygon constraints... but same idea.)



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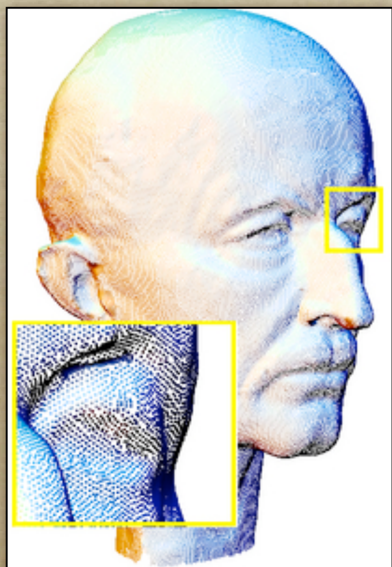
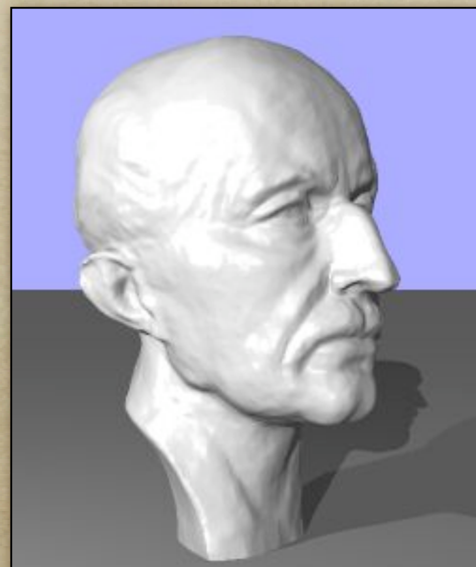
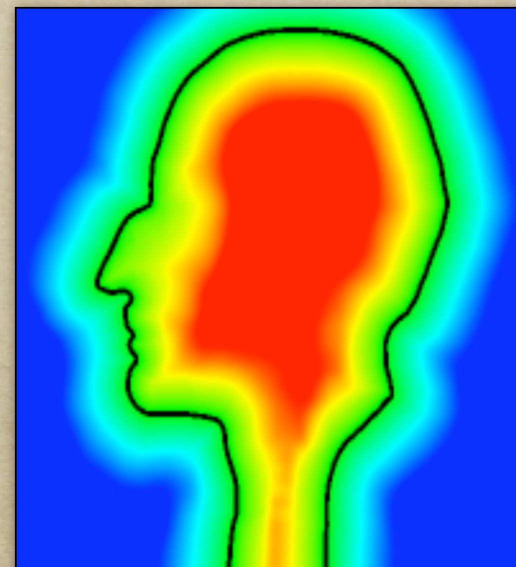
Partition of Unity Method







Ohtake, *et al.,* SIGGRAPH 2003

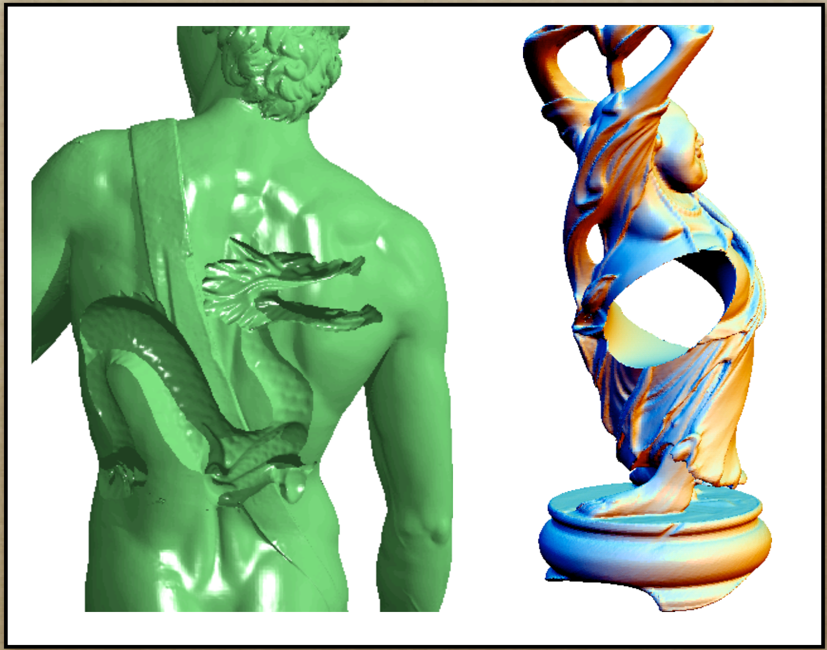
Partition of Unity is a special case of Moving Least-Squares

Partition of Unity Method

Ohtake, *et al.*, SIGGRAPH 2003

Editing Operations

- Implicit function can be
  - Combined w/ booleans
  - Warped
  - Offset
  - Composed
  - And more...



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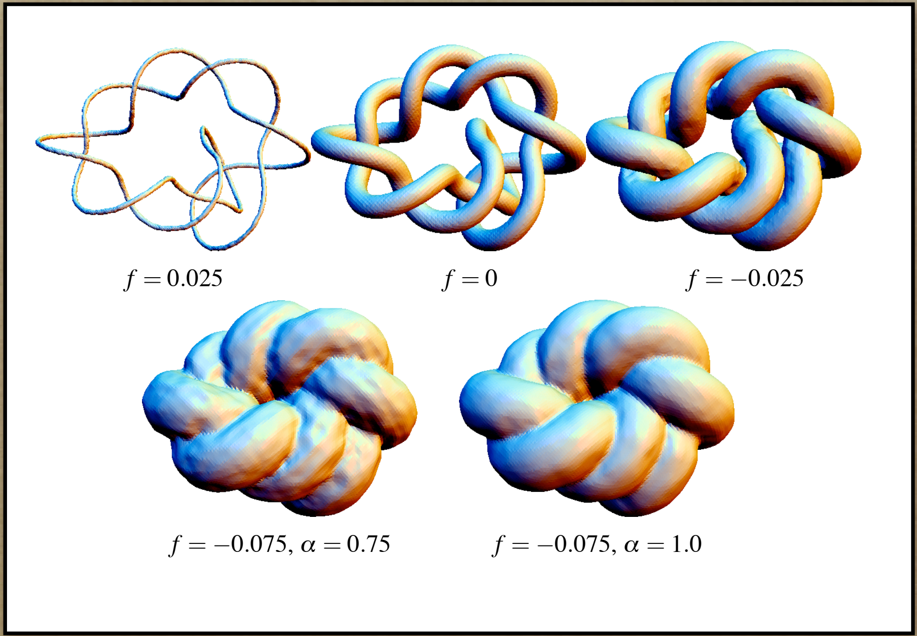

Ohtake, *et al.,* SIGGRAPH 2003

Editing Operations

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Ohtake, *et al.*, SIGGRAPH 2003

Smoothing

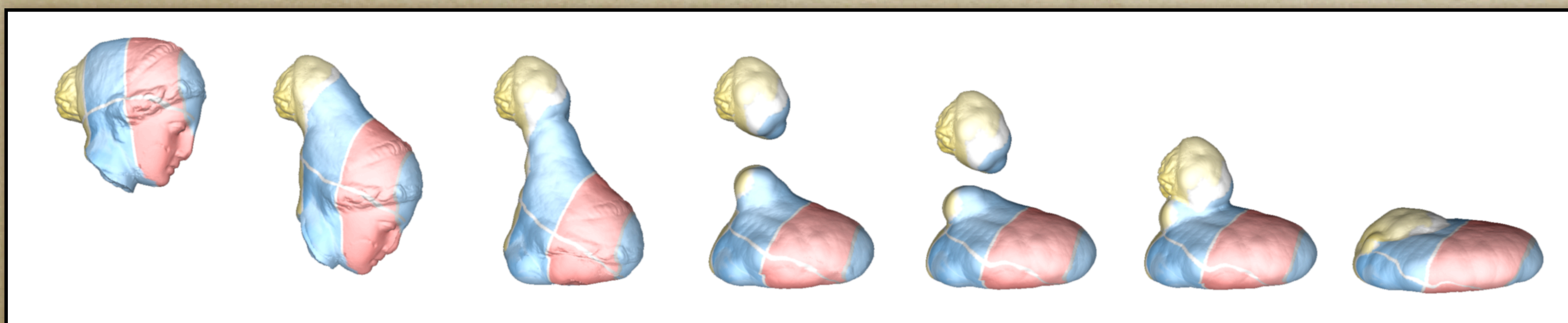
Simple Smoothing

Adjustment Smoothing

From Shen, *et al.*, SIGGRAPH, 2004.

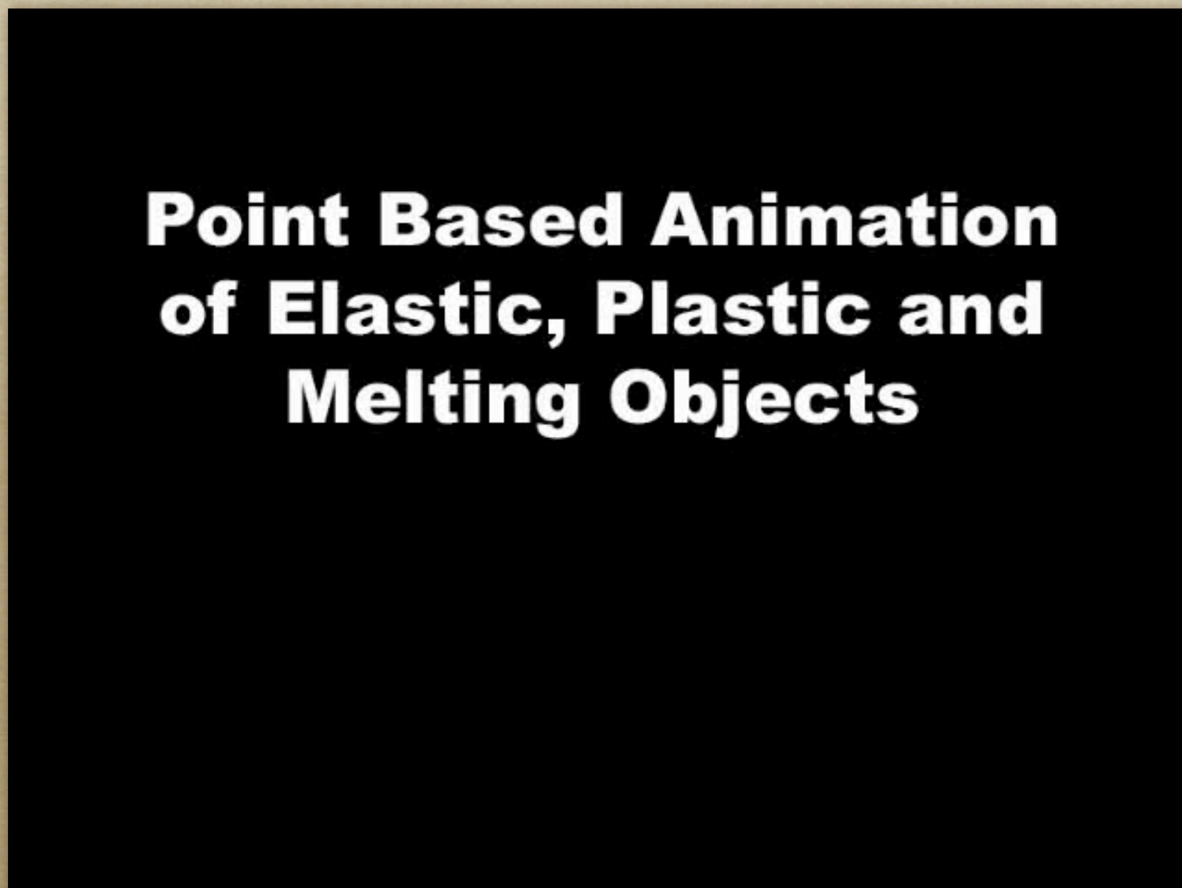
Point-Based Simulation

- MLS originated in mechanics literature
- Natural use in graphics for animation



From Mueller, *et al.*, SCA, 2004.

Point-Based Simulation



From Mueller, *et al.*, SCA, 2004.

Suggested Reading

- “QSplat: A Multiresolution Point Rendering System for Large Meshes” by Szymon Rusinkiewicz and Marc Levoy, SIGGRAPH 2000.
- “Multi-level Partition of Unity Implicits” by Yutaka Ohtake and colleagues, SIGGRAPH 2003.
- “Point Based Animation of Elastic, Plastic and Melting Objects” by Mueller and colleagues, SCA 2004.
- “Defining point-set surfaces” by Nina Amenta and Yong Joo Kil, SIGGRAPH 2004.
- "Interpolating and Approximating Implicit Surfaces from Polygon Soup" by Chen Shen, James O’Brien, and Jonathan Shewchuk, SIGGRAPH 2004.