Fluids

Full Resolution Simulation 120 \times 240 \times 120

Kim, Thuerey, James, and Gross, 2008
Fluids

Losasso, Talton, Kwatra, and Fedkiw, 2008
Fluids

Multiple Burst

Feldman, O'Brien, and Arikan, 2003
Two ways of representing flow
Two ways of representing flow

Particles
“Lagrangian”

Grid
“Eulerian”

J.-L. Lagrange
(dead now)

L. Euler
(also dead)
Smoothed particle hydrodynamics (SPH)

- Each particle has mass, position, velocity
- Particles represent samples of continuous underlying scalar/vector fields (density, velocity, etc.)

\[ \mathbf{v}(\mathbf{x}) = ? \]
SPH interpolation

- Evaluate the field anywhere by weighted averaging

\[ A(x) = \sum_i A_i \frac{m_i}{\rho_i} W(x - x_i) \]

Value at point \( x \)

Value at particle \( i \)

“Volume” of particle \( i \)

Smoothing kernel

Density:

\[ \rho_i := \rho(x_i) = \sum_j m_j W(x_i - x_j) \]
Particle-based fluids

• Each particle has a velocity

• Each time step:
  • Compute acceleration \( \mathbf{a} \) of each particle
  • Update velocities: \( \mathbf{v} = \mathbf{v} + \mathbf{a} \, dt \)
  • Update positions: \( \mathbf{x} = \mathbf{x} + \mathbf{v} \, dt \)
Gravity
Pressure

- Resists compression and volume change
- Force $\mathbf{f}^{\text{pressure}} = -\nabla p$
- In SPH, we’ll assume pressure proportional to density

$$p = k \left( \left( \frac{\rho}{\rho_0} \right)^7 - 1 \right)$$

Gas constant

Rest density
Pressure

Corner breaking dam with gas equation

130k particles, viscosity 0.1, pressure constant 500
Viscosity

• Resists relative motion within the fluid

\[ \mathbf{f}_{\text{viscosity}} = \mu \nabla^2 \mathbf{v} \]

Coefficient of viscosity
Surface tension

- Tries to minimize surface area
- Only relevant at small scales
- Hard to do correctly
Forces in a fluid

- Forces (per unit volume)

\[ f = \rho g - \nabla p + \mu \nabla^2 v \]

Gravity \hspace{1cm} Viscosity

Pressure

- How to evaluate \textit{gradients} of quantities?
Evaluating gradients with SPH

\[ A(x) = \sum_i A_i \frac{m_i}{\rho_i} W(x - x_i) \]

• So

\[ \nabla A(x) = \sum_i A_i \frac{m_i}{\rho_i} \nabla W(x - x_i) \]

• We just have to differentiate the kernel!

• Same thing works for higher derivatives (for viscosity).
Newton’s third law

• Forces between two particles should be equal & opposite

\[ \mathbf{f}_{i}^{\text{pressure}} = - \sum_{j} p_{j} \frac{m_{j}}{\rho_{j}} \nabla W(x_{i} - x_{j}) \]

\[ \mathbf{f}_{i}^{\text{viscosity}} = \mu \sum_{j} \mathbf{v}_{j} \frac{m_{j}}{\rho_{j}} \nabla^{2} W(x_{i} - x_{j}) \]
Newton’s third law

- Forces between two particles should be equal & opposite

\[ f_{i_{\text{pressure}}} = - \sum_j \left( \frac{p_i + p_j}{2} \right) \frac{m_j}{\rho_j} \nabla W(x_i - x_j) \]

\[ f_{i_{\text{viscosity}}} = \mu \sum_j (v_j - v_i) \frac{m_j}{\rho_j} \nabla^2 W(x_i - x_j) \]
Putting it all together

• For each particle:
  • Compute $\rho_i$ for each particle

• For each particle:
  • Evaluate net force $\mathbf{f}_i$
  • Compute acceleration $\mathbf{a}_i = \mathbf{f}_i / \rho_i$
  • Perform leapfrog integration
Particles

Particle view

Akinci, Ihmsen, Akinci, Solenthaler, and Teschner, 2010
Particles

Akinci, Ihmsen, Akinci, Solenthaler, and Teschner, 2010
Surface reconstruction

• Define a “color function” $c(x)$ that is 1 inside the fluid and 0 outside

  • e.g. do SPH interpolation as usual with $c_i = 1$ always

    $$c(x) = \sum_i c_i \frac{m_j}{\rho_j} W (x - x_j)$$

• Extract isosurface at $c = \frac{1}{2}$
Surface reconstruction

- Surface can look “lumpy” due to particle distribution
- Solution: use anisotropic kernels along surface

Isotropic kernels

Anisotropic kernels

Yu and Turk, 2010
Smoothed particle hydrodynamics!

Akinci, Ihmsen, Akinci, Solenthaler, and Teschner, 2010
References

• Müller, Charypar, and Gross, “Particle-Based Fluid Simulation for Interactive Applications”, 2003

• Becker and Teschner, “Weakly compressible SPH for free surface flows”, 2007

• Yu and Turk, “Reconstructing Surfaces of Particle-Based Fluids Using Anisotropic Kernels”, 2010